METHODS OF MEASURING ELECTRICAL RESISTANCE

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PREFACE.

THIS treatise contains a compilation of many methods of measuring electrical resistance, most of which are fully described. Some of the methods are new and are described here for the first time. Several are illustrated with records of sample measurements. While it is not claimed that the work is exhaustive, the author has selected for presentation all methods which in his judgment are useful, for commercial tests and measurements, for purposes of instruction in educational institutions and for application in technical and research laboratories. Rules for the estimation of errors are briefly considered in the first chapter. One chapter is devoted to methods of measuring temperature by means of resistance measuring apparatus, and in another chapter methods are considered for locating faults upon telephone and other land lines. While few descriptions of specific types of instruments are given, two chapters are devoted to a consideration of the broad principles which should apply when designing, selecting and using apparatus intended for the measurement of electrical resistance. An appendix contains data and information useful in connection with the subjects treated. Methods employed for the absolute determination of the ohm are not considered because few persons have occasion to make this determination.

In the examples recorded to illustrate specific methods, it may at times appear to some that the precision obtained is unsatisfactory. The measurements recorded, however, are real and not hypothetical cases, and they were made under such working conditions as ordinarily obtain. They are thought, therefore, to be more instructive than specially selected cases where the measurements have been made with unusual skill and care resulting in exceptionally high precision.

The author has felt justified in writing upon methods of measuring electrical resistance, because for over twenty years he has been engaged in electrical measurement, and for over seven years he was connected with The Leeds and Northrup Company

For $m = -1$

$$\frac{1}{1+a} = 1 - a; \quad \frac{1}{1-a} = 1 + a. \tag{4}$$

For $m = -2$

$$\frac{1}{(1+a)^2} = 1 - 2a; \quad \frac{1}{(1-a)^2} = 1 + 2a. \tag{5}$$

For $m = -\frac{1}{2}$

$$\frac{1}{\sqrt{1+a}} = 1 - \tfrac{1}{2}a; \quad \frac{1}{\sqrt{1-a}} = 1 + \tfrac{1}{2}a. \tag{6}$$

$$(1 \pm a)(1 \pm b)(1 \pm c) \ldots = 1 \pm a \pm b \pm c \ldots \tag{7}$$

$$\frac{(1 \pm a)(1 \pm c) \ldots}{(1 \pm b)(1 \pm d) \ldots} = 1 \pm a \pm c \ldots \mp b \mp d \ldots \tag{8}$$

For the geometrical mean of two quantities, which are very nearly alike, the arithmetical mean may be used. Thus

$$\sqrt{p_1 p_2} = \frac{p_1 + p_2}{2}. \tag{9}$$

If δ signifies a small angle measured in radians (1 radian = 57.2958 degrees) then,

$$\sin(x + \delta) = \sin x + \delta \cos x; \ \sin \delta = \delta, \tag{10}$$

$$\cos(x + \delta) = \cos x - \delta \sin x; \ \cos \delta = 1, \tag{11}$$

$$\tan(x + \delta) = \tan x + \frac{\delta}{\cos^2 x}; \quad \tan \delta = \delta. \tag{12}$$

Also, if a quantity a is very small compared with a quantity $x > 1$, then

$$\log_e(x + a) = \log_e x + \frac{a}{x}; \quad \log_e(1 + a) = a. \tag{13}$$

The true value of $(a + b)^n$ is given by the expansion

$$(a + b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{\lfloor 2} a^{n-2}b^2 + \cdots \tag{14}$$

The exact value of $(1 \pm a)^m$ when $a^2 < 1$ is given by the expansion

$$(1 \pm a)^m = 1 \pm ma + \frac{m(m-1)}{\lfloor 2} a^2 \pm \frac{m(m-1)(m-2)a^3}{\lfloor 3} + \cdots \tag{15}$$

Any quadratic may be put in the form $x^2 + px = q$. Its solution is then

$$x = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} + q}. \tag{16}$$

of Philadelphia, Pa. (but with whom he *now* has *no association*), which manufactures electrical resistance measuring apparatus. He hopes by recording the experience acquired he may benefit those who are interested in similar lines.

Doubtless this book is not free from errors and defects. Any reader noting such will confer a favor upon the author by pointing them out.

The author acknowledges his indebtedness to Mr. J. W. Wright of the Bell Telephone Company of Pennsylvania, for his careful reading of the chapter, "Elementary Principles of Fault Location," and for the valuable suggestions which he offered for its improvement. He adds his acknowledgment to The Leeds and Northrup Company for the loan of electrotypes. He further takes this opportunity to express gratitude to his wife, Margaret Stewart Northrup, for her encouragement to proceed with the work and for her unremitting assistance in the preparation of the manuscript.

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METHODS OF MEASURING ELECTRICAL RESISTANCE

CHAPTER I.

EXTENT, CHARACTER, AND PRECISION OF ELECTRICAL MEASUREMENT. THEORY OF ERRORS. OHMIC RESISTANCE.

100. **Electrical Measurement.** — The quantities to measure and the methods of making measurements are more numerous in electrical science than in any other, and when there is added the special tests required in connection with industrial practice, the extent of the subject is far too great for a full and adequate treatment in a single volume. The author has chosen, therefore, to present but one phase of the general subject — methods of measuring electrical resistance. We shall do well, however, to first consider briefly what is usually comprehended under the subject of electrical measurement, the nature of the problems involved and some of the fundamental principles which pertain to all kinds of electrical measurement. This will give a clearer understanding of the relation which methods of measuring resistance bear to electrical measurement in general.

In addition to several more or less fundamental quantities, industrial practice requires the determination or measurement of other quantities, such as: reactance; impedance; frequency of a periodically varying current or E.M.F.; phase differences; power factor; location of short circuits in coils; ratios of transformers; location of faults in linear conductors. Very many special determinations are also made, such as the calibration of instruments, the testing of conductors, conductor-insulation and all kinds of electric power machinery, etc. Special methods of measurement have been devised in many cases to meet these various requirements.

All the electrical quantities may be steady or constant in value, or they may vary in a determinate manner. In the latter case

the methods of measurement and the instruments employed are usually quite different from those in the former case. Electrical measurements are, therefore, ordinarily considered under direct-current measurements and alternating-current measurements. Currents and E.M.F.'s which vary rapidly in an entirely indeterminate manner are now studied and measured with the aid of the oscillograph.

The quantities which principally require consideration in ordinary industrial electrical measurements are the following:

Quantity	Symbol	Dimensions (El' mag. System)	Practical units
Quantity of electricity	Q	$L^{\frac{1}{2}}M^{\frac{1}{2}}$	Coulomb.
Potential difference	V	$L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-2}$	Volt.
Electromotive force	E	$L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-2}$	Volt.
Electric current	I	$L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$	Ampere.
Ohmic resistance	R	LT^{-1}	Ohm.
Resistivity	ρ	L^2T^{-1}	Ohm-centimeter.
Conductance	G	$L^{-1}T$	Mho.
Conductivity	σ	$L^{-2}T$	Mho per centimeter.
Capacity	C	$L^{-1}T^2$	Microfarad.
Inductance	L	L	Henry.
Specific inductive capacity	k	number	No name.
Strength of magnetic pole	m	$L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-1}$	No name.
Magnetic induction	B	$L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$	Gauss.
Magnetizing force	H	$L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$	Gilbert per Cm.
Magnetic permeability	μ	number	No name.
Electric energy	W	L^2MT^{-2}	Joule.
Electric power	P	L^2MT^{-3}	Watt.
Length	L	L	Centimeter.
Mass	M	M	Gram.
Time	T	T	Second.

101. Some General Principles. — Measurement always involves the process of finding, by direct or by indirect means, how many times some quantity, which we choose to call the unit, is contained in some other quantity of the same kind, the magnitude of which we wish to determine. In the actual process of measurement, the quantity, which is selected as the unit, must be something more than a mere abstraction or definition. It must be represented by a concrete thing. Thus the unit of length, in the metric system, is defined as the one ten-millionth of the earth's quadrant and is called the meter, but the real unit of length is an actual distance between two marks upon a par-

ticular bar of metal. The entire scientific world has agreed to call the distance between these two marks the real unit of length, which is called a meter. There are numberless, more or less accurate copies of this standard of length, and whenever an actual length measurement is made, the procedure consists essentially in finding how many times the length of one of these copies of the standard meter is contained in the length being measured. The concrete thing which embodies and represents the unit of definition is generally and properly called a standard. The standard may represent the unit exactly, as in the case of the meter, or it may be a known multiple or fraction of the unit represented. Thus the column of pure mercury of uniform cross-section which is 106.3 centimeters long and has a mass of 14.4521 grams, or a cross-section of 1 mm^2 is, when at a temperature of 0° C., an exact concrete realization of the unit of resistance, called the ohm. A standard cadmium cell, on the other hand, which bears a certified value of 1.0183 volts is a concrete realization which represents a known multiple of the unit of electromotive force.

It is desirable and convenient that a standard should represent the unit exactly or some simple even multiple of it, but the difficulties in the way of accomplishing this are often great, as in the case of the standard of E.M.F. just cited. For exact work it is customary to construct standards to equal the unit or its even multiple as nearly as possible, and furnish certificates giving the exact value in terms of the unit represented. Thus a standard one-microfarad condenser is seldom constructed to equal one microfarad closer than one-tenth or one-quarter of one per cent, tho condensers can be compared with one another much closer than this. The value stamped upon the standard is called the nominal value, and the certificate which goes with it states the amount, generally, or at least preferably, in percentage by which it is greater or less than its nominal value.

When a measurement is made, it is determined by some selected method of procedure how many times the magnitude of the standard is contained in the quantity measured. If this quantity is smaller than the standard, then, of course, it will be found to contain the standard a fractional number of times.

The measurement may be in error from two causes; either the standard may be larger or smaller than it is certified, in which

case the quantity being measured will be called smaller or larger than it really is, or the process of finding how many times the quantity measured contains the magnitude of the standard may be incorrectly applied. The responsibility of the first error rests with the standard and of the second error with the user of the standard.

In industrial measurements, the one engaged in making the measurements, that is, in finding how many times the magnitude of the standard he employs is contained in the quantity he is measuring, seldom undertakes a test of the precision of his standards. He relies upon others for this, and herein rests the vast importance and responsibility of a Standards' Bureau like the one we have in Washington. It can be shown that, in a final analysis, the accuracy of most of the measurements made in America rests upon the facilities, the intelligence, and conscientious care of this Bureau.

102. Comments on Accuracy and Method. — The precision with which any measurement is made may vary all the way from a rough estimation to the high refinement attained by a prolonged investigation where the total error may be reduced to a few parts in a million. This being so, the prominent question to be held before the mind, when starting a series of measurements, should be: Under all the circumstances of the case, what degree of precision is one justified in seeking? Assuming the accuracy of the standards the possible precision attainable is generally related very closely to the time spent upon the measurements, and the time which one is justified in using is governed by circumstances which should not be ignored. Suppose, for example, the object of a measurement is to determine the specific resistance of a sample of commercial graphite. With accurate standards and the expenditure of much time this might be determined to an accuracy, perhaps, of a twenty-fifth of one per cent. But there would be no justification in seeking such a high precision in this case because it would be without value in view of the variability of graphite. It is probable that different samples of graphite from the same supply would differ in resistivity by one per cent or more, and the same sample would certainly vary in resistivity from day to day by much more than a twenty-fifth of one per cent. On the other hand, if the object of a series of measurements is to determine the minute variations over a period of time which take

place in a manganin-resistance standard, the most painstaking care should be exercised to secure the necessary precision, and this care and the time spent would be entirely justified, if circumstances justified the research itself.

Again, great accuracy and painstaking care in finding how many times the magnitude of the standard available is contained in the quantity under measurement is not justified, if the uncertainty must remain great regarding the true value of the standard itself. Few lines of work require, as does electrical measurement, such discriminating judgment as to the relative importance of things. But no admonition or instruction can give this balanced judgment, so necessary to successful performance, as does practice. In this, as in most matters requiring skill, to measure well, one must practice measurement. When, by experience, a discriminating judgment has been acquired, it will be applied, not consciously, but quite instinctively.

In making electrical measurements much more mischief is likely to result from a careless commitment of gross errors than from failure to give attention to details and to deduce the most probable value from a set of observations. The gross errors may result from a misconception of the nature of the problem, from an entirely incorrect reading of the larger indications of an instrument, from mistakes made in ordinary arithmetic, or from an improper interpretation of units in calculating the results. The strained attention often required to read small decimals, frequently causes an entire loss of mental perspective as to the main features of the problem. Important points are overlooked while minutiæ are carefully observed. In no line of effort is a novice more apt "to strain at a gnat and swallow a camel," than when trying to make a refined electrical measurement.

It is generally wise, as a precautionary measure, after the apparatus is in adjustment and all the connections are completed, to make what might be termed a survey measurement, and then to deduce the results with a rough calculation, and consider well if these results look reasonable. If they do, and the outfit is seen to be in a proper, balanced, working condition, painstaking observations may then be undertaken and the data worked up. Nothing so assists in proving or disproving the reasonableness of the observations as plotting the data in a curve, and this, in almost every case, is to be recommended.

In nearly every extended series of observations there will be, in addition to accidental errors which are as likely to be plus as minus, certain systematic errors which escape observation and are not eliminated by taking the mean value of a number of readings. The novice, noting a fine agreement among his observations, is often deceived into supposing he has attained an accuracy far greater than the measurements really justify. The surest way to obtain enlightenment is to remeasure the same quantity by an entirely different method. The lack of agreement that is apt to result is often a disagreeable surprise and gives one, as nothing else can, a just estimate of how grudgingly Nature permits the real truth to be extracted. A close agreement in the results of measurements made by two or three entirely different methods gives, on the other hand, the highest assurance that a real precision has been attained.

When a quantity is to be measured with precision, it is generally wise to seek a method of measurement which will give the result as directly as possible and without the necessity of making a number of corrections. So-called "null" methods, in which the quantity being measured is balanced against some other quantity, are less rapid, as a rule, than deflection methods, in which the quantity is measured in terms of the deflection of some instrument, but ordinarily the former are much more accurate and there is less necessity of applying corrections. Null methods are generally to be preferred for all precision work. In the descriptions which follow, of the various methods of measuring electrical resistance, the relative advantages of the two methods will become obvious.

The question as to what sensibility the indicating or measuring instrument should have is an important one that must receive careful consideration. It may be said, however, that, in general, increased sensibility in the indicating instrument leads to a reduction in size and cost of all the rest of the equipment required. On the other hand, more care, time, and skill are required to work with sensitive instruments, and judgment must be constantly exercised to choose a sensibility best adapted to the particular problem in hand.

It should be borne in mind that the only electrical quantity which can be sold or exchanged for money is electrical energy, or electrical power expended for a given time. Electrical power

is composite, being the product of E.M.F. and the current which is in phase with the E.M.F. When the average power developed in a certain time is multiplied by the time, the total amount of electrical energy developed is obtained. Now energy has a real existence and a market value. Consequently, from the industrial standpoint, most electrical measurements have as their final object the precise determination of that which has money value, namely, electrical energy or watt-hours. While there are both scientific and practical considerations which make it necessary to measure separately such quantities as E.M.F., current, phase angles, resistances, etc., one should not lose sight of the industrial object to be attained, which is the proper and just charge for a quantity of electrical energy sold. Other considerations, of course, apply with those classes of measurement required in telephony and telegraphy, or those made solely for scientific investigation.

Finally, it is strongly recommended to all those engaged in electrical measurements that they give a careful study to the meanings and physical interpretations of the electrical and magnetic units in use. In this connection the writer would recommend the use and study of a little book called "Conversion Tables," by Dr. Carl Hering.

103. Elements of the Theory of Errors.* — There are, in general, two classes of errors; *systematic errors* and *accidental errors*. The former class often result from an incorrect value being assigned to the standards employed, from a faulty calibration of the scale of an instrument, or from the constant presence and influence of an unrecognized force, as, for example, the unrecognized presence of a thermoelectric force in the circuit of the indicating galvanometer. Systematic errors do not eliminate when the mean value of a series of observations is taken as the result.

The latter class generally result from inaccuracies in reading the instruments and from fluctuations above and below a mean value of some quantity which determines the readings of the instruments while the observations are being taken; for example, fluctuations in the E.M.F. employed when measuring resistances with deflection instruments. As accidental errors are as likely to be plus as minus, they tend to eliminate from the mean value as the number of readings is increased. But it may be added,

* Some of what follows under this heading is taken from "Instruments et Méthods de Mesures électriques Industrielles," par H. Armagnat.

that the theory of probability shows that the precision increases not directly with the number, but proportionally *with the square root* of the number of measurements.

The difference between the value found in a single measurement and the mean of the entire series of measurements is called the *apparent error*. If the sum of all the apparent errors, without regard to sign, be taken, and this sum be divided by the number of measurements, we obtain the *mean error*. The theory of probability shows that the mean error, obtained in this way, provided the errors are accidental and not systematic, is very closely the same as would be obtained if the mean error were found by taking the true value of the quantity instead of the mean value. Hence, in estimating the true value from a set of measurements, one can say that this true value is equal to the mean value found by the measurements plus or minus the mean error. For example, suppose one has made five measurements of a resistance, with the same apparatus and method, and all precautions have been taken to avoid systematic errors; the results would be expressed as follows:

No of Meas.	Ohms found	Apparent error
1	25.6	+0.04
2	25.3	−0.26
3	25.7	+0.14
4	25.5	−0.06
5	25.7	+0.14
	127.8 = sum	0.640 = sum
	25.56 = mean	0.128 = mean error

True value = 25.56 ± 0.128, which one would call
25.56 ± 0.13.

It is rare in electrical measurements that there is any need to apply the more exact methods for arriving at the most probably true result, which the theory of probabilities teaches us to apply, and the matter will not be further considered here.

The numerical difference between the result of a measurement and the true value of a quantity measured is called the *absolute error*. The ratio of the absolute error to the magnitude measured is called the *relative error*. One must take, however, in place of the true value, which is unknown, the mean value, for expressing

the relative error. Thus, in the example above, $\pm$ 0.128 is the *mean absolute error*, and $\pm \dfrac{0.128}{25.56} = \pm$ 0.00500 is the *mean relative error.*

If the relative error is multiplied by 100, it is then called the *per cent error.* Thus, in the above example, $\pm 0.00500 \times 100 = \pm 0.500$ of 1 per cent. Namely, the value obtained is equal to 25.56 ohms with a probable error of plus or minus one-half of one per cent.

The relative error or the per cent error is the error which is of interest, because the absolute error must always be considered in relation to the absolute value, if it is to have any physical meaning. Thus to measure 1000 ohms with a plus or minus error of 1 ohm is quite permissible but to measure 10 ohms with a plus or minus error of 1 ohm would be but rough work. In the former case the precision would be 0.1 of 1 per cent and in the latter case but 10 per cent.

In all that follows, unless specifically stated to the contrary, we shall, in speaking of errors, always refer to the relative error, or to the per cent error.

The result of a measurement is often given so as to be dependent upon several partial results, or separate measurements.

Let x represent the value sought, and y the phenomenon, as for example the deflection of a voltmeter, upon which the value depends, then x will be some function of y, or

$$x = F(y). \tag{1}$$

If an absolute error Δy is made in observing the phenomenon, the result will be in error some amount Δx, such that

$$x + \Delta x = F(y + \Delta y), \tag{2}$$

or

$$\Delta x = F(y + \Delta y) - F(y). \tag{3}$$

If the error made in y is small, we can treat the increment in y as a differential, in which case we can write

$$dx = F'(y)\,dy, \tag{4}$$

and the relative error will then be

$$\frac{dx}{x} = \frac{F'(y)\,dy}{F(y)}. \tag{5}$$

To illustrate Eq. (5); suppose it is required to determine, by measuring a current, the rise in the temperature T of a conductor, caused by the current I which it is made to carry. If we assume the

rise of temperature of the conductor to vary as the square of the current carried, we can write

$$T = KI^2, \text{ where } K \text{ is a constant.}$$

We then have

x is equivalent to T, y is equivalent to I,

$F(y)$ is equivalent to KI^2, dx is equivalent to dT,

and

$F'(y)\,dy$ is equivalent to $2\,KI\,dI$.

Hence, by Eq. (5),

$$\frac{dT}{T} = \frac{2\,KI\,dI}{KI^2} = \frac{2\,dI}{I}. \tag{6}$$

Eq. (6) shows that the relative error in the determination of the rise in temperature will be twice as great as the relative error made in measuring the current.

Eq. (5) may be extended to the case of several variables, and so permit us to estimate in advance the relative precision of the final result, when we know the magnitude of the errors that may be made in each of the separate elements measured and upon which the final result depends.

Let

$$x = F(u, v, w, \text{etc.}). \tag{7}$$

Differentiate this function in respect to each of the variables u, v, w, etc. Then we derive

$$\frac{dx}{x} = \frac{F'_u(u, v, w, \dots)\,du + F'_v(u, v, w, \dots)\,dv + F'_w(u, v, w)\,dw + \dots}{F(u, v, w, \dots)}. \tag{8}$$

Apply Eq. (8) to the case of a circuit in which we wish to measure the power consumed. If P is this power, R the resistance of the circuit, and I the current flowing, we have

$$P = RI^2. \tag{9}$$

If to determine P we have to measure R and I, we shall have

$$P = F(R, I);$$

hence, by Eq. (8),

$$\frac{dP}{P} = \frac{R\,2I\,dI + I^2\,dR}{RI^2} = \frac{2\,dI}{I} + \frac{dR}{R}. \tag{10}$$

Now, $\frac{dI}{I}$ is the relative error in measuring the current, which call E_I, and $\frac{dR}{R}$ is the relative error in measuring the resistance, which

call E_R. Then

$$\frac{dP}{P} = 2\,E_I + E_R. \tag{11}$$

Eq. (11) shows that, if $+\,0.33\frac{1}{3}$ is the per cent error made in measuring the current and $+\,0.33\frac{1}{3}$ is the per cent error in measuring the resistance, the per cent error in determining the power will be

$$100\,\frac{dP}{P} = 2 \times 0.33\tfrac{1}{3} + 0.33\tfrac{1}{3} = 1 \text{ per cent.}$$

If the per cent error in measuring the resistance happened to be $-\,0.33\frac{1}{3}$ then the per cent error in determining the power would be only $+\,0.33\frac{1}{3}$. But, as it is unknown whether the accidental errors are positive or negative, no greater precision can be assigned to the result than would be deduced upon the assumption that all the partial errors have a like sign.

Certain further conclusions can be drawn from Eqs. (5) and (8):

1st. The quantity Q_s being measured is *the sum* of two factors x, y, or

$$Q_s = x + y.$$

By Eq. (8) the relative error would be

$$\frac{dQ_s}{Q_s} = \frac{dx + dy}{x + y}. \tag{12}$$

It is to be noted here that the relative error in the result cannot exceed the relative error committed in either factor; for assume

$$y = 0, \quad \text{then} \quad \frac{dQ_s}{Q_s} = \frac{dx}{x}.$$

2d. The quantity Q_d is *the difference* of two factors x, y, or

$$Q_d = x - y.$$

In this case the relative error becomes

$$\frac{dQ_d}{Q_d} = \frac{dx - dy}{x - y}. \tag{13}$$

Here it is seen that the relative error is as much greater as the difference $(x - y)$ in the two quantities is smaller. This is why, in measuring a quantity which involves the difference of two factors, one will always obtain a result which is much less exact than is obtained in measuring each of the elements.

3d. The quantity Q_p is *the product* of two factors, or

$$Q_p = xy.$$

In this case the relative error will be

$$\frac{dQ_p}{Q_p} = \frac{dx}{x} + \frac{dy}{y}. \tag{14}$$

Thus the relative error in the result is equal to the sum of the relative errors committed in measuring each of the factors.

4th. The quantity Q_q is *the quotient* of two factors, or

$$Q_q = \frac{x}{y}.$$

In this case

$$\frac{dQ_q}{Q_q} = \frac{\frac{y\,dx}{y^2} - \frac{x\,dy}{y^2}}{\frac{x}{y}} = \frac{dx}{x} - \frac{dy}{y}. \tag{15}$$

Hence, in this case, the final relative error is equal to the sum of the relative errors made in each factor when these have opposite signs. But as it cannot be told what the signs will be, one must assume that the case may occur in which the signs are unlike.

5th. When Q'_p is *the power* m of a factor x, as $Q'_p = x^m$, the relative error will be

$$\frac{dQ'_p}{Q'_p} = \frac{mx^{m-1}\,dx}{x^m} = m\frac{dx}{x}. \tag{16}$$

Hence the relative error in the result will be m times as great as the relative error made in the factor.

6th. When Q_r is *the mth root* of the factor x, as

$$Q_r = x^{\frac{1}{m}},$$

the relative error will be

$$\frac{dQ_r}{Q_r} = \frac{\frac{1}{m}\left(x^{\frac{1}{m}-1}\right)dx}{x^{\frac{1}{m}}} = \frac{1}{m}\frac{dx}{x} \tag{17}$$

Hence, the relative error in the result will be $\frac{1}{m}$ of that made in the factor.

104. Application of the Theory of Errors. — Having in mind the above principles, we can reach a just estimate of the precision

which may be expected in the measurement of resistance by deflection methods.

In practice, the instruments most used are such as have a scale, like that of a Weston voltmeter — that is, a scale with a total of 150 divisions. While an attempt is often made to estimate the readings to one-tenth of a division, it is thought that one-fifth of a division is as close an estimation as can be relied upon. Assuming then that the quantity being measured is given directly by a full scale deflection, the per cent error can scarcely be less than $\frac{1}{150 \times 5} 100 = 0.13 +$ per cent, and this per cent error steadily increases as the deflection read becomes smaller. With a reading of ten divisions the probable error would be 2 per cent. An additional source of error will be introduced if the scale is not laid off so that the deflections indicated are proportional to the current passing thru the instrument. But in measuring resistances with Weston voltmeters and millivoltmeters this source of error can usually be disregarded. Not so, however, when the deflection instrument is a galvanometer with telescope and scale or lamp and scale. In such case the scale will probably have 250 divisions, each side of a central zero, and one might be led to expect higher precision than may be obtained with a pointer instrument of only 150 divisions. The scales of galvanometers, however, have divisions of uniform length, and as few galvanometers deflect exactly proportional to the current thru them, the deflections indicated are not apt to be accurately proportional. The advantage therefore of a longer scale may be offset by lack of proportionality in the deflections.

Further, in resistance measurements by deflection methods, the results in most cases are not given directly in terms of a single deflection but involve the *difference* of two deflections, and, as appears under case 2 (§ 103), the precision of the final result will be much less than the precision with which the individual deflections are read.

Again, the two or three readings which must be taken are not made simultaneously but *in succession*, and this procedure always involves the assumption that all conditions influencing the precision remain constant while the various readings are being taken. With a generator, subject to variations in speed, as the source of current, this assumption is hardly tenable.

The theoretical per cent error in any of the cases given in the following chapter for measuring resistances is easily deduced by making use of the principle expressed by Eq. (8) (§ 103). We proceed to apply the principle to the method given in par. 207, where two unknown resistances x_1 and x_2 in series are to be determined by three readings of a deflection instrument. Let one of the resistances be given by the expression

$$x_1 = \frac{R(D - d_1 - d_2)}{d_2}. \qquad (1)$$

If the resistance R is given and is not subject to variation, the three quantities which may vary and which are not independent are D, d_1 and d_2. If we apply to the above equation the operation expressed by Eq. (8) (§ 103), we obtain

$$\frac{dx_1}{x_1} = \frac{dD - dd_1}{D - d_1 - d_2} - \frac{dd_2(D - d_1)}{d_2(D - d_1 - d_2)}. \qquad (2)$$

Here dx_1, dD, dd_1, dd_2 are absolute errors.

These errors may assume either a positive or a negative sign, hence the relative error $\frac{dx_1}{x_1}$ will depend not only upon the magnitude of the errors dD, dd_1, dd_2 but also upon their sign. If the error dd_1 is equal to, and of the same sign as dD, the first term of Eq. (2) disappears.

It is essential, however, in computing the value of the relative error in the result, to assume that the errors made in the elements (namely, in this case, in D, d_1, d_2) have signs which are the least favorable to precision. Assume, then, that the error in d_1 is negative.

Now, in reading a voltmeter, it may be assumed that the conditions would be so arranged that the deflection D comes near the top of the scale, and it may be assumed that the readings can be made to $\frac{1}{p}$ of the largest deflection, or to $\frac{D}{p}$. Then the errors dD, dd_1, dd_2 may all be as large as $\frac{D}{p}$ and most unfavorable to precision as far as sign is concerned. With these assumptions we shall have

$$\frac{dx_1}{x_1} = \frac{\frac{D}{p} + \frac{D}{p}}{D - d_1 - d_2} + \frac{\frac{D}{p}(D - d_1)}{d_2(D - d_1 - d_2)} = \frac{D}{pd_2} \cdot \frac{2d_2 + D - d_1}{D - d_1 - d_2}. \qquad (3)$$

To illustrate Eq. (3) let us make use of the readings obtained in the measurement given in par. 207.

Assume that $\frac{D}{p} = \frac{100.8}{500}$, namely, that the readings can be made to $\frac{1}{500}$ of the greatest reading. Take the values $D = 100.8$, $d_1 = 72.2$, $d_2 = 14.4$. Then we have, by Eq. (3), as the maximum per cent error,

$$100\frac{dx_1}{x_1} = \pm \frac{100.8}{500 \times 14.4} \times \frac{2 \times 14.4 + 100.8 - 72.2}{100.8 - 72.2 - 14.4} \times 100$$
$$= \pm 5.6 \text{ per cent.}$$

The per cent error, actually made, was only $+0.88$ per cent. This may be due in part to a more accurate reading of the instrument than was assumed, and also to the fact that the errors cancelled each other; if the latter only were the case we should have

$$100\frac{dx_1}{x_1} = \pm \frac{D}{pd_2}\frac{D - d_1}{D - d_1 - d_2}100 = \pm 2.8 \text{ per cent.}$$

This last result shows that in the particular case of this measurement the readings must have been made within about 0.05 of a scale-division.

Returning to Eq. (1) (§ 104) we see from par. 207 that, if one of the resistances x_2 is infinity, the reading d_1 would be zero and we have

$$x_1 = \frac{D - d_2}{d_2}R.$$

This is similar to the case given in Eq. (8), par. 201.

If now in Eq. (2) we put $d_1 = 0$, we shall also have $dd_1 = 0$, whence,

$$\frac{dx_1}{x_1} = \frac{dD}{D - d_2} - \frac{dd_2D}{d_2\,(D - d_2)} = \frac{dDd_2 - dd_2D}{d_2\,(D - d_2)}.$$

If, as before, $dD = dd_2 = \pm \frac{D}{p}$, we have, by taking dd_2 with a minus sign and dD with a plus sign,

$$100\frac{dx_1}{x_1} = \frac{D}{pd_2}\frac{D + d_2}{D - d_2}100. \tag{4}$$

Eq. (4) expresses the theoretical maximum per cent error which would be obtained in the method described in par. 201 if the readings can be made to $\frac{1}{p}$ of the larger deflection. If Eq. (4) be

applied to the measurement of 100 ohms, as given in the table in par. 205, and it be assumed that $\frac{D}{p} = \frac{100}{1000}$, we obtain as the maximum per cent error 1.4 per cent, while the actual error was 0.76 per cent.

Enough has now been given to show the low accuracy that may be expected from deflection methods of measuring resistances. These methods, nevertheless, are of value in many situations.

If it be remembered that all copper and aluminum conductors, such as magnet coils in arc lamps, etc., change in resistance about 4 per cent for every change of 10° C. in temperature, it will be recognized that the rough methods of measuring resistances with deflection instruments are often quite as precise as the conditions demand, and because of their simplicity often more satisfactory than methods which are more refined.

The apparatus required is, furthermore, very generally available; the measurements may be made quickly and in places where more delicate apparatus could not well be used, and altho the precision attainable is low, it is sufficient to meet such requirements as arise in connection with the measurement of insulation resistances and the resistances of copper conductors, such as dynamo-field coils.

105. Comments on Ohmic Resistance. — An ohmic resistance, considered as a quantity to be measured, may be viewed in two ways. If we call E the drop of potential between two points in an electric circuit, and I the current flowing, we may write either

$$E = RI \tag{1}$$

or

$$R_1 = \frac{E}{I}. \tag{2}$$

In the first relation the drop of potential is expressed as proportional to the current where R is the constant of proportionality. From this viewpoint, R is considered a constant coefficient which pertains to a particular circuit and which, when multiplied into the current, will give the potential drop. The relation does not assume that R is in any sense a function of I. If a resistance, so viewed, is to have an exact meaning, it must possess, under conditions which are simple to state, a definite value. In the second relation, R_1 is viewed as a numerical quantity which merely

expresses the ratio of the actual potential-drop between any two points of an electric circuit to the actual current flowing in that portion of the circuit. Thus considered, no assumptions whatever are made respecting the constancy or nature of the quantity R_1. When this quantity is obtained by a measurement of the ratio $\frac{E}{I}$, it is customary to call it an ohmic resistance. Having been thus obtained it is often treated as if it might be used as in Eq. (1), and is often incorrectly considered a constant, which it is not unless specifications of the conditions under which it may be so considered are specifically stated.

As a matter of fact, in many cases in which resistances are measured, when the current is made appreciable the conductor heats, and the ratio $\frac{E}{I}$ changes. It is a change which is independent of external temperatures, which are always assumed to be specified when a resistance is designated or measured. This change of resistance with the current used in making the measurement is strikingly illustrated in measuring the resistance of carbon lamp-filaments and tungsten lamp-filaments. In some measurements made by the author the ratio $\frac{E}{I}$ for a carbon lamp-filament when cold (that is, with a current too small to heat it appreciably) was $\frac{20.6}{0.162} = 127.1$, and for the same filament, when hot, was $\frac{53.5}{0.505} = 105.9$. A tungsten lamp-filament in series with the carbon lamp-filament gave, cold,

$$\frac{10.9}{0.162} = 67.28,$$

and hot,

$$\frac{72.0}{0.505} = 142.5.$$

In the above cases the ratio $\frac{E}{I}$, or what would be termed the resistance, is very dependent upon the current used in making the measurements; in the one case diminishing and in the other case increasing with increase of current. If the measuring current were continually diminished, the ratio $\frac{E}{I}$ would approach a value, both in the case of carbon and tungsten filaments, which would be

constant for any particular external temperature. This constant value may be called the true ohmic resistance of the conductor at the external temperature at which the measurement is made. As the value of every resistance varies to some extent, and often very greatly, both with the external temperature and the measuring current used to determine it, it is seen that a resistance is only completely specified when both of these conditions are fully specified.

All conductors, used for conveying current, vary more or less as above and there is a very large class of resistance measurements where the quantity measured, and called a constant resistance, cannot be considered a constant quantity even approximately unless accompanied by a precise statement of the conditions under which the measurement is made. Most of the methods usually described for the measurement of resistance seem to assume this quantity as practically constant when only the temperature of its surroundings is maintained constant, and the custom is to treat the quantity from the viewpoint of Eq. (1) above. In many cases this procedure is justified, for it is true that by using special alloys and small measuring currents many kinds of resistance-determinations are made where, to a first approximation at least, the resistance is a constant quantity which may be multiplied into a current of any reasonable value and so give the potential-drop at its terminals.

Broadly speaking, however, metallic conductors should be divided into two classes — conductors intended to serve as definite resistance (as those used in resistance-standards, resistance-measuring instruments, rheostats, and wherever the flow of current is to be restrained) and conductors intended to convey electric power with as small a loss of power as possible. The former class of conductors is given intentionally a high resistivity and consists of alloys which change little in resistivity with the temperature-rise produced by the current or with temperature-changes in the surrounding medium. The latter class of conductors consists chiefly of the pure metals, copper and aluminum, and is given as low a resistivity as possible. In these conductors a large temperature coefficient cannot be avoided and they change greatly, about 0.4 of 1 per cent per degree C., with change of temperature from any cause. In measuring conductors of the first class, we may conveniently consider resistance as a constant property of a

circuit at a given external temperature which, when once determined, will give the potential-drop if multiplied by the current, it being merely assumed that the current is kept within reasonable limits. In measuring conductors of the second class, it is more convenient to regard resistance as a quantity which merely expresses the ratio $\frac{E}{I}\cdot$ If the resistance of such a conductor is determined in terms of the resistance of the conductor of the first class, the value obtained will be indefinite unless very complete statements are furnished respecting the external temperature and the current used in making the measurement. The difficulty of doing this has made it customary to determine, not the resistance of such conductors, but their conductivity in terms of the conductivity of a standard conductor of the same kind. By the methods employed in conductivity-measurements, both the temperature of the surroundings and the heating effect of the measuring current are caused to act alike upon the standard and the sample, and hence do not require special consideration or specification.

The standard methods employed for measuring resistance and conductivity differ considerably, and whether it is better to make a resistivity or a conductivity determination will usually depend upon the class of conductors under consideration. We shall consider first those methods of measuring resistance where, to a first approximation, resistance may be treated as a quantity which is constant under ordinary conditions, leaving to a later section a consideration of the methods employed in the measurement of conductors of the second class.

Resistances may be subdivided conveniently into medium, high, and low resistances, as the same methods of measurement are not equally well adapted to all three.

The electrical instrument which is most universally available is a voltmeter. These instruments are supplied with a fixed internal resistance the value of which is usually stated upon the instrument, or, if not stated, may be easily determined. As many kinds of resistance measurements may be made with this instrument alone, those methods which are useful will now be considered.

CHAPTER II.

RESISTANCE MEASURED WITH DEFLECTION INSTRUMENTS; VOLTMETER AND AMMETER METHODS.

200. Assumptions. — The methods generally involve the assumptions: (a) that the scale divisions of the deflection instrument are so laid off that the readings of the instrument are proportional to the current passing thru it, and (b) that the E.M.F., or the source of current, remains constant while taking successive readings, and (c) that the internal resistance of the source of current is negligibly small compared with the other resistances in the circuit.

The instrument generally used is a voltmeter, but it may be a millivoltmeter with a known resistance connected in series with it, or a galvanometer which has a proportional scale and a known resistance R.

201. Voltmeter Method. Circuit Includes a Known Resistance. Method I. — To meet the general case we shall suppose

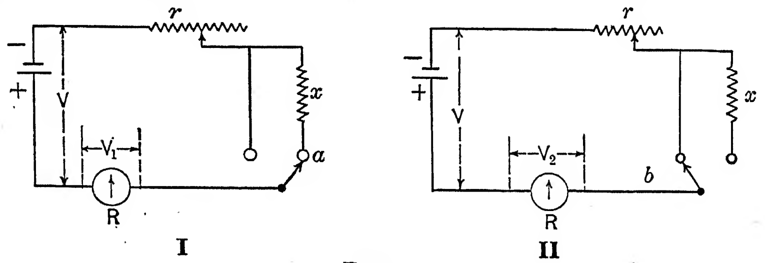

FIG. 201.

that there is a resistance r in the circuit as indicated in Fig. 201. With the connections as shown in I the current thru the deflection instrument will be

$$C = \frac{V}{R + r + x}, \tag{1}$$

where x is the resistance to be measured,

R the resistance of the instrument,

and V the E.M.F. of the source.

Also,
$$C = \frac{V_1}{R}, \tag{2}$$
where V_1 is the E.M.F. at the terminals of the instrument. From Eqs. (1) and (2)
$$V = \frac{V_1R + V_1r + V_1x}{R}. \tag{3}$$

With the connections as shown in II the current thru the deflection instrument will be
$$C' = \frac{V}{r + R} = \frac{V_2}{R}, \tag{4}$$
where V_2 is the E.M.F. at the terminals of the instrument, whence,
$$V = \frac{V_2r + V_2R}{R}. \tag{5}$$
From Eqs. (3) and (5) we derive,
$$x = \frac{(V_2 - V_1)(R + r)}{V_1}. \tag{6}$$

Since it is assumed that the deflections of the instrument used are proportional to the E.M.F.'s at its terminals, we can write $V_1 = k\,d_1$, $V_2 = k\,d_2$, where k is a constant and d_1 and d_2 are the deflections corresponding to the voltages V_1 and V_2. Hence Eq. (6) may be written
$$x = \frac{(d_2 - d_1)(R + r)}{d_1}. \tag{7}$$

Equation (7) shows first, that, if the law of the scale of the instrument is one of proportionality, it is not necessary that the deflections should indicate volts, millivolts or any particular quantity. It follows that for measuring resistances in this way one may use a voltmeter, a millivoltmeter with some extra known resistance in series with it, or a galvanometer. If a 150-volt voltmeter be used, then the current may be obtained from a 110-volt D.C. circuit, but if a millivoltmeter or a galvanometer be used, one or more cells of storage battery will suffice. In this method, as generally applied, the resistance r is zero, in which case
$$x = \frac{d_2 - d_1}{d_1}R. \tag{8}$$

The following actual sets of readings, taken by careful but untrained observers, will serve to illustrate the method above, as

expressed in Eq. (7), and to indicate the precision which may be expected under the arranged conditions. The deflection instrument was a Weston millivoltmeter, temporarily supplied with a series resistance obtained from a resistance box, such that $R = 102.3$ ohms (Fig. 201). The source of current was one cell of storage battery.

The resistances measured were accurate coils in a resistance box. r was taken 100 ohms.

The table below gives the results obtained.

d_1	d_2	x, given	x, observed	Per cent error
50.1	50.2	1	0.4	−60.0
47.9	50.2	10	9.7	−30.0
45.5	50.2	21	20.8	− 0.95
42.1	50.2	39	38.9	− 0.25
35.7	50.2	82	82.2	+ 0.25
33.5	50.2	101	100.8	− 0.19
28.3	50.2	157	156.6	− 0.25
19.5	50.2	321	318.5	− 0.77
12.1	50.2	638	636.7	− 0.17
6.6	50.2	1,283	1336.0	+ 4.1
3.6	50.2	2,559	2731.0	+ 6.7
1.9	50.2	5,117	5143.0	+ 0.55
1.0	50.2	10,241	9953.0	− 2.8

It should be noted that the error increases as the difference, $d_2 - d_1$, becomes smaller, and that a precision of better than one per cent may hardly be expected in the range 20 ohms to 5000 ohms. It may happen, however, that the only E.M.F. available is such as to deflect the instrument off its scale when connected directly to it and that it is necessary to include a resistance, as r or x, Fig. 201, in the circuit at all times. If this resistance r, as well as the resistance x is unknown, the values of r and x may both be obtained.

The following method may be easily applied, tho the formula to express the result is rather lengthy.

202. Voltmeter Method. Circuit Includes an Unknown Resistance. — Referring to Fig. 201, interchange the resistances r and x, and repeat the measurements as made in case (1). The deflection obtained when the circuits are arranged as in I will be the same as before, hence d_1 remains d_1, while the deflection obtained when the circuits are arranged as in II, but with r and x

interchanged, will be d_2'; then, we have, in the same manner that Eq. (7) (§ 201) was obtained,

$$r = \frac{(d_2' - d_1)(R + x)}{d_1}. \quad (1)$$

The resistance r may now be eliminated from Eq. (7) (§ 201) and Eq. (1) (§ 202), and we obtain

$$x = \frac{Rd_2'(d_2 - d_1)}{d_2 d_1 - d_2'(d_2 - d_1)}, \quad (2)$$

or, eliminating x,

$$r = \frac{Rd_2(d_2' - d_1)}{d_1 d_2' - d_2(d_2' - d_1)}. \quad (3)$$

203. Voltmeter Method. Circuit Includes a Known Resistance. Method II. — The usual method of making this measurement is to connect the known resistance r in series with the unknown resistance x and read the deflections obtained when first the unknown and second the known resistance are shunted with the deflection instrument. Make the connections as indicated in I and II, Fig. 203.

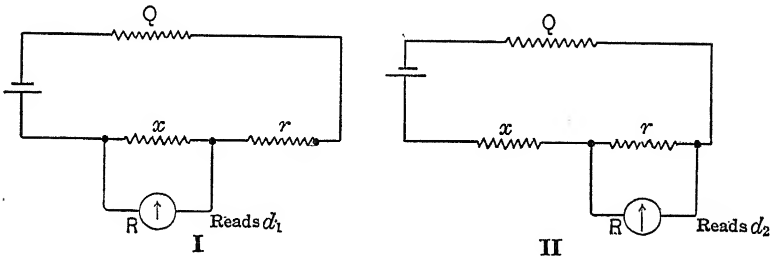

FIG. 203.

If the assumption is made that the resistance R of the deflection instrument is 500 or more times that of either the resistance r or the resistance x, then we have simply

$$\frac{x}{r} = \frac{d_1}{d_2}, \quad \text{or} \quad x = \frac{d_1}{d_2} r \quad (1)$$

where d_1 is the deflection with the connection I and d_2 the deflection with the connection II.

If one wishes, however, to measure the resistance of an incandescent lamp in this way, using a voltmeter, the above assumption would not be permissible for precise work, as the voltmeter would shunt from the resistance, at the terminals of which it is

connected, a portion of the current which may not be neglected. If the resistance R of the deflection instrument is known, and if the assumption be made that the main current is kept unchanged (as by means of a large resistance Q) when the voltmeter is connected, as in I and in II (Fig. 203), the value of x may be precisely expressed by the relation

$$x = \frac{d_1 r R}{d_2 (r + R) - d_1 r}. \tag{2}$$

Eq. (2) is easily deduced. If we put this expression in the form

$$x = \frac{d_1 r}{\frac{r (d_2 - d_1)}{R} + d_2},$$

it is seen that, when R is very great as compared with r, the expression takes the same form as Eq. (1). If r nearly equals x, the deflection d_1 will be nearly the same as the deflection d_2 and Eq. (1) again becomes applicable.

The above method, as expressed in Eq. 2, was applied to the measurement of the resistance of a 60-volt tungsten lamp operated first on 13 and then on 40 volts. A Weston voltmeter reading to 150 volts was used. The resistance R of the voltmeter was 15,660 ohms. The resistance r in series with the lamp was 260 ohms. The results obtained are exhibited in the table below:

d_1	d_2	x	Remarks
13.1 33.9	47.3 83.3	71.2 104.8	The resistance of the tungsten lamp increases 1.61 ohms per volt in range 13 to 40 volts.

It should be noted that, in order to apply this method of measuring the resistance of a lamp while burning, a known resistance r must be in series with the lamp, which will carry the same current as the lamp. It is also necessary to have for the measurement a voltage higher than the voltage which is to be applied at the lamp-terminals when the lamp is in service. Compare this measurement with another measurement made upon the same lamp by the method, par. 209 (Fig. 209).

204. Comparing Potential Drops with a Deflection Instrument; Special Case. — There is a special case in which, when two resistances are joined in series, the ratio of the two deflections

obtained (when the terminals of the instrument are first connected to the terminals of the one resistance and then to the terminals of the other resistance) gives the ratio of the resistances, whatever be the resistances themselves or the resistance of the deflection instrument.

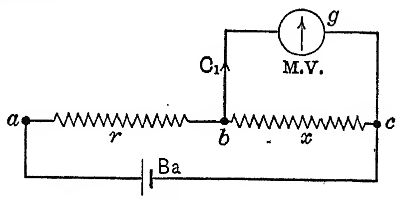

FIG. 204.

The necessary conditions which must be assumed to hold, to make the above statement rigidly true are: that (Fig. 204) the resistance of the battery Ba together with the leads from the battery to the points a and c is zero, and that the instrument M.V. may be joined to a point b which is a potential terminal common to both r and x. Both of these conditions can be often filled in practice very approximately.

Calling d_1 the deflection of the deflection instrument M.V. when joined to b and c, and d_2 the deflection when joined to a and b, and g the resistance of the deflection instrument, we have the following proof of the above statement. When the deflection instrument is joined to b and c, the current C_1 thru the instrument is

$$C_1 = \frac{V}{r + \dfrac{gx}{g+x}} \frac{x}{g+x}, \quad \text{or} \quad C_1 = \frac{Vx}{g\,(r+x) + rx}. \tag{1}$$

Similarly, when the instrument is joined to the terminals a and b, the current C_2 thru the instrument is

$$C_2 = \frac{Vr}{g\,(r+x) + rx}. \tag{2}$$

As the denominators of Eqs. (1) and (2) are the same, we have

$$\frac{C_1}{C_2} = \frac{x}{r},$$

and, if the instrument deflections are proportional to the current thru it,

$$\frac{C_1}{C_2} = \frac{d_1}{d_2},$$

hence,

$$\frac{d_1}{d_2} = \frac{x}{r}. \tag{3}$$

If there is resistance in the battery or the battery leads, relation (3) is not *rigidly* true, nor is it rigidly true if r and x are each provided with potential terminals so that the point b cannot be made common to both resistances. In cases, however, where the two resistances are considerably lower than the resistance of the deflection instrument the error resulting by applying Eq. (3) is too small to deserve consideration. Relation (3) would also be rigidly true under all circumstances in the single case when r and x are equal.

205. Voltmeter Method Using a Shunt. — It is sometimes desirable to shunt the deflection instrument with a known resistance S. In this case the unknown resistance may be determined in a manner similar to that given under method I, Fig. 201.

Thus, referring to Fig. 205, let $P = \dfrac{RS}{R+S}$ be the resistance of

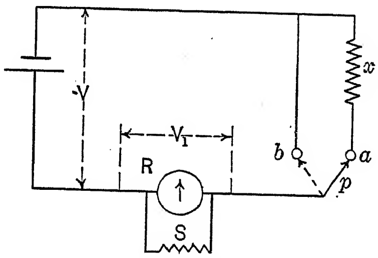

FIG. 205.

the instrument when shunted. With p joined to a, the current in the line will be

$$C = \frac{V}{P+x} = \frac{V_1}{P}, \tag{1}$$

where V is the potential of the source and V_1 the potential at the terminals of the instrument.

If the deflections of the instrument unshunted are proportional to the E.M.F. at its terminals, they will also be proportional when the instrument is shunted. Therefore, we can write $V = kD$, and $V_1 = kd$. Hence,

$$\frac{kD}{P+x} = \frac{kd}{P}, \quad \text{or} \quad x = \frac{P(D-d)}{d},$$

or

$$x = \frac{RS}{R+S}\,\frac{D-d}{d}. \tag{2}$$

The deflection D will be obtained by joining p to b.

The method as expressed in Eq. (2) is illustrated by the following measurements:

A millivoltmeter was used with resistance in series with it so that $R = 100.3$ ohms. The resistance R was shunted with a resistance $S = 10.0$ ohms, so that

$$P = \frac{10 \times 100.3}{10 + 100.3} = 9.093 \text{ ohms.}$$

One cell of storage battery was used for the source of current and the resistances measured were coils in a resistance box. The connections were made as in Fig. 205, and the results obtained are those given in the table below, calculated by the formula

$$x = 9.093\,\frac{D-d}{d}.$$

D	d	x true value	x found	Per cent error
100.1	90.3	1	0.987	−1.3
100.1	64.9	5	4.93	−1.4
100.1	31.6	20	19.71	−1.5
100.1	15.6	50	49.25	−1.5
100.1	8.4	100	99.24	−0.76
100.1	0.9	1000	1003.00	+0.3

206. Deflection Method. Resistance Measured by Substitution. — This is, perhaps, the simplest method employed for measuring a resistance, and is useful in certain cases, especially where insulation measurements are to be made.

For the sake of generality, suppose the resistance R of the deflection instrument is shunted with a resistance S.

Let $P = \dfrac{RS}{R+S}$ be the shunted value of the resistance of the instrument.

Let r be the resistance of the source of current and Q the known resistance. Then (Fig. 206a) if V is the E.M.F. of the source and C the current flowing, we have, with p joined to a,

$$C = \frac{V}{r + x + P} = k\,D \tag{1}$$

where k is a constant and D is the deflection obtained.

With p joined to b the current C' flowing will be

$$C' = \frac{V}{r + Q + P} = k\,d, \tag{2}$$

where d is the deflection obtained.

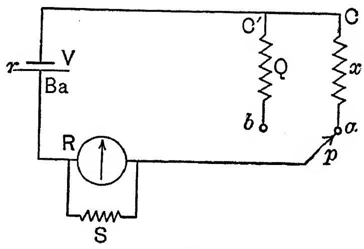

Fig. 206a.

From Eqs. (1) and (2)

$$x = \frac{d}{D}(r + Q + P) - (r + P). \tag{3}$$

The value of x cannot be obtained from Eq. (3) unless the resistance r of the source of current is either known or is negligible. This method is scarcely ever employed except in those cases where r may be neglected. With this assumed,

$$x = \frac{d}{D}(Q + P) - P. \tag{4}$$

In the practical application of this method to the measurement of high resistances the shunted resistance P of the deflection instrument may also be neglected, in which case

$$x = \frac{d}{D}Q. \tag{5}$$

The application of this method, and its modifications, to the measurement of the insulation resistance of cables is better reserved for a later chapter.

If the resistance Q is a rheostat which can be varied in small steps, then Eq. (4) may be greatly simplified and the measurement made with greater accuracy, if we so adjust Q that the deflection d is one half the deflection D. In this case

$$x = \frac{Q - P}{2}. \tag{6}$$

Or, if P is very small as compared with x or Q, we have

$$x = \frac{Q}{2}, \tag{7}$$

which, indeed, is obvious.

Again, if Q is varied until the deflection with p joined to Q is the same as the deflection with p joined to a, then $x = Q$. In making the measurement in this manner we substitute for the unknown quantity another quantity which is known and can be varied until the indicating instrument gives exactly the same indication whether the known or the unknown quantity is in circuit.

In this application of the substitution method no assumptions as to the proportionality, etc., of the indicating instrument are needed. Only two requirements need to be met for obtaining very high precision; first, that the indicating instrument shall be sensitive to small changes in the known quantity substituted, and second, that this quantity may be varied by very small steps.

The following method of measuring the resistance of an electrolyte is a good example of "the substitution method." This method was described by Prof. C. F. Burgess in Vol. XI, 1907, page 225, *Trans. of the Electrochem. Soc.* The method of Professor Burgess as given here has been somewhat modified in accordance with a suggestion made by Dr. Carl Hering.

In Fig. 206b is shown a glass tube, of known cross-section and with a graduated scale, immersed in an electrolyte, the specific resistance of which is to be measured. At the bottom of the glass tube is a fixed electrode, and the other electrode is fastened to a rod by which it may be set at various points, as p_1 and p_2, in the glass tube. V is a millivoltmeter and R is a resistance box adjustable in small steps. The millivoltmeter, resistance box, and the electrolyte in the glass tube are placed in circuit with a constant-potential storage battery Ba. R is then adjusted, with the electrode S set at p_1 until the millivoltmeter reads near the top of its scale. The electrode S is then moved up to p_2 and resistance is plugged out of the box, until the millivoltmeter reads the same as before. Then, without regard to the polarization of the electrodes, the resistance R_t plugged out of the box is equal to the resistance of the electrolyte between the points p_1 and p_2.

If s is the cross-section of the tube, and l is the length p_1 to p_2 and R_t is the resistance plugged out, the specific resistance of the

electrolyte, at the temperature t at which the measurement is made, is

$$\rho_t = \frac{sR_t}{l}. \tag{8}$$

Compare this method with the method of Kohlrausch given in par. 1120.

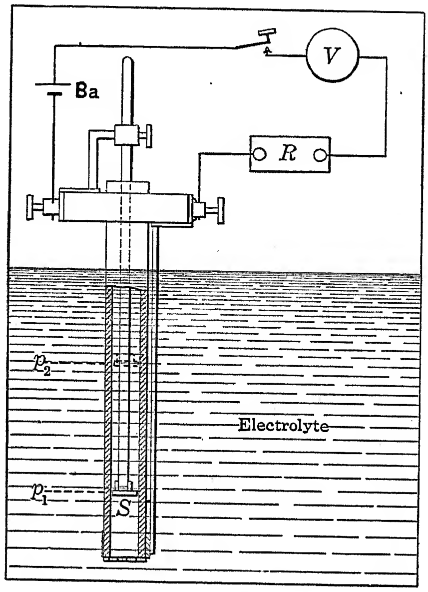

FIG. 206b.

207. Voltmeter Method. Circuit Forms Loop of Three Unknown Resistances, Two of which are to be Determined. — The following method is a more general case of the measurement of resistance with a deflection instrument and is especially useful for determining the insulation resistance of a two-wire interior wiring system, while the power is on. The method was first described by the author in the *Electrical World and Engineer*, pages 966–967, May 21, 1904.

The theory only of the method is given here, its application to insulation measurements of live wiring systems being left to a later chapter. Reference should be made to Fig. 207, I, II, and III, for the manner of making the connections and the meaning of the symbols used.

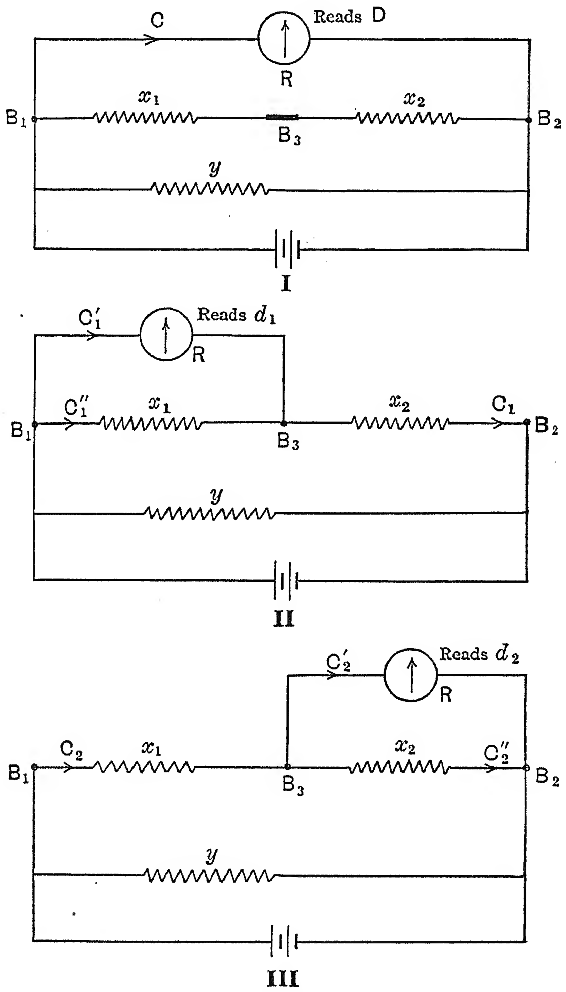

Fig. 207.

C_1, C_2, etc., are currents and R is the known resistance of the deflection instrument used. D, d_1, and d_2 are deflections each of which is assumed in all cases to be proportional to the E.M.F. applied at the terminals of the instrument. Measurements of this kind would ordinarily be made with a Weston voltmeter, reading to 150 volts, and 110 volts on the supply mains, or with a milli-voltmeter, having a known resistance of from 200 to 500 ohms in

series with it, and current supplied from one or two cells of storage battery.

If direct current is supplied to the system, a direct-current deflection instrument must be used, as a Weston voltmeter, a millivoltmeter with a series resistance, or a galvanometer of known resistance.

If the current is alternating, then an alternating-current Weston voltmeter may be used, or other alternating-current deflection instrument of the electrodynamometer-type with a scale marked to give indications which are directly proportional to the E.M.F. at the instrument terminals or to the current thru it.

The resistances x_1 and x_2 are determined by knowing R, the resistance of the deflection instrument, and by taking three instrument readings:

1st. Determine the deflection when the instrument is connected to B_1 and B_2 (Fig. 207, I). Call this D.

2nd. Determine the deflection when the instrument is connected to B_1 and B_3 (Fig. 207, II). Call this d_1.

3rd. Determine the deflection when the instrument is connected to B_3 and B_2 (Fig. 207, III). Call this d_2.

If in case 1 the deflection goes off the scale or in cases 2 and 3 the readings are but a very small fraction of the total scale, the method is not applicable, without modification, in the former case, and in the latter case the resistances x_1 and x_2 are too high to be satisfactorily measured by this method. Having taken the above three readings it will be shown that

$$x_1 = \frac{R\,(D - d_1 - d_2)}{d_2}, \tag{1}$$

and

$$x_2 = \frac{R\,(D - d_1 - d_2)}{d_1}. \tag{2}$$

As the resistance y does not enter into Eqs. (1) and (2) it may have any value without influencing the result.

It is to be noted that, as x_1, x_2, and y are in series and form a closed loop, this method determines the resistances of portions of a closed circuit without cutting it.

We further note from Eqs. (1) and (2) that

$$\frac{x_1}{x_2} = \frac{d_1}{d_2}, \tag{3}$$

if x_1 = infinity, the deflection $d_2 = 0$, and hence,

$$x_2 = \frac{R\,(D - d_1)}{d_1}. \tag{4}$$

e ordinary expression used in measuring a resistance with
ier, and has the same form as Eq. 8, par. 201.
ons (1) and (2) above are obtained as follows: Let k be
ant of the deflection instrument, such that the E.M.F.'s
strument terminals are proportional to the deflections.

$$C_1 = \frac{k\,D}{x_2 + \dfrac{Rx_1}{x_1 + R}}, \tag{5}$$

$$C_2 = \frac{k\,D}{x_1 + \dfrac{Rx_2}{x_2 + R}}, \tag{6}$$

$$C_1' = \frac{x_1}{R + x_1} C_1 = \frac{k\,d_1}{R},$$

$$C_1 = \frac{k\,d_1\,(R + x_1)}{x_1 R}. \tag{7}$$

$$C_2' = \frac{x_2}{R + x_2} C_2 = \frac{k\,d_2}{R},$$

$$C_2 = \frac{k\,d_2\,(R + x_2)}{x_2 R}. \tag{8}$$

have the two relations,

$$\frac{k\,D}{x_2 + \dfrac{Rx_1}{x_1 + R}} = \frac{k\,d_1\,(R + x_1)}{x_1 R} \tag{9}$$

$$\frac{k\,D}{x_1 + \dfrac{Rx_2}{x_2 + R}} = \frac{k\,d_2\,(R + x_2)}{x_2 R}. \tag{10}$$

: k cancels from equations (9) and (10) and x_1 and x_2 are
simple algebraic transformations, to have the values
ve in Eq. (1) and Eq. (2). If any one of the equations
id (4) are solved for R, it is seen that we have a method,
resistance is known, of determining the internal resist-
deflection instrument. The method is often useful for
e resistance of a galvanometer when no other instrument
e. As the E.M.F. of a single cell of battery will throw

a galvanometer off its scale, it is necessary to use a feeble E.M.F. which may be obtained by opposing two nearly equal cells, or by taking the drop between two near points on a low-resistance wire.

The above method is illustrated by the following measurements: The deflection instrument used was a millivoltmeter with a resistance in series with it, so that [Eqs. (1) and (2)] $R = 102.3$ ohms. The other quantities in the table below have the same meanings as in Eqs. (1), (2) and in Fig. 207. The source of current was a single cell of storage battery and the resistances measured were manganin resistance coils. For the sake of generality a resistance y (Fig. 207) of 200 ohms was used to shunt the resistances x_1 and x_2. The results obtained are given in the table below.

D	d_1	d_2	x true value	x found	Per cent error
100.5	31.5	63.0	$x_1 = 10$	9.74	-2.6
			$x_2 = 20$	19.48	-2.6
100.8	72.2	14.4	$x_1 = 100$	100.88	$+0.88$
			$x_2 = 20$	20.12	$+0.6$

It should be noted that in the first case the difference $D - (d_1 + d_2) = 6$ divisions, only, and in the second case it equals 14.2 divisions and that the precision is, as should be expected, better in the latter case.

The theoretical precision which can be obtained by this method is fully discussed in par. 104.

208. Limitations of Voltmeter Methods. — It remains to point out the limitations of voltmeter methods when applied to the measurement of resistances of over a megohm.

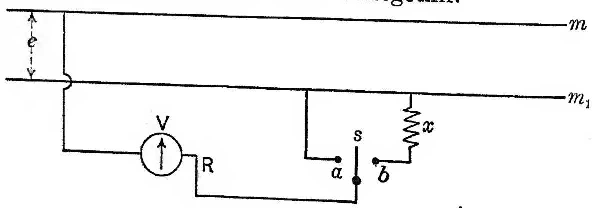

FIG. 208

Let m and m_1 be two direct-current mains between which the potential is e.

Let V be the voltmeter of internal resistance R and let x be a resistance (over a megohm) which is to be measured. With the

connections made as in Fig. 208, first put the switch s on a and read the deflection of the voltmeter, which call d_2, then on b and read the deflection, which call d_1. By the theory and formula given in par. 201, Eq. (8), we have

$$x = \frac{d_2 - d_1}{d_1} R. \tag{1}$$

Ordinary conditions would be met by assuming that the mains furnish 110 volts, that the voltmeter has one division to the volt and reads 150 volts at the limit of its scale, and that it has a resistance of 100 ohms to the volt or a total of 15,000 ohms. We shall then have $d_2 = 110$ and $R = 15{,}000$.

Solving Eq. (1) for d_1, we find

$$d_1 = \frac{R\,d_2}{x + R}$$

If we assume that $x = 10^6$ ohms, or one megohm, we find $d_1 = 1.625 +$ divisions.

The expression for calculating the maximum percentage error is,

$$E_p = \frac{d_2}{p\,d_1} \frac{d_2 + d_1}{d_2 - d_1} 100. \tag{2}$$

[See Eq. (4), § 104] where $\frac{1}{p}$ is the fractional part of the larger deflection which can be read. Suppose that the readings are taken with great care so that they are accurate to within 0.05 of a division, then

$$\frac{1}{p} = \frac{1}{20 \times 110} = \frac{1}{2200},$$

and we find by Eq. (2) that

$$E_p = \frac{110}{2200 \times 1.625} \times \frac{110 + 1.625}{110 - 1.625} \times 100 = 3.17 \text{ per cent.}$$

This is a fairly representative case and it shows that one megohm is about the largest resistance which can be measured with a 150-volt voltmeter, with even very moderate precision. If the voltmeter is a 300-volt instrument and this potential is available the accuracy would of course be correspondingly increased.

209. Resistance Measured with a Voltmeter and an Ammeter. — Connect the ammeter in series with the resistance and the volt-

meter in shunt, as indicated in Fig. 209. Calling R the resistance of the voltmeter, x the resistance to be measured, I the current

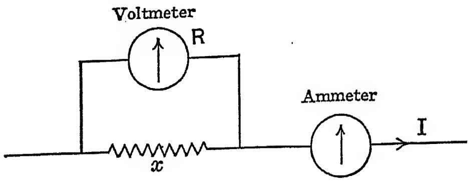

FIG. 209.

measured by the ammeter, and V the potential at the terminals of the voltmeter, we have, by Ohm's law,

$$\frac{Rx}{R+x} = \frac{V}{I}. \quad \text{Hence,}$$

$$x = \frac{RV}{IR - V} = \frac{V}{I - \frac{V}{R}}. \tag{1}$$

It appears from Eq. (1) that if R is very large as compared with V the term $\frac{V}{R}$ can be neglected.

As the above method is often used to measure the resistance of an incandescent lamp while carrying full load current, the following example is given to show the importance of the term $\frac{V}{R}$ in this case: Let the current I be measured with an ammeter, which has a full scale value of 1 ampere, and let the voltage drop over the lamp be read with a 150-volt voltmeter which has 100 ohms resistance to the volt or a total of 15,000 ohms. If the line current is 0.5 ampere, then the true resistance of the lamp, if the voltmeter reads 110 volts, is,

$$x = \frac{110}{0.5 - \frac{110}{15{,}000}} = 223.1 \text{ ohms.}$$

If the last term in the denominator is neglected we obtain for the value of the resistance,

$$x' = \frac{110}{0.5} = 220 \text{ ohms.}$$

This result is 1.39 per cent lower than the true value, which is 223.1 ohms.

The practical way to measure the resistance of a 110-volt incandescent lamp by this method, using Weston instruments, would be to divide the volts read by the current and add 1.4 per cent to the result.

A trial was made of this method using the same tungsten lamp and voltmeter employed in the test to illustrate the method shown in Fig. 203. The resistance of a carbon 60-volt lamp joined in series with the tungsten lamp was determined at the same time. The resistance R of the Weston voltmeter was 15,660 ohms. The ammeter was a Weston instrument reading 1 ampere for a full scale. The results were calculated by Eq. (1), par. 209, and are exhibited in the table below:

I	V	X	Remarks
0.479 0.317	65.8 32.7	138.5 103.8	*Tungsten Lamp.* The resistance of the tungsten lamp increases 1.05 ohms per volt in range 33 to 66 volts.
0.620 0.270	65.7 32.8	106.6 122.3	*Carbon Lamp.* The resistance of the carbon lamp decreases 0.477 ohm per volt in range 33 to 66 volts.

Calculating the resistance of the tungsten lamp at 33.9 volts from the resistance found at 32.8 volts on the assumption that the resistance increases 1.05 ohms per volt, we find this resistance to be 105.06 ohms. By the method of par. 203, we found the resistance at this voltage to be 104.8 ohms. The two measurements are thus seen to be in agreement within 0.25 of 1 per cent.

210. Remarks Upon the Methods of Chapter II. — The methods given above for measuring resistance with deflection instruments of convenient type are the ones which are most often used and are best suited to practice. They may be modified and extended in a variety of ways, but the fundamental principles involved would not be altered. The methods as given are typical and will cover almost every case that may arise. They are sufficient to illustrate the principles involved, and, if these are understood, modifications may easily be made to meet any special requirements. In considering the theoretical precision of the deflection methods given above, reference should be made to the elementary discussion of the theory of errors in Chapter I.

211. Ohmmeters and Meggers. — These instruments are devices for reading resistances directly. Ohmmeters are of two general types: those which read a resistance by the deflection of a pointer over a scale, this deflection being proportional to the resistance, but independent, within wide limits of the value of the testing current employed; and those which operate on the principle of a slide-wire bridge, which is balanced, the balanced condition being indicated by a galvanometer or a telephone. The scale under the slide wire is laid off in ohms so that the resistances are read directly in ohms.

The first type is constructed by the maker so as to operate directly by simply connecting the resistance to be read to two binding posts and reading the deflection of a pointer. A description of them involves describing not methods of measurement, but instruments, and they will not be further considered here.

The second type should be included under a description of "balance" methods of measuring resistances.

"Megger" is a trade name given to a type of deflection ohmmeter, which is very fully described in trade publications. It is claimed that they give all results which can be obtained with voltmeters in the measurement of resistances and with greater facility and precision. This claim is probably justified. If such an instrument is available it should be used in preference to a voltmeter for many kinds of resistance measurements.

The "Megger" was designed and patented by Mr. Sidney Evershed of London, for the rapid determination of high resistances. Following is a very brief description of this instrument and some of the claims made for it.

It is a direct-reading ohmmeter with a direct-current hand-driven dynamo mounted in the same case. The instrument is arranged for easy portability and rough usage. The scale is graduated in ohms so that no calculations are required. The hand-driven dynamo delivers current at 100, 250, or 1000 volts (according to the range and sensibility of the particular type used) and this makes the instrument independent of outside current supply. The instrument is similar in principle to a differential galvanometer so devised that moderate change of dynamo voltage does not vary the scale reading. The entire instrument weighs about 20 lbs. It is claimed to have a range from 0.01 to 2000 megohms. Two scales of high-range meggers are shown full size

in Fig. 211. From an inspection of these scales one may estimate the possible precision attainable on the assumption that the instrument itself is accurately calibrated.

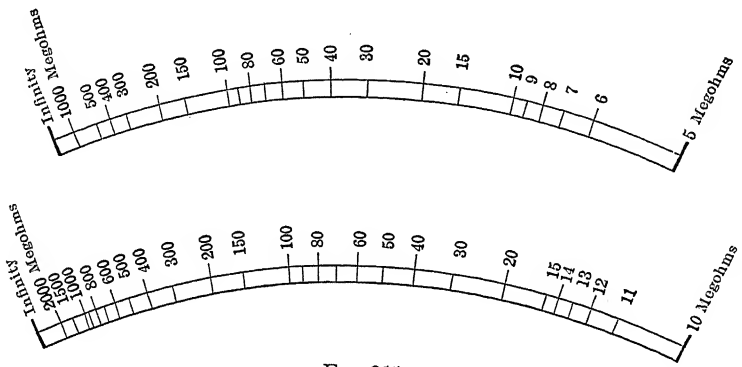

FIG. 211.

The instrument is sold in America by James G. Biddle, Philadelphia, Pa., and is fully described in his catalogue No. 740. This being an instrument rather than a method of measurement, the author must refer the reader to the above-mentioned pamphlet for a more detailed description.

CHAPTER III.

NULL METHODS. RESISTANCE MEASURED BY DIFFERENTIAL INSTRUMENTS.

300. **Remarks on Null Methods.** — When some quantity of known value is varied in a known way until it is made equal to, or to some known multiple of, a quantity being measured, and when the attainment of this condition is indicated by some detector then showing no deflection, the measurement is said to be made by a "null" or "balance" method. The most familiar example of the above is weighing with a chemical balance. Here a mass is the unknown quantity. Known masses are placed in the pan until the balance, which is here the detecting instrument, shows no deflection. Other examples are: The measurement of resistance with a Wheatstone bridge, where known resistances are varied until they are made a known multiple of an unknown resistance. The zero deflection of a galvanometer indicates in this case when the known resistances have been perfectly adjusted; the measurement of potential differences with a potentiometer, where a known potential difference is varied in a known manner until it equals an unknown potential difference under measurement. The zero deflection of a galvanometer, the detecting instrument, shows in this case also when the equality is attained.

Null or balance methods of measurement are generally more sensitive and accurate than deflection methods: 1st, because it is unnecessary, as in deflection methods, to know the constant of the detector; 2d, because it is unnecessary to know the law of the deflection of an indicating instrument, or to rely upon the accuracy or the constancy of the calibration of a scale; 3d, because quantities of the same kind are simply matched against, or compared with, one another; 4th, because it is easier to detect a small departure from the zero of an indicating instrument than it is to read the exact extent of a deflection.

For reasons of the above character, weighing, bridge methods of measuring resistance, and potentiometer methods of measuring potential differences are the most accurate used.

Balance or null methods of measurement have, however, the disadvantages of requiring more apparatus, of consuming more time, and of requiring more manual manipulation than do deflection methods. If the quantity under measurement is subject to fluctuations, which it is desirable to note, then strictly null methods are less suitable than deflection methods. In case the quantity being measured is fluctuating, it is generally impossible to perform the manual manipulations with sufficient rapidity to maintain the balanced condition and, even if this can be done, to record the settings.

There are methods, which we shall discuss later, which measure the quantity by approximately matching it with a known quantity, deflections being used to determine the small differences. These balance-deflection methods are of great value in industrial electrical measurements, as they combine precision and speed.

The strictly null methods are chiefly valuable in connection with standardization measurements. Those which are useful in the measurement of medium resistances will receive full attention. The methods to be discussed now are those which make use of a differentially wound galvanometer and those which employ some form of the network of resistances generally known as the Wheatstone bridge.

The differential methods have certain advantages in some important commercial applications and therefore deserve a careful consideration.

301. Properties of Differential Circuits. — Almost any instrument which deflects with the passage of an electric current thru it may be differentially wound; namely, so wound with a double winding that a current thru one winding is exactly neutralized, in its action to produce a deflection, by an equal current thru the other winding. A differential winding may be applied to milliammeters, voltmeters, galvanometers of the moving-magnet or of the moving-coil type, and to telephones. When applied to direct-current instruments the circuits have certain interesting and useful properties, to a consideration of which we now proceed. The properties to be discussed are quite independent of the type of the differential instrument. This may be a moving-magnet or a moving-coil galvanometer, but for definiteness let us fasten our attention upon a differentially wound D'Arsonval galvanometer.

One assumption must *always* be made, and a second assumption generally. The *first* is that equal currents, but opposite in their electromagnetic action, in the two windings shall be without influence upon the movable system. The *second* is that the resistances between binding posts of the two windings shall be the same. A *third* assumption is sometimes required for certain classes of refined work. It is, that the resistances of the two windings shall remain constant under temperature changes in the room and with varying quantities of current in the windings. This last assumption can only be met, as a rule, with difficulty or at the sacrifice of sensibility, as when coils are wound with wire of high resistance to secure a negligible temperature coefficient.

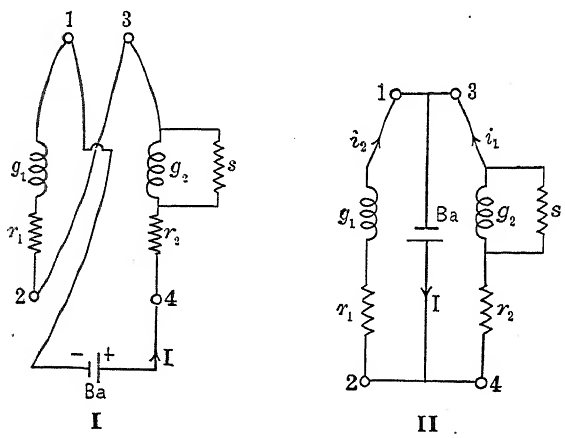

Fig. 301a.

To test if the instrument is truly differential join the two windings in series as shown at I, Fig. 301a. Then, if the number of turns in the windings g_1 and g_2 are equal and wound in opposite directions, the *same* current traversing these windings should produce no turning moment on the system. It is found in practice, however, that howsoever carefully the coil is wound to be made differential, some tendency will remain for the system to turn. Suppose the winding g_2 slightly predominates. If a very high resistance S is shunted around this winding, it may be adjusted until the system is entirely without tendency to rotate, whatever value the current I has; provided this current is not so large as to greatly heat the windings.

Next join the windings in parallel, as in II, Fig. 301a, and vary a small resistance r_1 in series with g_1, or a resistance r_2 in series with g_2, until $i_1 = i_2$, when the system will again give no deflection. With these two adjustments it is possible to make the system differential and to give the two windings exactly the *same* resistance — namely, the total resistance from post 1 to post 2 will equal the total resistance from post 3 to post 4. The above adjustments should be made by the instrument maker and are explained here only to show their possibility.

If the windings g_1 and g_2 are of copper and differ very much in resistance, then changes in temperature will destroy the relation between the resistance of g_2 and the shunt S and the adjustment to differentiality will not be permanent. With proper original construction, however, such a defect should not exist.

It will now be assumed in what follows that the instrument is truly differential and that the two windings from binding post to binding post have the same resistance. It cannot be assumed that the resistances of the windings will *remain constant* when these are of copper, but if they change with temperature they will change alike; because the two windings are made by winding a double wire and are therefore intimately associated, and hence always at the same temperature.

We proceed, first, to prove an interesting and useful property of the circuits of a differential galvanometer when these are made up in the manner indicated in Fig. 301b.*

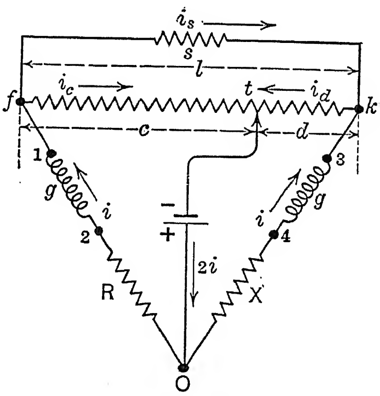

FIG. 301b.

Let 1, 2, and 3, 4 be the binding posts of the differential instrument (made strictly differential by means of adjustments as described above). Let g and g be its two windings so disposed that equal currents i and i flowing in the direction indicated by arrows, produce no deflection.

Let fk be a uniform slide-wire resistance along which a sliding

*Described by author in the *Trans. of the Electrochemical Society*, May, 909, Vol. XV, p. 340.

contact t may be moved, and let this slide wire be shunted by a resistance S.

Let R be any fixed resistance from 0 to f and X any resistance from 0 to k which may be varied.

Now, it is evident that if R and X are nearly equal there will be some position for t on the slide wire fk such that the currents i and i thru the two windings will be equal and the system give no deflection.

If l is the length of fk (and is proportional to its resistance), we wish to determine how the distance c of t from f varies when X varies. If c varies uniformly with variations in X we have a means of measuring small changes in X with high precision.

The signification of all the symbols used will be easily understood by a reference to Fig. 301b, without further explanation.

By Kirchoff's laws,

$i - i_c - i_s = 0$ are the currents which enter and leave f, (1)

$i + i_s - i_d = 0$ are the currents which enter and leave k, (2)

$iX + i_d d = E$ is the E.M.F. in circuit $OktO$, (3)

$iR + i_c c = E$ is the E.M.F. in circuit $OftO$, (4)

$i_s S + i_d d - i_c c = 0$ is the E.M.F. in circuit $fSkf$. (5)

Subtracting (4) from (3) gives

$$i\,(X - R) + i_d d - i_c c = 0. \tag{6}$$

From (1) $i_c = i - i_s$. (7)

From (2) $i_d = i + i_s$. (8)

Putting the values of i_c and i_d in (6) gives

$$i\,(X - R) + di + di_s - ci + ci_s = 0, \tag{9}$$

$$\text{or} \quad i_s\,(c + d) + i\,(X - R) - i\,(c - d) = 0. \tag{10}$$

Putting the values of i_c and i_d in (5) gives

$$i_s S + di + di_s - ci + ci_s = 0,$$

$$\text{or} \quad i_s = \frac{i\,(c - d)}{S + c + d}. \tag{11}$$

Also from (10) we have

$$i_s = \frac{i\,(c - d) - i\,(X - R)}{c + d}. \tag{12}$$

Equating the second members of (11) and (12) we derive

$$X = \frac{R\,(c + d + S) + S\,(c - d)}{S + c + d}. \tag{13}$$

In (13) replace d by its value $l - c$ and we obtain

$$X = \frac{2\,cS}{S+l} + \frac{R\,(l+S) - lS}{S+l}. \qquad (14)$$

The last term of (14) is a constant.
Call this K and we have, finally,

$$X = \frac{2\,cS}{S+l} + K. \qquad (15)$$

Differentiating (15), we find

$$\frac{\delta X}{\delta c} = \frac{2\,S}{S+l},$$

$$\text{or } \delta c = \frac{S+l}{2\,S}\,\delta X. \qquad (16)$$

Eq. (16) shows that c varies proportionally to X and *only*

$$\frac{S+l}{2\,S} \text{ as rapidly as } X.$$

This result means that in the case of a uniform slide resistance, the slide resistance may be given any convenient value greater than the minimum allowed. It may then be shunted to give the particular resistance range desired with any chosen length of slide resistance.

By minimum resistance allowed, we are to understand a resistance which is large enough to permit a balance to be obtained with the sliding contact upon the slide resistance fk when X changes from its least to its greatest value.

For example, if when X has a minimum value the contact t is at point f for a balance, it must be possible to obtain a balance with the contact t at, or to the left of, the point k when X has its greatest value.

Putting Eq. (16) in the form

$$\delta c = \frac{1 + \dfrac{l}{S}}{2}\,\delta X,$$

we see that when S is infinity, that is, when the shunt is absent, the variations in c are one half as great as the variations in X.

302. Illustration of the Practical Advantages of Differential Circuits.—To see the advantages of the above properties of differential circuits let the diagram, Fig. 301b, be reconstructed as given in Fig. 302.

In Fig. 302, 1, 2, 3 are three binding posts of the differential apparatus and 4, 5 are the terminals of a resistance X which is supposed to be placed at a distance, which may be very great, from the differential instrument. If the resistance of the lead wire A is made *equal* to the resistance of the lead wire B, Eq. (16) (§ 301) still holds. For, giving A a resistance and B an equal resistance is not different from increasing by an equal amount the resistance of each of the windings g and g, and as the resistance of these windings does not appear in either Eq. (15) or Eq. (16) (§ 301) the

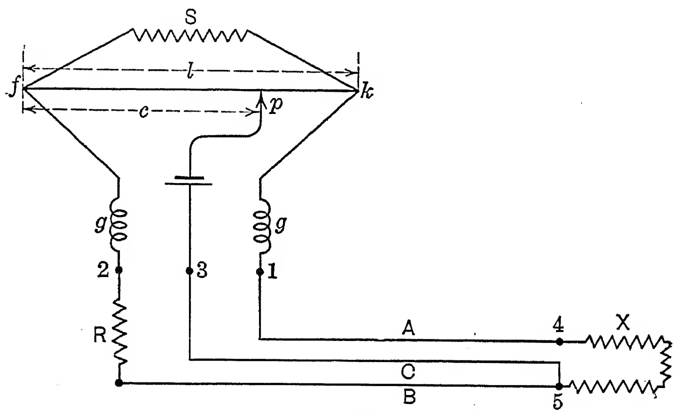

FIG. 302.

equal resistances A and B do not enter these equations. In other words the resistance X, and changes in the resistance X, may be determined by knowing the constant resistances R, l, S and the resistance c which is varied for obtaining a balance; the resistances g and g, A, B, and C not appearing.

The chief industrial application of the above principles is in the measurement of temperature with electrical-resistance thermometers which may be placed at a distance from the reading instrument. The resistances of the lead wires — which only need to be three in number—can vary (provided the A and B leads are equal in resistance) without affecting the accuracy with which the resistance of the thermometer is determined, and hence the temperature which is a function of its resistance.

Again, the method may be employed for the determination of any unknown resistance X. If the contact p is set at the middle of slide resistance fk and the resistance R is varied until a balance

is obtained, we have, without regard to the resistance of the lead wires, $X = R$.

If R is made equal to X at a particular temperature, then small variations in X, due to temperature changes, can be accurately determined by moving p over the slide resistance fk till the galvanometer is balanced. The variations in X are then given by the relation

$$\delta X = \frac{2S}{S+l}\delta c. \tag{1}$$

If fk is a slide wire of uniform resistance, the variations in c can be read directly on a millimeter scale and thus the curve giving the relation between the resistance and the temperature of a wire can be very accurately determined. As S can be given any value, let it equal infinity, then $\delta X = 2\,\delta c$. This shows that when the method is used for determining temperature coefficients, the highest value of c (which cannot exceed that of l) will not be greater than one half the total change which takes place in X with the greatest variation in temperature employed. For example, suppose it is required to determine the temperature coefficient of a copper coil which has a resistance of 10 ohms at 0° C. in the range 0° to 100° C. At 100° C the resistance of the copper coil would be about 14.2 ohms. Then its increase in resistance is 4.2 ohms and the resistance of the slide wire fk could not be less than 2.1 ohms. The resistance R would be so chosen that the galvanometer would be balanced when X was at 0° C. and p is set at f, or when $c = 0$. Let it next be required to determine the temperature coefficient of a 1-ohm coil in the same range. The increase in resistance of this coil in the 100° C. range would only be 0.42 ohm and the distance to move on the slide wire would only correspond to 0.21 ohm or 0.1 of its length. If, however, the slide wire is shunted with a resistance of a proper value (and R is always so adjusted that with the 1-ohm coil at 0° C. we have $c = 0$ for a balance), we can again have $c = l$ when the 1-ohm coil is at 100° C., and thus determine the temperature coefficient of 1 ohm with the same percentage precision as 10 ohms.

To find the proper value to give S we assume an approximate increase in the resistance of the 1-ohm coil when the temperature increases 100° C. We then solve Eq. (1) for S and find

$$S = \frac{l\,\delta X}{2\,\delta c - \delta X} \tag{2}$$

Since δX is to be 0.42 ohm, and l has previously been made 2.1 ohms, and δc is to be practically the whole length of the slide wire, or 2.1 ohms, we derive

$$S = \frac{2.1 \times 0.42}{2 \times 2.1 - 0.42} = 0.233 + \text{ohm}.$$

It should be noted in the above methods that the only movable contact is in the battery circuit, and hence variations in its resistance in no wise affect the readings.

Analysis shows that, for obtaining greatest sensibility, each winding of a differential instrument should have a resistance which is approximately equal to the resistance external to that winding. Or, approximately, we should have

$$g = X + \frac{l}{2}. \tag{3}$$

303. Differential Galvanometer Used with Shunts. — This method is perhaps better applied with a differential galvanometer

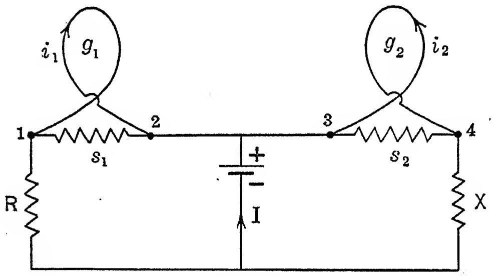

FIG. 303.

of the moving-magnet type. Some of these galvanometers are made with a bell-shaped magnet hanging in a hole in a copper sphere to damp the oscillations of the system. The current coils are two in number and can be individually moved along a horizontal bar so as to approach or recede from the system. When the same or equal currents are sent in opposite directions thru the two coils, one or both of these can be moved into a position where the galvanometer is perfectly differential. If this adjustment is made and it be assumed that the resistance of its two coils are alike and known, and if they can be relied upon to remain constant, we can, by the following well-known method, measure any resistance X. Let the circuits be arranged as in Fig. 303.

Here g_1 and g_2 are the two windings or coils supposed to be alike in resistance, so $g_1 = g_2 = g$, the resistance of each. Each of these coils is shunted with resistances S_1 and S_2 respectively. R is a fixed resistance and X a resistance to be measured.

Denote by I the current in the battery circuit and let i_2 denote the current in the right-hand coil and i_1 the current in the left-hand coil. To find an expression for the value of X, when the galvanometer is balanced, we make use of the following law of the division of currents:

Let a circuit divide into two branches of resistances x and y, and let the resistance x carry a current i_1 and the resistance y, a current i_2 and let $i_1 + i_2 = I$ be the total current; then

$$i_1 = \frac{y}{x+y} I \quad \text{and} \quad i_2 = \frac{x}{x+y} I;$$

that is, the current in either branch is equal to the resistance of the other branch divided by the total resistance around the circuit times the total current.

Applying this principle to the circuits shown in Fig. 303, we have

$$i_1 = \frac{X + \dfrac{S_2 g_2}{S_2 + g_2}}{X + R + \dfrac{S_1 g_1}{S_1 + g_1} + \dfrac{S_2 g_2}{S_2 + g_2}} \, \frac{S_1}{S_1 + g_1}, \tag{1}$$

and

$$i_2 = \frac{R + \dfrac{S_1 g_1}{S_1 + g_1}}{X + R + \dfrac{S_1 g_1}{S_1 + g_1} + \dfrac{S_2 g_2}{S_2 + g_2}} \, \frac{S_2}{S_2 + g_2}. \tag{2}$$

If the galvanometer is so adjusted that $g_1 = g_2 = g$, and we alter S_1, S_2, and R until there is no deflection, we get from Eqs. (1) and (2)

$$X = R \frac{1 + \dfrac{g}{S_1}}{1 + \dfrac{g}{S_2}}. \tag{3}$$

The method expressed by equation (3) can only give accurate results on the assumption that the resistance g is either the same at different temperatures, or is known at the temperature at which the measurement is made. To realize the first assumption the

windings must be made of some low-temperature coefficient wire, as manganin, which has a high resistance, and to realize the second assumption requires the taking of accurate temperature readings of the coils. Moreover, it would in this case be impossible to make the shunts S_1 and S_2 bear any fixed relation to the resistance of the windings. From these considerations the method is seen to be inferior to the Wheatstone-bridge methods of measuring resistance which are in vogue. But, with galvanometer windings of manganin and shunts of suitable values, the precision and the range of resistance-measurement possible can be made equal to that of the ordinary post-office type of Wheatstone bridge. For example, suppose

$$S_1 = \frac{g}{99} \quad \text{and } S_2 = \text{infinity},$$

then, by Eq. (3), $X = 100\ R$, and if R can be varied in steps of 1 ohm from 0 to 10,000 ohms we can measure a resistance $X =$ 1,000,000 ohms. Or, suppose

$$S_2 = \frac{g}{99} \text{ and } S_1 = \text{infinity},$$

then, by Eq. (3), $X = 0.01\ R$, and if $R = 1$ ohm, $X = 0.01$ ohm.

The above method is thus seen to require, for precision and range, a specially constructed galvanometer and shunts which have values of at least $\frac{1}{9}$ and $\frac{1}{99}$ of the resistance of a coil winding and a 10,000-ohm rheostat variable in steps of 1 ohm. It has, therefore, little to recommend it in competition with the bridge methods to be described, and is given here chiefly to complete the treatment of the differentially wound instrument as used for resistance-measurements.

304. The Differential Telephone. — The magnet of a telephone may be wound with a differential winding. This instrument can then be used to indicate when there is an equality between two currents which are alternating or unsteady. The two wires of the differential winding should be twisted together and wound bifilar upon the magnet. The theory of this instrument is complicated, and the circumstances under which it can be used to advantage are limited, therefore we shall not give further discussion to it here.

CHAPTER IV.

THE WHEATSTONE–BRIDGE NETWORK. SLIDE–WIRE BRIDGE METHODS.

400. **Network of the Wheatstone Bridge.** — The Wheatstone bridge is a network of *six* conductors and should be distinguished from the Kelvin double bridge (to be discussed later) which is a network of *nine* conductors. While the Wheatstone net or bridge may assume many forms, the essential electrical properties are the same in all.

This network of six conductors may be represented in three ways which are equivalent. They are presented in I, II, and III, Fig. 400a.

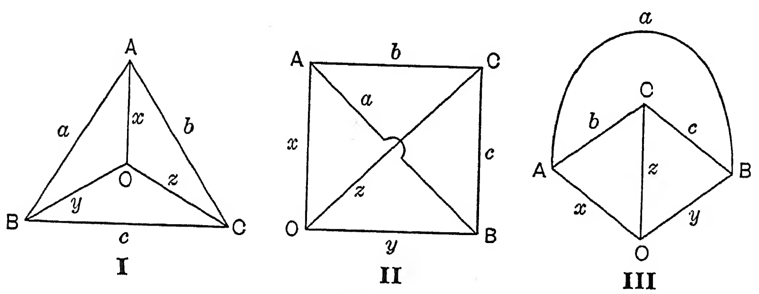

Fig. 400a.

In the three representations like letters designate the same conductors and the same points of junction. Following the lettering of the diagrams, it will be seen, from a well-known property of the bridge, that, if the resistances b, c, x, y have the relation

$$yb = cx \qquad (1)$$

there will be no current in a if there is an E.M.F. in z, and, conversely, there will be no current in z if there is an E.M.F. in a. Under these circumstances the two conductors a and z are said to be *conjugate* conductors. Other pairs of conductors can become

conjugates. By referring to diagram I we can write, from the symmetry of the diagram, the following relations:

If $yb = cx$, then z and a are conjugates.
If $za = by$, then x and c are conjugates.
If $xc = az$, then y and b are conjugates.

The above relations belong to what we shall term the "first property" of the Wheatstone bridge.

We shall presently show that if any two conductors are conjugates, in whatever part of the network an E.M.F. be placed, the current which flows through one of the conjugates is independent of the resistance of the other conjugate. For example, if in diagram III a and z are conjugates and an E.M.F. is placed in branch c, the current which will flow in z will be the same whatever may be the resistance of its conjugate a. This principle is made use of later in determining the internal resistance of a battery and the resistance of a galvanometer. We shall term this the "second property" of the Wheatstone bridge. The above-mentioned "first" and "second" properties of the Wheatstone bridge may be deduced as follows:

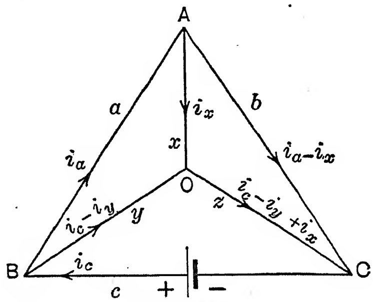

FIG. 400b.

If, in Fig. 400b, we place an E.M.F. in the branch BC we shall have the distribution of currents shown in the diagram, where

i_c is the current in c,
i_a is the current in a,
i_x is the current in x,
$i_c - i_y$ is the current in y,
$i_a - i_x$ is the current in b,
and $i_c - i_y + i_x$ is the current in z.

As before (Fig. 400a), x, y, z, a, b, c designate ohmic resistances. If it is true that x and c are conjugates, then the E.M.F. in c will

produce no current in x, and hence we shall have $i_x = 0$. Assume, then, the current i_x is zero and determine the relation of the resistances necessary to produce this result. With $i_x = 0$, the points A and O will be at the same potential, and we shall have by Ohm's law the fall in potential from B to $A = ai_a$ and the fall in potential from B to $O = y\,(i_c - i_y)$ and these will equal each other, or

$$ai_a = y\,(i_c - i_y). \tag{2}$$

Likewise (remembering that we have assumed $i_x = 0$) we shall have

$$bi_a = z\,(i_c - i_y). \tag{3}$$

Hence, taking the ratio of Eq. (2) to Eq. (3), we obtain

$$\frac{a}{b} = \frac{y}{z}, \quad \text{or} \quad za = by,$$

which is the relation necessary to make x and c conjugates. The above proves the "first property" of the bridge.

To prove the "second property" assume that an E.M.F. exists in some branch of the bridge, as AC (Fig. 400b). Then a current will flow in the branch BC as well as in the branch AO. Assume now a counter E.M.F. to be introduced into the branch BC. This E.M.F. cannot produce any current in the conjugate AO. In particular choose this counter E.M.F. such as to just reduce the current in BC to zero. But when an E.M.F. is introduced into BC, which reduces the current in this branch to zero, we have done the equivalent of opening the circuit BC. Hence it follows that the current in AO resulting from an E.M.F. in AC is unaffected by any change in the resistance of BC.

The mathematical relations which hold for the Wheatstone bridge when this is unbalanced, namely, when no two conductors are conjugates, are complicated and require lengthy calculations to derive. These relations are given in such standard works as Kempe's "Handbook of Electrical Testing." They are rarely made use of in the practical employment of the Wheatstone bridge, and we do not consider that it would be advisable to give a general treatment of these relations here. We shall, therefore, proceed at once to a consideration of those features which are useful to know and understand in applying the Wheatstone bridge to such electrical measurements as arise in practice.

Represent the bridge as in Fig. 400c. Here K_1 and K_2 are the two conductors which are to become conjugates when the bridge

is balanced. If a, b, c, d represent the resistances of their respective arms, this condition is filled when $\frac{a}{b} = \frac{c}{d}$. If any three of these four resistances are known the other is given by this relation.

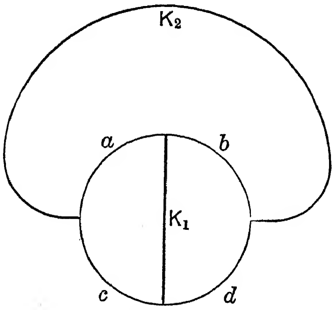

FIG. 400c.

In conformity with custom we shall call the two adjacent arms which do not contain the unknown resistance the "ratio arms" and the arm adjacent to the unknown resistance the "rheostat" of the bridge. It is evident that the bridge may be balanced either by varying the ratio arms or by maintaining the latter fixed and varying the rheostat. When the former plan is followed, the bridge usually takes the form of a so-called slide-wire or slide-resistance bridge and when the latter plan is followed the bridge is usually a so-called plug-type or dial-type bridge.

401. Uses of the Slide-wire Bridge. — The slide-wire bridge (often referred to as the "meter bridge" because a slide wire a meter long stretched over a meter scale is used) was one of the earliest if not the earliest form of a Wheatstone bridge.

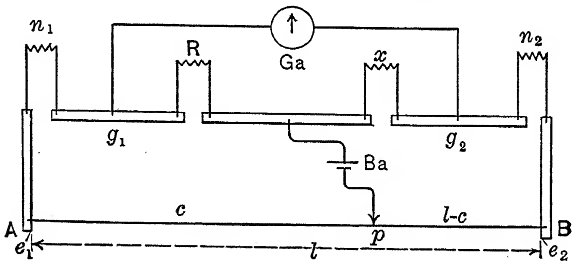

FIG. 401a.

This form of the Wheatstone bridge is diagrammatically shown in Fig. 401a. Here R is a known resistance, which may be given various convenient values, x is the resistance to be measured and n_1 and n_2 are two resistances which may be inserted at the ends of the slide wire.

To use this bridge, a resistance R is chosen which, preferably, is as nearly as possible equal to the resistance x to be measured.

The resistances n_1 and n_2 should be chosen low enough so that the bridge may be balanced by sliding the sliding contact p to some point on the bridge wire. The battery and galvanometer may be located as shown in the diagram but the position of these may be reversed. If l is the length of the bridge wire and c the *distance* from A to p in millimeters, and if n_1 and n_2 are resistances determined *in equivalent millimeters of length of bridge wire* we have for a balance

$$\frac{R}{x} = \frac{n_1 + c}{n_2 + l - c},$$

or

$$x = \frac{n_2 + l - c}{n_1 + c} R. \tag{1}$$

In practice the two resistances n_1 and n_2 would be made equal and may then be called n, and l would be 1000 mm.
Then

$$x = \frac{n + 1000 - c}{n + c} R. \tag{2}$$

The value which may be given to the resistances n and n will depend upon how nearly it is possible to choose the resistance R to be like the resistance x. Since the greatest value c can have is 1000, and a balance still be obtained with the slider upon the wire, we assume c to have this value and solve Eq. (2) for n and thus find,

$$n = \frac{1000\,x}{R - x}, \tag{3}$$

as the maximum value n may have. Suppose it is possible to always choose R so that the value of the resistance x which we have to measure, will never be less than $\frac{R}{2}$. Then we could make

$$n = \frac{1000 \frac{R}{2}}{R - \frac{R}{2}} = 1000.$$

Namely, n could equal the resistance of 1000 mm of bridge wire. If the resistance R is obtained from a small plug resistance box, this is almost always practicable. Using, then, this value of n, Eq. (2) becomes

$$x = \frac{2000 - c}{1000 + c} R. \tag{4}$$

The effect of using resistances n and n each of which are equal in resistance to 1000 mm of bridge wire, is, virtually, to make this wire three times as long. Furthermore, by care in construction, the resistances n and n may be made to include the unavoidable resistances of the joints and copper straps between the points e_1 and g_1, and e_2 and g_2 (Fig. 401a). When, as in the more ordinary use of the slide-wire bridge, the resistances n and n are assumed equal to 0, and the formula

$$x = \frac{l - c}{c} R = \left(\frac{1000}{c} - 1\right) R \tag{5}$$

is used, errors are sure to result from a neglect of the small and unknown resistances which exist between the galvanometer terminals g_1, g_2 and the ends of the wire e_1, e_2 where the scale begins and ends. The use of the resistances n and n limits the range of the bridge (in the case of Eq. (4), from $x = 2\ R$ to $x = 0.5\ R$), but gives greater precision than can be obtained without them. When these resistances are not used and the connecting resistances g_1 to e_1 and g_2 to e_2 are kept very small by a good construction of the bridge, the range of measurement is, theoretically, from zero to infinity, tho in practice the precision for either extreme is very low. If the method expressed in Eq. (5) is used, it is always desirable to choose R as nearly equal to x as possible so as to bring the balance point near to the center of the wire. It should be noted that Eq. (5) can be written,

$$x = (\text{reciprocal of scale reading} \times 1000 - 1)\ R.$$

Thus by using a table of reciprocals, or a slide rule, the values of x, when a number of measurements are to be made, may be calculated simply and rapidly. For R one would choose values, as 1, 10, 100, or 1000, etc.

In the above methods employment is made of a slide wire of high resistance alloy one meter in length. It is now possible to obtain alloys of practically zero temperature coefficient and having 60 times the resistivity of copper. If the slide wire is No. 24 B. & S. gauge its resistance per meter might be made about 5 ohms.

It is very desirable to be able to produce a uniform slide resistance of 100 or more ohms. This is accomplished by certain well-known makers, by winding on a long mandril of about 3 mm diameter an *insulated* resistance wire and then withdrawing the

mandril. This leaves a long helix of insulated wire with the turns close together. This helix is laid in a groove in a strip of hard

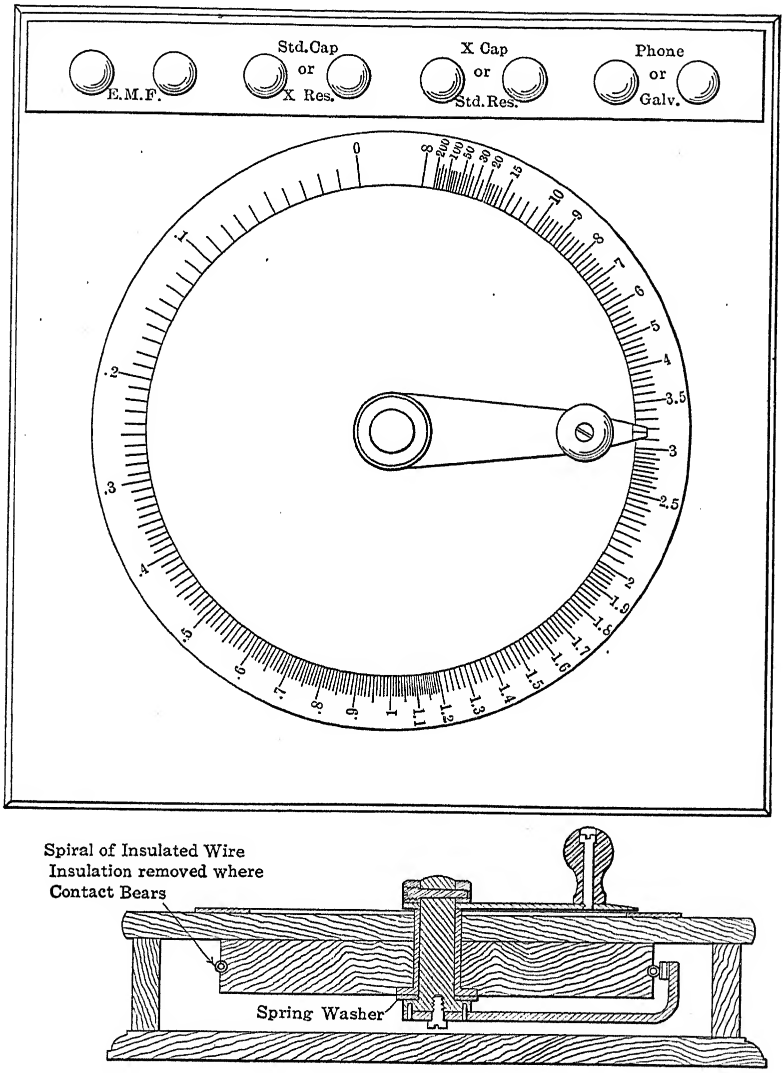

Fig. 401b.

wood or hard rubber, or in a groove in the periphery of a disk of hard rubber and cemented in place with thick shellac. When the

shellac is dry and hard, the insulation is removed along one side of the helix parallel with its axis. A sliding contact is arranged to slide over the bared portion of the helix so as to make contact at any point along its length. In this manner a substantial slide resistance may be made which can be given a resistance of as much as 1000 ohms to the meter. By rubbing one side of the helix with emery cloth such a slide resistance can be made very uniform in resistance from one end to the other.

When such a slide-wire resistance is used, the bridge wire is usually made circular in form and the contact is moved around the periphery of the circular disk by means of a handle at its center. The disk supporting the slide wire is placed underneath a rubber plate, and a circular scale and a pointer which moves with the contact are placed above the rubber plate. The construction is illustrated in Fig. 401b.

If the scale above the rubber plate is properly laid off, in the manner indicated in the figure, it is only necessary to multiply the scale reading by the value of the fixed resistance R which, being chosen 1, 10, 100, or 1000 ohms, makes the instrument direct reading. Hence it becomes a slide-resistance ohmmeter. When mounted with a small portable pointer galvanometer it becomes a portable ohmmeter and is a convenient laboratory tool. The instrument as regularly made is capable of giving measurements of an accuracy of 0.25 of 1 per cent when a standard is chosen which will bring the reading not far from the center of the scale.

In the classes of slide-wire bridge measurements as described above, sufficient sensibility may be obtained from a pointer galvanometer of from 30 to 100 ohms resistance of the types made by Paul, of London; The Leeds and Northrup Company, of Philadelphia, Pa.; or by Edward Weston.

The type of slide-wire bridge measurements, described in connection with Fig. 401a, with a portable pointer galvanometer used as a detecting instrument is very suitable for instruction in the use of the Wheatstone-bridge principle and is to be recommended for college laboratories.

402. Comparison of Resistances by Modified Slide-wire Bridge. — This method, which is applicable to low-resistance conductors, is explained as follows (Fig. 402): Let 1–2 be a linear conductor of uniform resistance, the resistance per unit length of which we wish to determine, and let 3–4 be a linear conductor

of uniform resistance, the resistance per unit length of which is given. R_1 and R_2 are ratio coils.

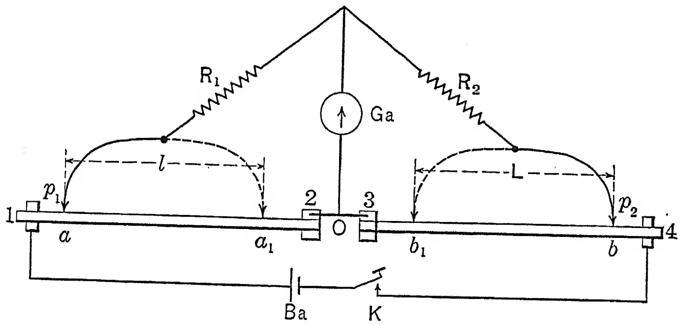

FIG. 402

The two conductors are joined together, no special regard being taken to make this a low-resistance contact. The battery is joined to the ends 1 and 4 with a key K in circuit. With contact p_1 set at a, move contact p_2 to some point of balance b. Next move contact p_1 to a_1 which is some accurately measured distance l from a. Now move contact p_2 to some point b_1 where a balance is again obtained and accurately measure or read off on a scale beneath the conductor the distance L of b_1 from b. When a balance is first obtained the relation holds:

$$\frac{R_1}{R_2} = \frac{\text{resistance } a \text{ to } O}{\text{resistance } b \text{ to } O}.$$

When the balance is next obtained the relation holds:

$$\frac{R_1}{R_2} = \frac{\text{resistance } a_1 \text{ to } O}{\text{resistance } b_1 \text{ to } O}.$$

Hence,

$$\frac{a \text{ to } O}{b \text{ to } O} = \frac{a_1 \text{ to } O}{b_1 \text{ to } O}, \quad \text{or} \quad \frac{a \text{ to } O}{a_1 \text{ to } O} = \frac{b \text{ to } O}{b_1 \text{ to } O}.$$

It is proved in algebra that, if

$$\frac{x}{y} = \frac{w}{z}, \quad \text{then} \quad \frac{x - y}{w - z} = \frac{x}{w}.$$

Now $(a \text{ to } O) - (a_1 \text{ to } O) = (a \text{ to } a_1)$

and $(b \text{ to } O) - (b_1 \text{ to } O) = (b \text{ to } b_1)$;

hence,

$$\frac{a \text{ to } a_1}{b \text{ to } b_1} = \frac{a \text{ to } O}{b \text{ to } O} = \frac{R_1}{R_2},$$

or,
$$x = r\frac{R_1}{R_2}. \qquad (1)$$

Eq. (1) states that the *resistance* x of a length l of the rod 1–2 is $\frac{R_1}{R_2}$ times the *resistance* r of a length L of the rod 3–4. If this last is known the other is given, and the method enables one to compare in a very simple way and with inexpensive apparatus the resistance of a given length of one conductor, as a rod of aluminum, with that of another conductor, as a rod of copper.

If the conductor used as a standard has the same temperature coefficient as the conductor which is being compared with it, no regard has to be taken of temperature other than to be sure that the two rods are at the *same* temperature.

With a D'Arsonval galvanometer of such a sensibility that 10^{-8} ampere will produce a deflection of one scale-division with the scale at a meter distance, as the detecting instrument, the ratio coils R_1 and R_2 may assume resistances considerably higher than 100 ohms, so that the contact resistances of the sliders p_1 and p_2 can be entirely neglected.

This method gives results in low-resistance measurements similar to those obtained with a Kelvin double bridge to be later described. But in this method a balance must be obtained *twice*, instead of only once as with the Kelvin double bridge.

The method is very conveniently applied when the conductor 3–4 is a 5-ohm meter bridge, the wire of the meter bridge resting upon a meter scale marked off in millimeters. The resistance of this wire per centimeter of length must be accurately known. If the conductor 1–2 is of low resistance, then R_1 would be made smaller than R_2, so that a length l on 1–2 of, say, 50 cms. would correspond in resistance with a length of, say, 75 cms. on the bridge wire.

This method may likewise be applied to the measurement of low resistances between potential points. In this case p_1 would first be set on one potential point, and a balance obtained with p_2 at b, then on the other potential point and a balance be again obtained with p_2 at b_1, the ratio $\frac{R_1}{R_2}$ being so chosen that L would be a considerable proportion of the length of the bridge wire. Then if x is the resistance sought between potential points, and if ρ is the resistance of the bridge wire 3–4 per centimeter of length,

we have as above,

$$x = \frac{R_1}{R_2} L\rho, \tag{2}$$

which gives x in ohms, when L is in centimeters.

403. **The Carey-Foster Method.** — The Carey-Foster method of using a slide-wire bridge, while being a very elegant precision method, is not as much employed in this country as formerly. It, nevertheless, deserves a somewhat extended consideration. Tho the method was originally devised for the measurement of very low resistances, it is even better adapted to the accurate comparison of medium or even high resistances. The success with which the Carey-Foster method may be applied to precision work in comparing resistances depends a good deal upon how well the apparatus which is needed for carrying out the method is designed and made. We can only give here the theory of the method referring the reader to trade publications for a description of the mechanical features of the Carey-Foster bridges which are upon the market.

The unique feature of the Carey-Foster modification of the Wheatstone slide-wire bridge consists in interchanging the standard resistance, which is in one arm of the bridge, and the resistance under comparison which is in the adjacent arm. A reading is taken of the position of the contact on the slide wire necessary for a balance before and after the resistances are interchanged. By this procedure all resistances other than those being measured, as well as all constant thermal E.M.F.'s in the bridge circuits, are eliminated.

The connections for the Carey-Foster bridge method are shown in Fig. 403a. S is a standard resistance coil and S_1 is a coil of approximately the same value, which is to be compared with S. R_1 and R_2 are two fixed resistance coils of nearly equal value. a_1 and a_2 are the number of units of length of the portions of the bridge wire to the left and right respectively of the galvanometer contact p. If ρ is the resistance of one unit of length of the bridge wire, then ρa_1 and ρa_2 represent the resistance of the bridge wire to the left and to the right of the galvanometer contact. n and n_1 may be taken to represent all resistances of unknown value in the two bridge arms. As above stated, the Carey-Foster method provides for eliminating these unknown resistances by taking two readings of the position of the galvanometer contact

p on the bridge wire, one reading when the coils S and S_1 have the position shown in Fig. 403a and one when these coils are interchanged. The bridge is mechanically so constructed that this interchange of the standard coil and the coil under comparison can be quickly and easily made.

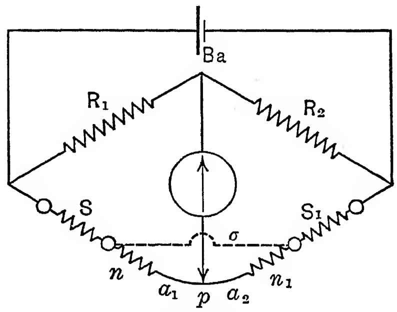

FIG. 403a.

The dotted line σ, Fig. 403a, represents a low-resistance shunt to the bridge wire and its leads, which may or may not be used. When used it has the effect of magnifying the distances that the contact p must be moved along the bridge wire to obtain a balance in the two positions which the coils S and S_1 are made to assume. The general theory of the method is the same whether this shunt is or is not used.

When S and S_1 have the positions shown in Fig. 403a, and the contact p is so chosen that no current flows in the galvanometer circuit,

$$\frac{R_1}{R_2} = \frac{S + n + \rho a_1}{S_1 + n_1 + \rho a_2}, \tag{1}$$

where ρ is the resistance per unit length of the bridge wire.

S and S_1 are now interchanged and a balance is again obtained by sliding p to a new position on the bridge wire. Calling the two lengths of bridge wire, which are now to the left and to the right of the slider a_1' and a_2',

$$\frac{R_1}{R_2} = \frac{S_1 + n + \rho a_1'}{S + n_1 + \rho a_2'}. \tag{2}$$

Changing the forms of Eqs. (2) and (3) by adding unity to each side, we obtain

$$\frac{R_1 + R_2}{R_2} = \frac{S + S_1 + \rho\,(a_1 + a_2) + n + n_1}{S_1 + \rho a_2 + n_1} \tag{3}$$

and $$\frac{R_1 + R_2}{R_2} = \frac{S + S_1 + \rho\,(a_1' + a_2') + n + n_1}{S + \rho a_2' + n_1}. \tag{4}$$

Changing the position of the galvanometer contact p does not alter the total resistance of the bridge wire, hence,

$$\rho\,(a_1 + a_2) = \rho\,(a_1' + a_2'). \tag{5}$$

The numerators of the second members of Eqs. (4) and (5) are, therefore, equal, and, as the second member of each of the two equations is equal to the same quantity, the denominators of the two equations are equal.

Thus, we have

$$S_1 + \rho a_2 + n_1 = S + \rho a_2' + n_1,$$

or

$$S_1 = S - \rho\,(a_2 - a_2'), \tag{6}$$

or, as

$$\rho a_2 - \rho a_2' = \rho a_1' - \rho a_1 = \rho\,(a_1' - a_1),$$

$$S_1 = S - \rho\,(a_1' - a_1). \tag{7}$$

Eqs. (6) and (7) show that the difference in the resistance of the standard resistance S and the resistance S_1 under comparison is equal to the resistance of a certain length of bridge wire.

If, once for all, the resistance ρ per unit length of the bridge wire be determined, then the difference in the resistance of any coil under comparison and a standard coil is given in ohms with great accuracy.

There are several methods known for determining the value of ρ. A simple but inferior method is to balance two standard coils the difference of whose resistances S and S_1 is accurately known, then

$$\rho = \frac{S - S_1}{a_2 - a_2'}. \tag{8}$$

On account of differences of temperature and temperature changes in the coils S and S_1 it is not simple to accurately determine the difference in their resistances. We have, therefore, found the following method of determining the value of ρ, by means of four readings, to be very accurate and satisfactory:

It is necessary for the determination to use a coil S_1 the value of which does not need to be known, but which is sufficiently near the resistance of coil S to make it possible to obtain a balance with p (Fig. 403b) not far from the center of the bridge wire. An ordinary resistance spool or resistance box will answer very well. The resistance may be varied by shunting or by any rheostat

method until a value is reached by trial which will balance the standard S with p near the center of the bridge wire.

C is another coil or box of resistance coils with which S may be shunted by closing the contact K. This shunt coil should have about 100 times the resistance of S and its resistance need not be known to a high degree of accuracy.

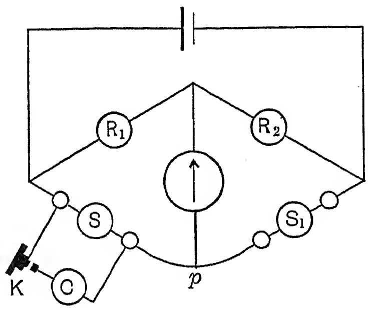

FIG. 403b.

The four readings are taken as follows:

Reading 1 is taken with K open and S and S_1 in the positions shown in Fig. 403b. Call this reading a_1. Reading 2 is taken with S, together with coil C, and S_1 interchanged and K open. Call this reading a_2. S, together with coil C, and S_1 are now interchanged again so that they have their original positions. K is now closed, shunting S with C. Call the resistance of S when shunted S_1'. Reading 3 is then taken, which we will call a_1'. Lastly S together with coil C is interchanged with S_1, K is closed, and reading 4 is taken, which we call a_2'.

The value of ρ is then deduced as follows:

Before S was shunted

$$S_1 = S - \rho\,(a_2 - a_1). \tag{9}$$

After shunting S

$$S_1 = S_1' - \rho\,(a_2' - a_1') \tag{10}$$

Eliminating S_1 from Eqs. (9) and (10), we derive

$$\rho = \frac{S_1' - S}{a_2' + a_1 - a_1' - a_2}. \tag{11}$$

Since $S_1' = \dfrac{CS}{C + S}$, the value of ρ may also be expressed

$$\rho = \frac{S^2}{(C + S)\,(a_2 + a_1' - a_1 - a_2')}. \tag{12}$$

When low-resistance coils are being compared the distance that the galvanometer contact p, Fig. 403a, moves over the bridge wire in obtaining a balance with S and S_1 in the two positions is often very small.

In order, therefore, that the same slide wire may serve for comparing coils of both high and low resistance it has been found advantageous to be able to shunt the bridge wire with a low-resistance shunt. This shunt is represented in dotted line in Fig. 403a and called σ.

The lower the resistance of this shunt the greater is the range of motion of p over the bridge wire. If the bridge wire is calibrated (that is the value of ρ determined) after being shunted, no error is introduced by such shunting, provided the resistances of the leads n and n_1 to the bridge wire do not alter after the calibration is made. This possible error is avoided by making these leads of heavy copper cable.

The bridge may be provided with three bridge wires of different resistances, any one of which may be used at will. Two shunts may be provided for the wire of lowest resistance, and thus there is altogether the equivalent of five different bridge wires. This arrangement adapts the apparatus to making direct comparisons with a wide range of resistances.

The bridge, as mechanically designed, often consists of two separate units.

One unit, which we may designate "the coil holder," consists of a hard-rubber base upon which are mounted massive copper bars for holding and connecting the ratio coils and the resistance standards, also the commutating device for interchanging the standard and the coil under comparison.

The other unit, which may be called the "bridge," consists of a hard-wood base upon which are fastened the three slide wires and three scales. A sliding contact maker is also a part of the "bridge."

The two units are joined together electrically by low-resistance cables.

The Carey-Foster method, as above described, is especially useful:

(1) For comparing with fundamental standards of resistance, coils of approximately the same resistance. For this purpose the bridge is adapted to comparing coils with standards of resistance.

(2) Coils may be compared with the standards when they differ in value from the standards very considerably, provided one is supplied with a resistance set of only very moderate accuracy. If the resistance coil has a value higher than the standard, it is shunted with the coils of a resistance box until the value of the

shunted combination is sufficiently near that of the standard to enable an exact balance to be obtained with one of the bridge wires. If the coil under comparison has a lower value than the standard, then the standard is shunted. The exact method of procedure will be explained later.

(3) A valuable application of the method is the obtaining of temperature coefficients of resistance coils or specimens of wire of any kind, and exceedingly accurate results can be obtained.

(4) The method is very useful in adjusting a number of resistance spools to the same value. For this a bridge wire is used, of the same kind and size as the wire of the resistance spools. A single reading will then give at once without any calculation the exact length of wire that must be cut off to make the resistance of the spool equal to that of the standard.

(5) A Carey-Foster bridge, together with a few reliable standard resistances, enables one to check up the coils of a Wheatstone bridge or standard resistance box with great accuracy.

(6) Very low resistances, such as the contact resistance of plugs in a resistance box or the resistance of lead wires, are conveniently measured with the Carey-Foster bridge. For such cases a thick metal bar of known resistance may occupy the place of a standard resistance coil.

When one wishes to compare with a standard resistance another resistance differing from it considerably, we may do so by shunting the standard resistance, if it has the higher value or the resistance to be compared, if it has the higher value. The procedure is then the same as in other cases.

Suppose first that the resistance under comparison is greater than the standard. Then if we shunt this resistance with a known resistance σ we have

$$\frac{S_1\sigma}{S_1+\sigma} = S - \rho D,$$

where $$D = a_2 - a_2',$$

or we obtain $$S_1 = \frac{\sigma\,(S - \rho D)}{\sigma - S + \rho D}. \tag{13}$$

If, second, the resistance is less than the standard, the standard may be shunted with a resistance σ and we have

$$S_1 = \frac{S\sigma}{S+\sigma} - \rho D. \tag{14}$$

The value of σ does not need to be known with great accuracy if its value is considerably greater than the resistance which it shunts. Thus if $\sigma = 100\,S$, and is in error one-tenth of one per cent, the calculated value of S will be in error only about one-hundredth of one per cent.

It may happen that we wish to compare a coil of, say, nominally 10,000 ohms resistance with a standard coil of that resistance. In general the coil to be compared will differ so much from the standard that a balance cannot be obtained with the slider on the bridge wire. In this case a variable resistance may be inserted in series with the coil of lower resistance and adjusted until a balance is made possible with the slider on the bridge wire. This added resistance, if not known, can then be measured and its value added or subtracted as may be required from the coil being compared.

Again it may be required to measure a resistance which is lower than the total length of one of the two bridge wires. Such a case would be where we wish to obtain the value of the lead wires connecting the mercury cups of the bridge to a coil in a resistance box. This case is easily met by connecting together one set of mercury cups by a copper bar of negligible resistance, or a bar of resistance metal of negligible temperature coefficient and known resistance.

Such a bar, then, takes the place of a standard coil, serving, in fact, as a standard of zero or very low resistance. If R_0 is the resistance of this short-circuiting rod, and R_m the low resistance being measured we have the same relation as that expressed by Eq. (6), namely,

$$R_m = R_0 - \rho\,(a_2 - a_2').$$

Following are a few specimen readings which will serve to illustrate a satisfactory method of procedure. Some readings are also given below which were taken in calibrating one of the bridge wires. The numerical calculation of the final result is also given.

a_1'	a_1	$a_1' - a_1$	Temp. st'd.	Temp. coil, a_1'	Temp. coil, a_1	Mean temp.	Res., st'd.	Res. of coil at observed temp.
512.0	510.5	+ 1.5	23.8	19.4	19.6	19.5	100.0068	100.0062
471.0	554.0	− 83.0	23.8	26.5	26.5	26.5	100.0068	100.0403
453.0	586.0	−133.0	24.6	30.4	30.2	30.3	100.0070	100.0607

Here column (1) gives the readings in millimeters of the length of scale to the left of the contact when an end A of the commutator

is towards the ratio coils. Column (2) gives the scale readings after reversing the commutator. Column (3) gives the difference between the two sets of readings, regard being had to the sign. Column (4) is the observed temperature in degs. C. of the standard. Column (5) is the observed temperature of the coil at the time of taking reading a_1'. Column (6) is the temperature of the same at the time of taking reading a_1. Column (7) is the mean of the two temperatures. Column (8) is the resistance of the standard corresponding to the observed temperatures in column (4), these resistances being calculated using the certified value and temperature coefficient of the standard, or more conveniently read from a curve plotted to show the relations between resistance and temperature of the standard. Column (9) gives the calculated resistances at the different mean temperatures of column (7) of the coil under comparison. When a sufficient number of readings are taken for different temperatures a very accurate curve showing the changes in resistance due to temperature changes may be plotted.

With the particular bridge wire used in obtaining the above results $\rho = 0.000404$ ohm per millimeter of wire, and thus, if S be the resistance of the standard given in column (8) for particular temperatures, the results in column (9) are obtained from the relation $S_1 = S - 0.000404\,(a_1' - a_1)$. See Eq. (7), par. 403. The actual readings and the resistances used for obtaining the above value of ρ are as follows:

10-ohm standard, No. 1592, called S_1. 10 ohms shunted with 1000 ohms, called S_1'.

$$S_1' = \cfrac{1}{\cfrac{1}{10.00206} + \cfrac{1}{1000}} = 9.90300,$$

$$S_1' - S_1 = 9.90300 - 10.00206 = -\,0.09906.$$

a_1	a_2	a_1'	a_2'	Temp. standard	Res., st'd.	Res. shunted.
526.5	497.0	649.0	374.0	30.6	10.00206	10.000
526.5	497.5	648.5	374.5	"	"	"
526.1	497.5	648.5	374.5	"	"	"
526.1	497.5	648.5	374.5	"	"	"
526.3	497.4	648.4	374.4	mean readings.		

$$\rho = \frac{S_1' - S_1}{a_2' + a_1 - a_1' - a_2} = \frac{-\,0.09906}{374.4 + 526.3 - 648.4 - 497.4},$$

or

$$\rho = \frac{-0.09906}{-245.1} = 0.000404.$$

404. Galvanometer Resistance. Measured, Using the "Second Property" of the Bridge.—This method, called Kelvin's method, offers a good example of the employment of the "second property" of the bridge. The method would be employed where only one galvanometer, the one whose resistance is to be determined, is available. The method is equally applicable for determining the resistance of a millivoltmeter.

Make the connections as shown in Fig. 404.

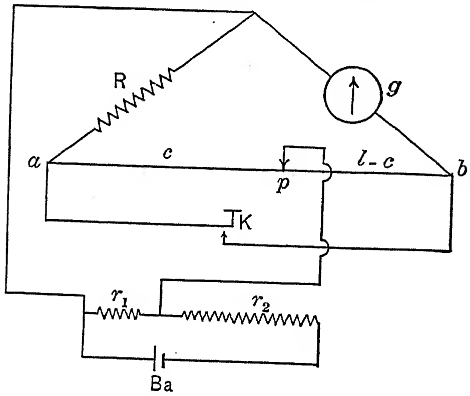

FIG. 404.

The resistance R should be chosen about equal to the resistance g of the galvanometer. The E.M.F. of the battery Ba will generally be too great to be applied directly to the bridge, as the galvanometer will deflect off its scale. The E.M.F. may be reduced to $\frac{r_1}{r_1 + r_2}$ of its value in the manner indicated in Fig. 404. It should be so adjusted that the deflection of the galvanometer (or other deflection instrument having a resistance g to be determined) remains upon the scale at all times. K is a key. It will be found that the deflection of the galvanometer is varied when the sliding contact p is moved along the slide wire ab and the deflection will also vary when the key is closed, except for *one* position of the slider p. The measurement is made by finding this position of p where the deflection of the galvanometer remains unaltered when the key K is open or closed. When this position

is found the battery circuit and the key circuit are conjugates, and then by the "second property" of the Wheatstone bridge (§ 400) we have

$$\frac{R}{g} = \frac{c}{l - c}, \quad \text{or} \quad g = \frac{l - c}{c} R, \tag{1}$$

when l is the length of the bridge wire and c is the distance from end a. This method, due to Lord Kelvin, gives very good results when correctly applied. The source of E.M.F. applied to the bridge and the key may be interchanged when the same results follow.

A trial was made of the above method in measuring the resistance of a Weston millivoltmeter. The connections were made as in Fig. 404. A slide-wire meter bridge was used, the resistance of the slide wire being 0.1397 ohm per centimeter (a much higher resistance than is usual in this type of bridge).

Ba was a storage cell. R was selected 10 ohms. The value found for the resistance corresponding to the 200-millivolt scale was

$$g = \frac{l - c}{c} R = \frac{100 - 49.1}{49.1} 10 = 10.366 + \text{ohms.}$$

A second trial was made in which R was again chosen 10 ohms, but use was made of extension coils each of 100 ohms. These were inserted at the ends of the bridge wire, as shown in Fig. 401a above. Remembering that the resistance of the bridge wire had been found to be 0.1397 ohm per centimeter the result obtained becomes

$$g = \frac{n + l_1 - c_1}{n + c_1} R = \frac{100 + (100 - 36) \times 0.1397}{100 + 36 \times 0.1397} 10 = 10.372 \text{ ohms.}$$

Here l_1 = the total length of the bridge wire in centimeters multiplied by the resistance of the wire per centimeter, and c_1 = the length in centimeters from left end of wire of sliding contact multiplied by its resistance per centimeter. This last result differs from the first by about 0.06 per cent which shows that both methods are fairly precise.

The method is too insensitive, as applied above, for measuring the high resistance of a voltmeter.

405. Calibration of Bridge Wire. — When a slide-wire bridge is used and high precision is required in resistance measurements it is necessary to calibrate the slide wire for uniformity of resistance. A manganin wire, if carefully selected and handled, will

uniform in resistance. Nevertheless this fact should be ...d, and, if it is found not to be uniform, corrections should ...d in as simple a manner as possible. We may make the ...n and obtain an expression which will give the values of ...tions to apply as follows:

. 405a, ρ, ρ, etc., represent ten or more resistance coils ... exactly alike in resistance. They may have, conven-

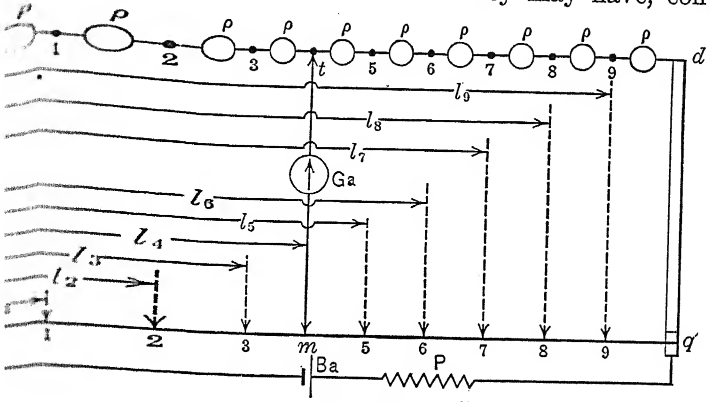

FIG. 405a.

...sistance of about 100 ohms each. The absolute resist-...ese coils does not need to be known and it is easy to ...umber of such coils to be equal in resistance with the ...heatstone bridge of very simple construction. One of ...al wires of each spool may be shortened or lengthened ...ntil each coil matches some one taken as a standard. ...ols are wound with manganin wire the difficulty of ...g the temperature sufficiently constant while the adjust-...being made is not great.

...minals of these coils, n in number and joined in series, ...ted by means of heavy leads cp and dq to the terminals ...ge wire to be calibrated. It is assumed that this wire ...ded into divisions of known length by means of a scale lying underneath, the wire. A cell of battery, with ...ance P in series, is joined to the ends of the bridge points p and q (not at c and d). A galvanometer has ...al connected to a traveling contact and the other is arranged so that connection may be made at the ...nts between the coils, as 1, 2, 3, etc.

Then, if the contact t is at point 1 a balance of the galvanometer will be obtained when the sliding contact is at a distance l_1 from the end of the wire, or, in general, when the contact t is at any point m' between the coils, a balance will be obtained when the sliding contact is at a distance l_m from the end p of the wire. As the coils ρ, ρ, etc., are all of equal resistance it is evident that the wire becomes in this way divided up into n lengths of equal resistance. If the wire is not uniform in resistance these n lengths will not be equal.

Let R = the total resistance of the wire from p to q and
L = the total length of the wire,

then $\frac{m}{n}R$ will be $\frac{m}{n}$ of the total resistance.

If we call W_1 the resistance of $\frac{1}{n}$ of the length of the wire or, in general, W_m the resistance of $\frac{m}{n}$ of the length of the wire, we shall have

$$W_m = \frac{m}{n}R + \delta r_m, \quad \text{or} \quad W_m = \frac{m}{n}R - \delta r_m,$$

according as $W_m >$ or $< \frac{m}{n}R$. Choosing the first case

$$\delta r_m = W_m - \frac{m}{n}R. \tag{1}$$

Here δr_m is the small difference in resistance between $\frac{m}{n}$ of the length of the wire and $\frac{m}{n}$ of the total resistance of the wire.

Again, let l_m = the distance from end p at which a balance is obtained when the terminal t is between the mth coil and the $(m + 1)$th coil. Then

$$l_m = \frac{m}{n}L + \delta l_m, \quad \text{or} \quad l_m = \frac{m}{n}L - \delta l_m.$$

Choosing the first case

$$\delta l_m = l_m - \frac{m}{n}L. \tag{2}$$

Here δl_m expresses the difference in the lengths (at points 1, 2, 3, etc. . . . n) where a balance comes on the actual wire and where it would come if the wire were uniform.

It is also evident that within an error of the second order which need not be considered,

$$\delta r_m = \frac{\delta l_m}{L} R. \tag{3}$$

The results expressed in Eq. (2) should be plotted in a curve as follows:

Divide the axis of ordinates into distances l_1, l_2, l_3, . . . l_n, corresponding to the various positions found upon the wire where a balance was obtained. At each of these points raise ordinates

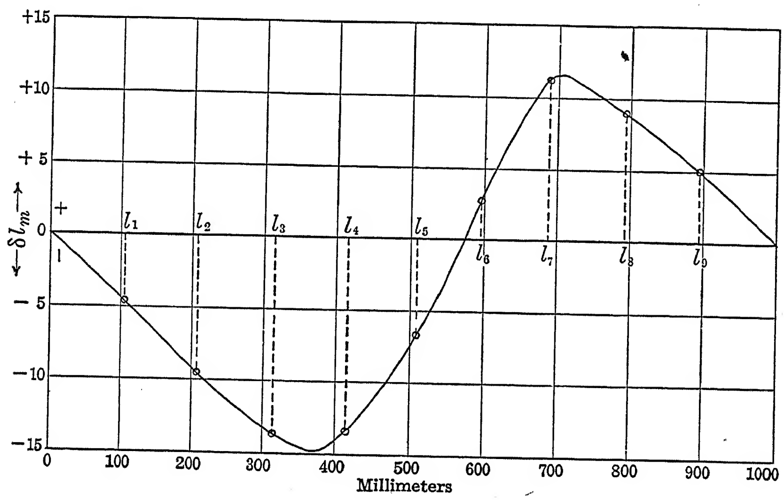

FIG. 405b.

equal to δl_1, δl_2, δl_3, etc., to $\delta l_{(n-1)}$. The values of δl_m may have different signs and some of the ordinates may extend below the axis. Thru the ends of the ordinates draw a smooth curve. From this curve values which lie between the determined values of δl_m may be taken.

Fig. 405b shows such a curve drawn from actual observations made upon a manganin wire 1 meter long. This wire had been scraped in its middle portion for about one third of its length in order to exaggerate its inequality.

We can now find an exact expression for the value of any unknown resistance when measured with a slide-wire bridge in which the calibrated wire is used.

In measuring the resistance X with the slide-wire bridge in the manner indicated in Fig. 405c, we have

$$X = \frac{W_m}{R - W_m} r, \tag{4}$$

where R is the total resistance of the wire and r the known resistance.

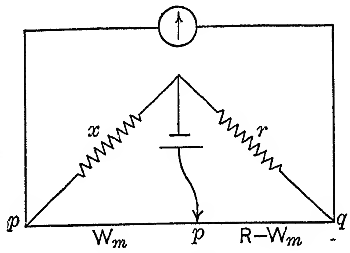

FIG. 405c.

From Eq. (1) $$W_m = \frac{m}{n} R + \delta r_m.$$

Hence $$X = \frac{mR + n\delta r_m}{nR - (mR + n\delta r_m)} r. \tag{5}$$

Placing in Eq. (5) the value of δr_m given in Eq. (3) we have

$$X = \frac{m + \frac{n}{L}\delta l_m}{n - (m + \frac{n}{L}\delta l_m)} r. \tag{6}$$

Since n and L occur as a ratio we can express them in any units we choose provided the same units are used for each. Take $L = 1000$, corresponding to 1000 millimeters for a slide wire 1 meter long, and take n equal 1000. Then

$$X = \frac{m + \delta l_m}{1000 - (m + \delta l_m)} r. \tag{7}$$

To use Eq. (7) proceed as follows:

Obtain a balance of the bridge by sliding the contact p along the wire. Note the distance in millimeters (or in divisions equal to $\frac{1}{1000}$ of the length of the wire, if this is not 1 meter long) from the left-hand end of the wire to the point of balance. This distance will be m. From the curve find the value of δl_m which corresponds to m scale divisions (to millimeters in the case of a

wire 1 meter long). Add this value to the reading m, and call the result a. Then

$$X = \frac{a}{1000 - a} r. \tag{8}$$

The quantity $\frac{a}{1000 - a}$ may be taken from the table (Appendix I, 1) as in other cases. If a calibration of a slide wire is made carefully in this way and used, a slide-wire bridge may become an accurate device for measuring resistance.

A trial was made of the application of this correction. The wire used was the same one for which the calibration curve is shown plotted in Fig. 405b. The known resistance r was given different values and a resistance was measured using the connections shown in Fig. 405c. The values obtained for X were calculated by Eq. (7) and are exhibited below.

r in ohms	m in millimeters	δl_m	$m + \delta l_m$	X, measured	X, true value	Per cent error
25	790.8	+ 8.95	799.75	99.84	100	− 0.16
100	507.1	− 7.00	500.10	100.04	100	+ 0.04
200	348.2	− 14.80	333.40	100.03	100	+ 0.03
300	261.5	− 11.70	249.80	99.89	100	− 0.11

The failure to get higher precision resulted from the crudeness of the apparatus. This consisted of a wire stretched upon a wooden meter stick with no provision for the sliding contact other than the blade of a knife held in the hand.

406. The "Kelvin-Varley Slides."—This is a device whereby the slide wire of a slide-wire bridge may be replaced with sets of resistance coils. The latter are so disposed that without an excessive number of coils the ratio $\frac{a}{1000 - a}$ may be obtained and a be varied in steps as small as desired. This device connected in a Wheatstone-bridge network is represented in Fig. 406. In the arrangement shown there are in row 1, 11 coils, each of resistance r. Spanning always two coils are the two contacts p_1, p_1, which may be moved together over the studs or blocks to which the coil terminals are joined. In row 2 there are also 11 coils, each of resistance $\frac{r}{5}$, and two traveling contacts p_2, p_2,

which always span two coils of this row. In row 3 there are 10 coils, each of resistance $\frac{r}{25}$ and one traveling contact p.

Now, it is evident, if the resistance from contact p_1 to p_1 of all below these contacts is equal to $2r$, that two coils of row 1 are always shunted with a resistance equal to these two coils. Thus the resistance from p_1 to p_1, when the contacts bear upon

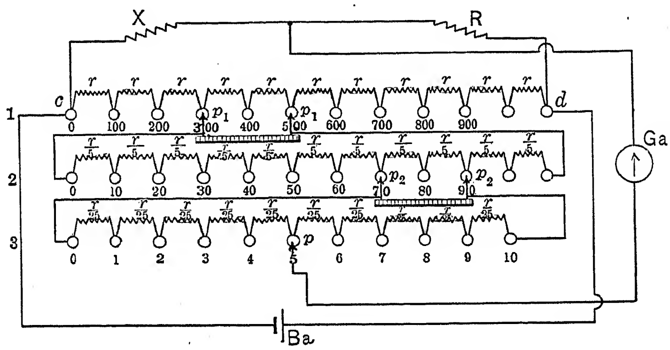

FIG. 406.

the studs, will be r. There will be thus in row 1 ten equal resistances, each of value r. Similarly, if the resistance from contact p_2 to p_2 of all below these contacts is $\frac{2r}{5}$, then there will be ten resistances in row 2, each equal to $\frac{r}{5}$. Lastly, in order that the total resistance of the 10 coils of row 3 shall equal $\frac{2r}{5}$, each of the 10 coils must have a resistance $\frac{r}{25}$. With this arrangement the total resistance from c to d will be $10r$ ohms. The value of the reading a will now be given by the positions occupied by the left-hand contacts. With the positions shown in the figure the reading is $a = 375$ and the ratio $\frac{a}{1000 - a}$ becomes

$$\frac{375}{1000 - 375} = 0.6000 \text{ (see table, Appendix I, 1).}$$

The same principle of subdivision may be indefinitely extended to read in steps as small as desired. The one represented in the

diagram reads to 1 part in 1000. The Kelvin-Varley slides is a device equivalent to a long slide wire. It may be given any accuracy of adjustment desired. The cost of the mechanical construction required and the many coils which must be adjusted have limited the use of this otherwise excellent arrangement.

CHAPTER V.

WHEATSTONE-BRIDGE METHODS. VARIABLE RHEOSTAT. ARRANGEMENTS OF RESISTANCES. PER CENT BRIDGE. SUGGESTIONS FOR USING BRIDGE.

500. Wheatstone-bridge Methods with Variable Rheostat. — To carry out these methods, arrangements are provided for setting the ratio arms to a chosen fixed ratio, and for varying the resistance in the rheostat arm until a balance is obtained.

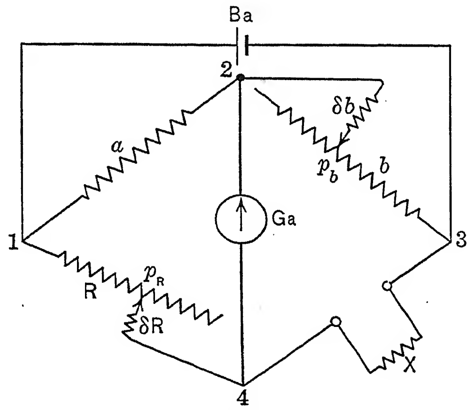

Fig. 500.

To change the setting of the ratio and to change the resistance in the rheostat involves, by all ordinary methods, contact resistances in at least two arms of the bridge. This is a possible source of error which will be better understood from a consideration of Fig. 500.

In this figure let a and b represent the two ratio arms, R the rheostat and X the resistance to be determined. To change the ratio a to b (in bridges of ordinary construction) it is necessary to change the value of at least one of these resistances. This may be accomplished by moving a point of contact, as p_b, to different positions along the arm 2 to 3. But there will be some contact resistance, however good the construction, at the point p_b.

Call this contact resistance δb. Likewise to change the value of the rheostat arm 1 to 4 will require with any type of construction, at least one movable contact, as p_R, which will have some contact resistance which we may call δR.

The equation of balance of the bridge, then, becomes

$$\frac{a}{b+\delta b} = \frac{R+\delta R}{X},$$

$$\text{or } X = \frac{(b+\delta b)(R+\delta R)}{a}. \tag{1}$$

If we expand the second member of Eq. (1) and neglect the product $\delta b\ \delta R$ we find

$$X = \frac{b}{a}R + \frac{b\delta R + R\delta b}{a}. \tag{2}$$

The last term of Eq. (2) represents a necessary error in estimating the value of X, which is due solely to contact resistances in two arms of the bridge. In the special type of construction described in par. 510, the contact resistance δb is done away with by shifting the galvanometer terminal to different positions along a or b, these being soldered together at joint 2. The error which still remains is the second term of the right-hand member of the relation

$$X = \frac{b}{a}R + \frac{b}{a}\delta R. \tag{3}$$

This source of error should be reduced to a minimum by a proper mechanical construction of the rheostat. To accomplish this and at the same time make a rheostat which may be conveniently varied in small steps thru a wide range has given opportunity for a wide variety in design. The resistance coils or units may be arranged in numerous ways and the mechanical construction for putting various resistance values in circuit may be greatly varied. Both brass blocks, to be connected by plugs, and brushes which can be moved over studs arranged in the arc of a circle, called a dial, are extensively used. We shall now describe the various methods of arranging the coils, but will omit a description of mechanical constructions.

501. Arrangements of Resistances in Wheatstone-bridge Rheostats. — This subject was discussed by the author in an article published in the *Electrical Review*, June and July, 1903,

and much of what follows under this heading and the next is take from that publication:

The fundamental purpose of a Wheatstone-bridge rheostat, whether employing plugs and blocks or dials and sliding contacts, is to provide means of obtaining the largest possible number of values from the fewest possible number of accurately adjusted resistance units and to do this without introducing into the circuit objectionable contact resistances.

Where the number of coils is made greater than the least number required theoretically, it is done to give some convenience of working or to increase the ease or simplicity with which the values are added up.

The theoretical combinations of resistance coils which are possible are given by the following relations: If there are n coils these can be joined in various combinations, in series, or in parallel, or in parallel and series or again in mixed arrangements. The total number of combinations that can be formed, using the coils singly and by joining them in series combinations, is

$$T = 2^n - 1. \tag{1}$$

The same total number of combinations can be formed if the coils are taken singly and joined in all parallel combinations, but since the coils used singly are the same for both arrangements, we have as the total number of combinations possible for n coils used singly and joined in series and in parallel combinations

$$N = 2^n - 1 - n + 2^n - 1 \quad \text{or} \quad N = 2^{n+1} - (n + 2). \tag{2}$$

We will not, in general, by combining coils in all these ways, obtain as many different values of resistance as there are combinations of coils, for many of the combinations of coils give the same resistance values even tho the resistance of each coil be different. Thus, the much-used set of coils, of 1, 2, 3, 4 ohms, respectively, can be joined in series in $T = 2^4 - 1 = 15$ different ways, but in 5 cases the same values of resistance are repeated, giving but 10 different values of resistance for this series of coils. The total number of different values of resistance which can be obtained from a given number of coils becomes in any particular case a complex problem, if the number of coils be large.

If coils are arranged in all possible ways, we can get from 2 coils 4 arrangements, from 3 coils 17 arrangements and from 4 coils 106 arrangements. Thus it is often possible, when only a few

standards of resistance are at hand, to obtain, nevertheless, a large variety of values.

502. Rheostat Coils; Classical Arrangements. — To arrange resistances in combinations so a series of values can be quickly and easily added up has led to the employment of at least five methods, tho only three of these are now in common use.

Siemens' Plan. — The first plan is due to Siemens and consists in joining a series of coils between blocks, the coils having the values 1, 2, 4, 8, 16, 32, etc.

Resistance is thrown in circuit by removing plugs from between the blocks. This plan employs the smallest possible number of coils for attaining a given range of values, but, as the summing up of the values is not an easy matter, this method is now obsolete.

The 1, 2, 3, 4 Plan. — By this plan all values from 1 to 10 ohms, in steps of 1 ohm, are obtained by the coils 1, 2, 3, 4; all values from 10 to 100 ohms, in steps of 10 ohms, by the coils 10, 20, 30, 40; all values from 100 to 1000 ohms in steps of 100 ohms, by the coils 100, 200, 300, 400. By this arrangement of coils there must be as many plugs as coils, the required values being obtained by withdrawing plugs. As many plugs will be withdrawn as there are coils in the circuit.

The 1, 2, 2, 5 Plan. — This arrangement of coils is used in precisely the same way as the one above. Most resistance boxes in which plugs are removed to throw resistance in the circuit are based on one or other of these two plans.

The 1, 1, 3, 5 Plan. — 1, 1, 3, 5; 10, 10, 30, 50; etc., will give ten consecutive values in each unit's place, like the two above. This arrangement, so far as the author knows, has not been used in any resistance boxes placed upon the market.

There are no other four numbers which add up to ten which will give ten consecutive values.

The Decade Plan. — In this arrangement, as ordinarily applied, there are 9 or 10 one-ohm coils for the units' place, 9 or 10 ten-ohm coils for the tens' place, 9 or 10 one-hundred-ohm coils for the hundreds' place, and so on. Each series of coils of the same value is designated a decade. The connections as usually made are as shown in Fig. 502.

It is apparent from this diagram that any value in any one decade can be obtained by *inserting* between a bar and a block

one, and only one, plug. It also appears that if several decades are in series any value up to the limit of the set can be read off directly from the position of the plugs, without any addition whatever.

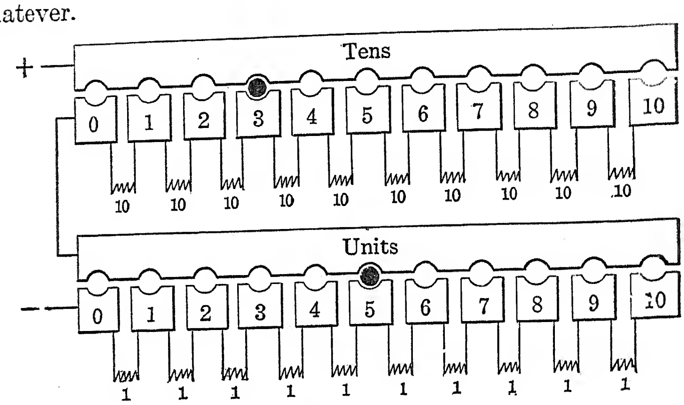

FIG. 502.

It is evident that in the first four plans the values of resistance required are obtained by *withdrawing* plugs which must be laid aside, with liability of being lost, and one must make sure that *all* the remaining plugs are well seated so as not to introduce unknown contact resistances. Furthermore, as many plugs must be employed as there are resistance units and to obtain any given value may require the manipulation of a large number of plugs. In the decade plan, upon the other hand, there is but one plug used to a decade and this is always in service and hence not readily mislaid. The use of only one plug to the decade makes it easy to ascertain that this is tightly fitted in its place. When the value is finally obtained, by manipulating only the one plug to the decade, this value is readily read off without any mental summing up of values. Again the decade plan alone permits of obtaining a succession of values by means of sliding contacts or dial switches, a method which is becoming deservedly more appreciated.

These and other obvious advantages of the decade plan are in part offset by the necessity, if the decades are arranged as in Fig. 502, of using a larger number of resistance units than is required by the previous plans.

503. Northrup's Four-coil Arrangement. — An arrangement of coils has been devised by the author which secures to the decade

plan the further advantage of requiring no more coils than the 1, 2, 3, 4 or the 1, 2, 2, 5 plans. This arrangement may be explained as follows: Let the terminals of the 1 ohm and 2 ohm coils, and the points of union of the other coils be numbered (1), (2), (3), (4), (5), as shown in Fig. 503a. The current enters at point (1) and leaves the coils at point (5), traversing 1, 3′, 3, 2 = 9 ohms in all. If this series is multiplied by any factor n, then $n\,(1 + 3' + 3 + 2) = n\,9$ ohms. It will be seen that if the

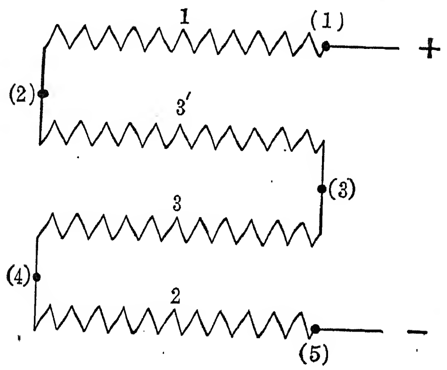

FIG. 503a.

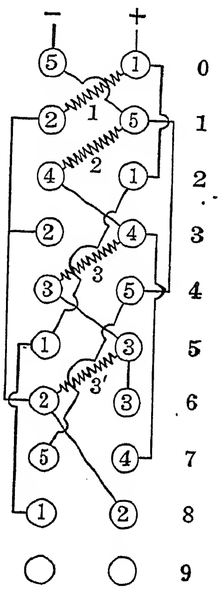

FIG. 503b.

points (1) and (5) are connected, all the coils are short-circuited, and the current will traverse zero resistance. If the points (2) and (5) are connected, the 3′, 3 and 2 ohm coils will be short-circuited and the current will traverse 1 ohm. By extending the process so that we connect two, and only two, points at a time, it is possible to obtain the regular succession of values n (0, 1, 2, 3, 4, 5, 6, 7, 8, 9), the last value being obtained when no points are connected. The following table shows the points which must be connected to obtain each of the above values and the coils which will be in circuit for giving each value:

Value	Points connected	Coils used
0	(5–1)	0
1	(2–5)	1
2	(4–1)	2
3	(2–4)	1, 2
4	(3–5)	1, 3′
5	(1–3)	3, 2
6	(2–3)	1, 3, 2
7	(5–4)	1, 3, 3′
8	(1–2)	3′, 3, 2
9	(0)	1, 3′, 3, 2

Fig. 503b shows the method of connecting these points two at a time, with the use of a *single* plug.

The circles in the diagram represent two rows of ten brass blocks each. To the first two blocks at the top of the rows, the points 5 and 1 are connected, to the second two the points 2 and 5 are connected, and so on, no points being connected at the last pair of blocks. It is evident that if a plug be inserted between the blocks 1 and 5, the points 1 and 5 are connected, giving the value 0; if between the blocks 2 and 5, the points 2 and 5 are connected, giving the value 1, and so on. The value 9 is obtained when the plug is disposed of by being inserted in the last pair of blocks which have no connections.

The only combinations of *four* coils which will give the decade in the above manner are the coils n (1, 3, 2′, 2), n (1, 1′, 4, 3) and n (1, 3, 3′, 2) where n may have any value.

The above method of obtaining the decade with only four coils, as well as the ordinary decade arrangement, can be applied to a dial or sliding brush construction. To accomplish this, 20 studs, or ten pairs, are arranged in a circle. To make the required connections, the pairs of studs at opposite ends of a diameter of the circle must be successively joined together. This can be done by rotating, with a handle at the center of the circle, a single connecting bar or brush which will join successively pairs of studs to give the values 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The 20 studs are fastened to the top surface of a hard rubber plate and there extends down from each stud thru the plate a shaft slightly longer than a resistance-unit or spool. On four of these shafts (beneath the rubber plate) four resistance spools are mounted — the resistance values of these spools for a units' dial being 1, 2, 3, 3′ ohms; for a

tens' dial 10, 20, 30, 30′ ohms; for a hundreds' dial 100, 200, 300, 300′ ohms, etc. The ends of the wire of the spools are joined, one to the end of the shaft on which a spool is mounted, and one to the end of an adjacent shaft. The ends of other shafts are cross-connected in the manner shown in Fig. 503c. This gives a view of the connections as seen from underneath the

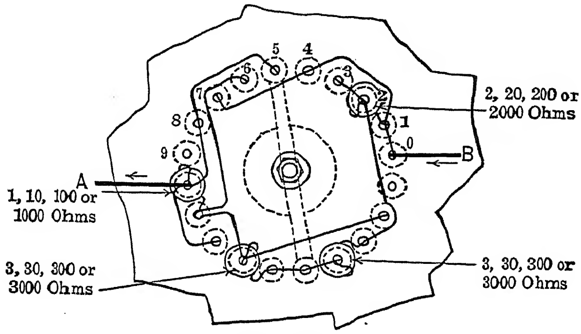

FIG. 503c.

rubber plate. The circles in full line represent ends of resistance spools. The circles in dotted line represent the brass studs upon the upper side of the rubber plate. The larger circle in dotted line represents a handle at the center of the circle of studs located above the upper side of the plate. The bar or brush shown in dotted line connects one pair of studs after another as the handle is turned. The resistance value between the points A and B with the bar in the position shown in Fig. 503c is 5 ohms.

Among other advantages of this method of arranging four resistance units to give the decade in dial form may be mentioned the following: The traveling bar or brush contact can be rotated continuously in either direction, as there are no electric connections made to it. Hence, one can pass directly from the value 0 to 9 by turning back one stud. The construction is very economical and there are only four coils in place of nine to put and keep in accurate adjustment. It is easy to construct the traveling bar of a number of thin copper leaves, which make a right angle turn at each end, so as to give an end bearing of many copper leaves upon the faces of the brass studs, thus securing a certain and low-resistance contact with the studs. The disadvantages are slight, one being that 20 instead of 10 studs, as in

the ordinary arrangement of coils, are required. Also, at those times where the contact, in passing from one stud to the next, joins both together, the resistance value is not that of the stud touched which reads the lower value, but is some odd value of resistance. Hence, in using the dials the battery or galvanometer key should be open while the dial is being turned. This objection has no weight in Wheatstone-bridge work, but it has some weight in certain classes of work where it is desirable to follow rapidly changing resistances with the battery and galvanometer keys closed.

On the whole this type of dial construction, used in an ordinary Wheatstone bridge, is economical, accurate, and highly satisfactory in service.

504. **Five-coil Combinations.** — The author pointed out * that by using five coils the decade, 0 to 9 inclusive, can be obtained in a manner similar to that used for obtaining the decade with four coils, as described above, from the values,

$$n\,(1, 1, 2, 2, 3),$$
$$n\,(1, 2, 2, 2, 2),$$
and
$$n\,(1, 1, 1, 1, 5).$$

Also that eleven values, namely 0 to 10 inclusive, may be obtained from the following arrangements of five coils:

$$n\,(1, 2, 3, 2, 2),$$
$$n\,(1, 5, 1, 1, 2),$$
$$n\,(1, 3, 1, 3, 2),$$
$$n\,(1, 1, 1, 3, 4),$$
$$n\,(1, 1, 1, 4, 3),$$
$$n\,(1, 1, 4, 1, 3),$$
$$n\,(1, 3, 1, 3, 2),$$
$$n\,(1, 2, 1, 4, 2).$$

It was further pointed out that by traveling a single contact, in the manner used with the four-coil arrangement, the general method may be indefinitely extended. Any number of successive values may be obtained with the use of many less coils than the values obtainable. Thus from the seven coils,

$$n\,(1, 1, 3, 1, 3, 3, 2),$$

we can get fifteen consecutive values inclusive of 0. It was also

* *Electrical Review*, July 18, 1903, Vol. 43, page 75.

pointed out that in the above methods a sliding contact or a dial switch may be used in place of a plug for making the connections.

505. Decade System of Feussner. — In Fig. 505 is shown a

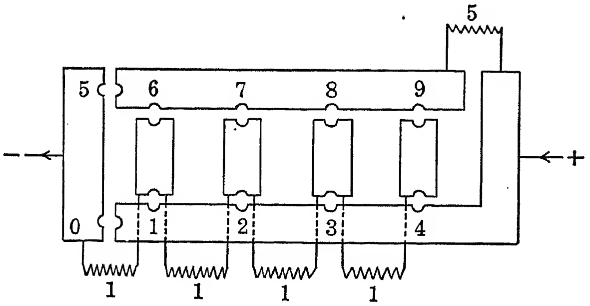

FIG. 505

disposition (credited to M. Feussner) of blocks and resistance units, whereby the values 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 may be obtained by using five coils and only one traveling plug. The resistance units used are n (1, 1, 1, 1, 5) where n has values 1, 10, 100, 1000, etc. An examination of the figure makes the system easy to understand. The four-coil decade being still better, this method is not likely to be much used in this country.

506. Decade System of Irving Smith. — In Fig. 506 is shown a disposition of 5 coils consisting of n (1, 2, 2, 2, 2) where n has values 1, 10, 100, 1000, etc., whereby the decade may be obtained by traveling a single plug. The system is easily understood from an examination of the figure.

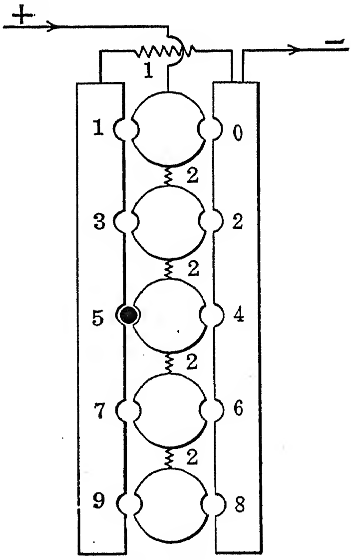

FIG. 506.

Mr. Smith has used this system with a dial construction to which it is well adapted.

507. Multiple Arrangements. — If it be required to obtain a very low contact resistance of the plugs, to enable the last decade

row to be varied with some precision in steps of 0.01 ohm or even 0.001 ohm, then a multiple arrangement of coils may be employed with advantage. The principle whereby regularly increasing values of resistance are obtained by joining coils in multiple is given in Fig. 507.

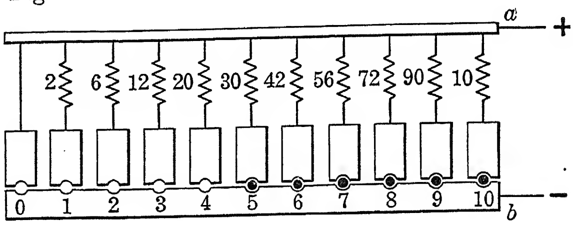

FIG. 507.

It may easily be shown that when ten coils have values n (2, 6, 12, 20, 30, 42, 56, 72, 90, 10) ohms, where n has any value, their resistance when all are joined in multiple is n ohms. Also the nine coils n (6, 12, 20, 30, 42, 56, 72, 90, 10) have the resistance $2n$ ohms when joined in multiple, the eight coils n (12, 20, 30, 42, 56, 72, 90, 10) the resistance $3n$ ohms, the seven coils n (20, 30, 42, 56, 72, 90, 10) the resistance $4n$ ohms, etc., to n (10) which has the resistance $10n$ ohms. Thus, with ten coils arranged as in the figure and having the above values, all values in steps of n (1) ohms from 0 to $10n$ ohms may be obtained. Thus, referring to the figure, if the plugs 0, 1, 2, 3, . . . 10 are all in, the resistance from a to b is 0 ohm, if 1, 2, 3, . . . 10 are all in, the resistance is 1 ohm. If plugs 0, 1, 2 are removed, the resistance is 3 ohms, etc., that is, the resistance obtained between a and b will always be that which is stamped opposite the last plug toward the left, which is not removed. Since the current passes thru all the plugs not removed, in parallel, the contact resistance is greatly reduced and this contact resistance decreases as the resistance value plugged decreases.

This method is an excellent arrangement when it is required to obtain ten regularly ascending values of very low resistance, as, for example, 0.001, 0.002, 0.003 . . . 0.01 ohm. Greater precision of adjustment can be gotten and maintained by this parallel arrangement of coils, of relatively high resistances, than from coils arranged in series.

If the entire rheostat is based upon this principle and has several

decades it would only be required to have the values run from 0 to 9 n ohms in the decades, except the one of lowest denomination. For obtaining the succession of values from 0 to 9 n ohms the coils so joined in parallel are n (2, 6, 12, 20, 30, 42, 56, 72, 9) ohms. This decade plan is deserving of more attention than it has received, for excellent results may be obtained for accurately varying resistance in steps as low as 0.001 ohm.

The above described selections and dispositions of resistance units for Wheatstone-bridge rheostats comprehend those known which are of interest. We shall describe now the methods in use for arranging the resistances in the ratio arms of a bridge.

508. Arrangements of Resistances for the Ratio Arms of Wheatstone Bridges. — The most simple disposition of ratio

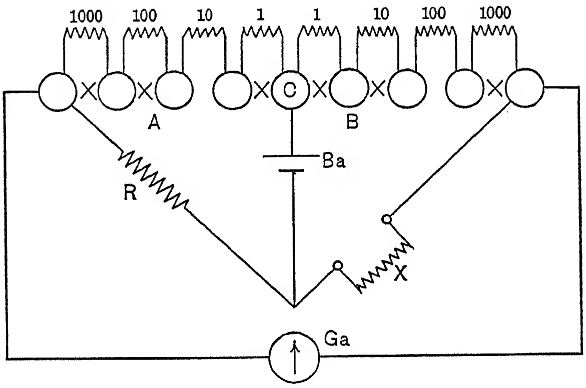

FIG. 508a.

coils, and one which is found in most inexpensive Wheatstone-bridge boxes, made for laboratories and, the use of students, is shown in Fig. 508a. The required ratio, as 10 to 100, is obtained by withdrawing a plug from each of the arms A and B. The contact resistance of the remaining plugs in each arm enters into the ratio and there is no provision made for reversing the two arms.

In Fig. 508b is shown how connections and plugs may be disposed so that the ratio arms may be reversed.

This is the classical arrangement which is used in the so-called "English Post Office" bridge. By changing two plugs from holes 1, 2, to holes 3, 4, the positive pole of the battery is joined to the end of A and the negative pole to the end of B or vice

versa. By so reversing the ratio arms and twice balancing the bridge and then taking the mean value of X to be the true value, any errors of adjustment in the ratio coils is eliminated *when unity ratio is used.* However, it is always desirable so to choose the values in the ratio arms that a large portion of the rheostat will be required for getting a balance, and it is only when the ratio is unity that this can be done with the ratio coils used both direct and reversed. This requirement limits the usefulness of reversible ratio arms to those cases where unity ratio may be used. Thus, suppose the capacity of the rheostat is 10,000 ohms, and may be varied in steps of 1 ohm. To set the bridge to an accuracy of 0.1 of 1 per cent would require that at least 1000 ohms

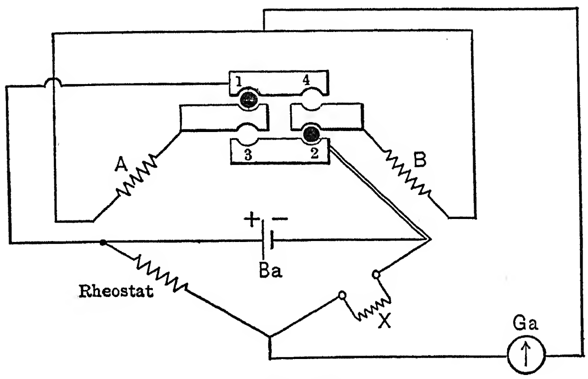

FIG. 508b.

of the rheostat be brought into service. If a resistance of 900 ohms is to be measured, a ratio of 10 to 1, with the ratio arms direct, would utilize 9000 ohms of the rheostat, but when the ratio arms are reversed only 90 ohms of the rheostat could be utilized, which would necessitate a setting that might be less accurate than 0.5 of 1 per cent. In such a case one would either have to abandon the advantage of reversible ratio arms and set the ratio 10 to 1, or use unity ratio and reverse the ratio arms, in which case the rheostat setting would be 900 ohms, which, being variable in steps of 1 ohm, would permit settings to 1 part in 900. The advantage, then, of reversible ratio arms is chiefly confined to the measurement of resistances of the same order of magnitude but smaller than the total resistance of the rheostat. When the instrument maker can be trusted to accurately adjust the ratio

coils in a Wheatstone bridge the reversible feature is scarcely worth its extra cost as applied to the "Post Office" type of bridge.

509. Schöne's Arrangement of Ratio Arms. — This very superior disposition of ratio coils was described by O. Schöne, in "Zeitschrift für Instrumentenkunde," May, 1898. It is now extensively used in America and by its superiority deserves to supersede all other arrangements of resistances for reversible ratio arms.

According to this arrangement all the ratio coils have one of their terminals joined to a common bar connector which corresponds to the block marked C of Fig. 508a. The other terminal of each coil is joined to a separate block. The scheme is given in Fig. 509.

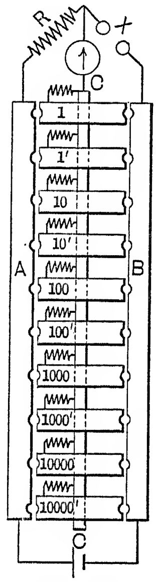

FIG. 509.

The bar A on one side of these blocks is joined to the rheostat R, and the bar B, on the other side, to an X post.

In the ordinary use of this arrangement two plugs only are used. One plug is inserted between the bar A and one of the blocks 1, 1′, 10, 10′, etc., of the central row, and the other plug is inserted between the bar B and any one of the blocks of the central row, except the one which the other plug joins to bar A.

The construction generally embodies two ratio coils of each value. Referring to Fig. 509, if one wishes to obtain a unity ratio, as 1000 to 1000′, one plug would be inserted between the block 1000 and the bar A, and the other plug between the block 1000′ and the bar B. This disposition of the plugs joins the end of the 1000 ohm coil to the rheostat and the end of the 1000′ ohm coil to the X post. If, now, one plug is inserted between the 1000′ block and bar A, and another plug between the 1000 block and bar B, the ratio arms become reversed; that is, the 1000′ ohm coil is joined to the rheostat, and the 1000 ohm coil to the X post.

When uneven ratios are used the same ratio can be obtained by four different combinations. If we wish to obtain the ratio 1 to 10, we can plug between A and 1 and B and 10 and get 1 to 10, or between A and 1′ and B and 10 and get 1′ to 10, or between

A and 1 and B and 10′ and get 1 to 10′, or between A and 1′ and B and 10′ and get 1′ to 10′.

To obtain the reciprocal set of ratios, like the above, we would plug A and 10, B and 1, and get 10 to 1; A and 10′, B and 1, and get 10′ to 1; A and 10, B and 1′, and get 10 to 1′; A and 10′, B and 1′, and get 10′ to 1′.

By using more than two plugs and connecting certain of the coils in parallel combinations, a large number of other ratios may be obtained. For example, we can plug between A and 100 and A and 100′, and between B and 1000 and get the ratio 50 to 1000, or we can plug between A and 1000 and A and 1000′ and between B and 100 and get the ratio 500 to 100.

With this arrangement of ratio coils it is seen that errors due to plug contacts become practically nil, because only two plug contacts enter the circuit, while with even ratios it is only the *difference* in the resistance of the two plug contacts which affects the results.

510. Nonreversible Ratio Arms Adjustable without Contact Resistances. — A very excellent arrangement of variable, but

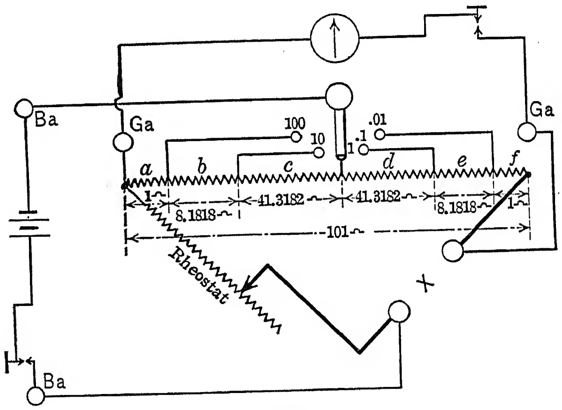

Fig. 510.

nonreversible ratio arms, which involves no contact resistances in the ratio arms, is shown in Fig. 510. The ratio values may be varied by moving a brush contact, which is joined to the battery, over studs as indicated in the figure.

To calculate the odd values required for ratio coils the solution must be found for equations of the form,

$$\frac{a}{b+c+d+e+f} = \frac{1}{100},$$
$$\frac{a+b}{c+d+e+f} = \frac{1}{10},$$
$$\frac{a+b+c}{d+e+f} = 1,$$
$$\frac{a+b+c+d}{e+f} = 10,$$
$$\frac{a+b+c+d+e}{f} = 100.$$

In the case selected we have five equations and six unknowns, hence some one of the quantities, as a, must be assumed as known. If we choose $a = 1$ ohm, then the solution of the above equations gives

$$a=1, \quad b=8.1818, \quad c=41.3182, \quad d=41.3182, \quad e=8.1818, \quad f=1.$$

We note that $a = f$, $b = e$, and $c = d$, and therefore there are but two odd values of resistance to adjust to give the five different ratio settings, 100, 10, 1, 0.1, 0.01.

The method may be indefinitely extended and is an excellent arrangement to use with bridges in which the rheostat values are varied by means of dials and sliding-brush contacts, for then *all* resistance changes in the box can be effected with dials and no plugs are required. By this arrangement the ratio arms, being free from contact resistances, give ratios just as accurately as the coils are adjusted.

511. Wheatstone Bridge Arranged for Reading in Per Cent. — It frequently happens that the problem of very rapidly measuring a large number of resistance units of even values as 1, 10, 100 ohms, etc., is presented. Instrument makers have this to do in checking up the precision of the coils in resistance boxes. In such cases it is of little interest to know the absolute number of ohms by which any coil is in error, the important question being what is the *per cent* accuracy of any coil.

To meet the above requirements of a Wheatstone bridge, the author devised the method and connections given below in Fig. 511. Bridges were constructed, embodying the connections of Fig. 511 and placed in continuous service, which would give by a direct reading the per cent value of any coil being measured in terms of the standard employed. The readings of the last of the four

dials used were in steps of 0.001 of 1 per cent, and the range of the bridge was from 95 per cent to 106 per cent of the standard.

As a method may be applied for eliminating the lead resistances, the per cent bridge may be used advantageously for resistances from 1 ohm up, with errors not exceeding 0.001 of 1 per cent.

The diagram, Fig. 511, is practically self-explanatory.

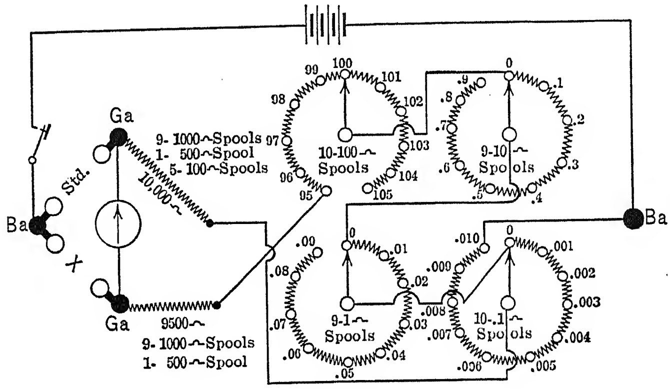

Fig. 511.

To use this method, one first connects the lead wires together, which go to the coil to be measured. All the bridge dials are set so as to read an even 100 per cent. Then the lead wires which go to the standard resistance are also joined together at the ends which connect to the standard, and they are varied in length until the galvanometer shows a balance. This means that the lead resistances to the X coil are equal to the lead resistances to the standard coil, and as the X coil is never very greatly different in value from the standard, the lead resistances eliminate.

The bridge requires for its practical use a complete set of resistance standards, against which to match the resistance coils to be measured.

This form of the Wheatstone bridge is of great value to the instrument maker who has many coils to measure with both rapidity and precision.

512. Remarks upon the Use of the Wheatstone Bridge. — In arranging to use a Wheatstone bridge with accuracy, speed, and convenience, one should select with care a suitable galvanometer or other detector for indicating when the bridge is balanced. In addition to its greater sensibility a galvanometer is more advantageous than detectors of the telephone type, in that the deflec-

tions are to the right or to the left, according as the rheostat adjustment is higher or lower than the setting required for a balance; whereas with the telephone the sound increases equally and without distinction for a departure from the setting of the rheostat in either direction from that which gives a true balance. Furthermore, when a telephone is used, the current through the bridge arms must be made variable to cause a sound in the telephone, and correct values of resistance will only be obtained by meeting the condition, not always possible of fulfillment, that the four arms of the bridge are without appreciable capacity or inductance. These considerations practically necessitate, for the general use of the Wheatstone bridge, the employment of a galvanometer as the instrument to show when the bridge is balanced.

The variety and the types of bridges and the methods of their employment are so great that no general rules can be laid down as to what kind of galvanometer will best serve the purpose. However, a few general considerations may be mentioned. Except for special requirements, a mirror galvanometer of the D'Arsonval type, having a resistance of from 100 to 500 ohms, will be found convenient, and will have ample sensibility if it will show a deflection of one division on a scale 1000 divisions from the mirror with a current of 0.005 microampere. The galvanometer should be just aperiodic to save time in waiting for the deflections to return to zero and increased satisfaction will be found in working in proportion as the period of the galvanometer is made shorter. A galvanometer of the D'Arsonval type having a period of three seconds, a resistance of 200 ohms and a sensibility of 200 megohms, can easily be constructed and will admirably meet nearly all the requirements of Wheatstone-bridge work of high precision. It should be recalled that a galvanometer sensibility is expressed in megohms when, with the scale at 1000 scale divisions from the mirror, the sensibility S_m is the number of megohms which must be in the galvanometer circuit, so that, with an E.M.F. of 1 volt in the circuit, there will result a deflection of one scale division. One should distinguish sensibility from "figure of merit" which is defined by the equation

$$F = \frac{S_m}{T^2\sqrt{R}},*$$

* See par. 1504, Eq. (11), also article by Edwin F. Northrup in the *Journal of the Franklin Institute*, Oct., 1910, entitled "The Comparison of Galvanometers and a New Type of Flat-coil Galvanometer."

where T is the undamped complete period of the galvanometer and R the resistance of its coil.

For most uses of the slide-wire Wheatstone bridge and other types in which the coils are adjusted to an accuracy of not better than 0.05 of 1 per cent, it is unnecessary to use a reflecting type of galvanometer, with either telescope and scale or lamp and scale. A small pointer galvanometer, of 100 or 200 ohms resistance, having a sensibility such that with 1 volt and 250,000 ohms in circuit the pointer will deflect 1 millimeter on its scale, will be found amply sensitive and very convenient to use.

The relative positions of the battery and the galvanometer in the Wheatstone-bridge circuits should be chosen to meet the condition that the terminals of the galvanometer shall connect such junction points of the four arms of the bridge as will make as nearly as possible the resistance external to the galvanometer equal to the resistance of the galvanometer itself. For example, if the resistance X is 10 ohms, and the ratio arms are made 100 ohms to 1 ohm, the terminals of the galvanometer, if this has a resistance of 100 ohms or more, should be connected, one to the junction point of the 100 with the 1000 ohms coil of the rheostat, and the other to the junction point of the 1 with the 10 ohms coil. Maxwell gives, in his "Electricity and Magnetism," Vol. I, par. 348, the following rule: "Of the two resistances, that of the battery and that of the galvanometer, connect the greater resistance so as to join the two greatest to the two least of the four resistances." In modern practice, one generally uses a battery of 4 or 6 volts, and then reduces the current in the bridge circuit to a suitable value by the use of a resistance in series with the battery, and the position occupied by the galvanometer should be chosen with reference to its own resistance only, as compared with the resistances of the bridge arms. The object to be obtained is that the circuit external to the galvanometer should be as nearly as possible that of the galvanometer itself, without regard to the battery resistance. Hence the rule at the beginning of the paragraph. The object for this choice of position of the galvanometer is to give the arrangement the maximum sensibility, but, with the current-carrying capacity of the manganin coils now in use, of practically zero temperature coefficient, and with the high sensibility of easily obtainable galvanometers, the sensibility is generally adequate

however the position of the galvanometer is chosen, and the importance of fulfilling the above conditions is slight.

The safe watt capacity of the coils of a Wheatstone bridge will vary from one-quarter to four watts per coil according to its construction. If this watt capacity of a coil is greatly exceeded the coil may be heated to a point where the resistance is permanently changed, even though the insulation is not charred. It should always be remembered that the watt load put on any coil in a bridge is equal to the square of the potential applied at its terminals divided by its resistance, and that, as a rule, this quantity should never exceed 1 watt. Unless one should forget and make connections which would bring an excessive voltage at the terminals of a coil, it is always well in order to avoid this danger to keep an external resistance in circuit with the battery. This will limit, for any connections of the bridge, the flow of current to a safe amount.

Even though the rheostat of a bridge is incapable of being varied by very small steps, one can measure resistances with exactness by making use of the deflections of the galvanometer after a balance has been obtained within an adjustment of the smallest step of the rheostat. The procedure is as follows: The current furnished by the battery being assumed constant and the deflections of the galvanometer proportional to the current through it, one takes note of the permanent deflection of the galvanometer when the resistance of the rheostat required for a balance is set too small, in a final adjustment, by the smallest step in the rheostat. Call R this resistance and d the deflection. Then increase the resistance of the rheostat by one of its smallest steps, say one ohm, and observe the deflection then obtained which will be in the opposite direction to the one previously obtained. Call this deflection d'. The true value of X will then be given by the relation

$$X = \frac{a}{b}\left(R + s\frac{d}{d + d'}\right), \tag{1}$$

where a and b are the resistances in the ratio arms, and s the value in ohms of the smallest step in the rheostat.

When the resistance to be measured is wholly unknown one should proceed, in seeking a balance, in a systematic manner. To avoid violent deflections of the galvanometer this may be temporarily shunted with a low resistance, which shunt is removed when

a balance is nearly obtained. It is well to start with unity ratio and with zero resistance in the rheostat. A quick tap of the key will cause a moderate deflection of the shunted galvanometer in one direction. 1000 ohms may now be put in the rheostat and the key be again tapped. A deflection in the opposite direction will now indicate that the resistance lies between 0 and 1000 ohms. 500 ohms should now be plugged in the rheostat and, if the deflection is like the first one when the key is tapped, the resistance is known to lie between 500 and 1000 ohms.

By proceeding in this manner the resistance is narrowed down, with only a few trials, very close to its actual value. One should now choose the value of the ratio so that in obtaining a final balance the largest possible portion of the rheostat is brought into service. The final balancing is made with the shunt removed from the galvanometer and the procedure to be followed is precisely that adopted for weighing with a delicate balance. With a galvanometer in which the coil is visible the preliminary balancing is usually effected by directly observing the movements of the coil. In the final adjustments only is it necessary to observe the movements of the coil by looking thru the telescope, or by observing the spot of light on the scale.

No definite limitations can be laid down for the useful resistance range of a Wheatstone bridge, as this depends upon the range of its rheostat and upon the number and precision of the coil-values provided in its ratio arms. Ordinarily, Wheatstone bridges should be considered adaptable for the fairly accurate measurement of resistances which lie between 1 and 1,000,000 ohms, though this range is often exceeded in both directions with high-class bridges.

The precision of measurements possible with a Wheatstone bridge depends upon a variety of circumstances, such as, the value of the resistance being measured, the accuracy of the coils in the bridge, the possibility of reversing the ratio arms to eliminate their error, and the care with which contact resistances are allowed for, or guarded against. In routine work, for resistances of the same order of magnitude as the total resistance of the bridge rheostat, a precision of 0.04 of 1 per cent may be considered fairly good, tho the author owns a bridge which can be relied upon to measure resistances in the range from 10 to 10,000 ohms to an accuracy better than 0.02 of 1 per cent.

Since the advent of manganin coils with their practically zero temperature coefficients, little regard need be given to the interior temperature of the bridge. The temperature of the resistance being measured, however, unless this is also of a zero temperature coefficient material, must be carefully observed.

CHAPTER VI.

THE MEASUREMENT OF LOW RESISTANCE.

600. **Introductory Statement.** — When one is about to make an electrical measurement, it is often not possible to choose the best method because the apparatus for this is not available. For this reason it is desirable to be acquainted with alternative methods, and this consideration leads us to describe several methods for measuring low resistances, tho the one known as the "Kelvin-double-bridge" method is preëminently the most accurate and elegant, and, when apparatus suitable for its application is to be had, should be chosen in preference to any other.

While there is no sharp distinction between medium and low resistance we may, for convenience, consider any resistance which is less than one ohm as low. Ordinary methods, applicable to medium resistances, fail to give precision with low resistances either on account of contact resistances which are likely to enter the circuit which contains the resistance being measured or because a low resistance is often a short conductor, and errors in the determination of the exact length measured are apt to enter. Both these sources of error are avoided by providing the resistance measured and the standard with which it is compared with potential points. The resistance which is determined is the resistance which lies between two potential points, when the lines of current flow thru the low resistance have a particular distribution. It should be remarked, that, if the resistances of several conductors are given, each provided with fixed potential points to which connections may be made, these conductors cannot be joined in series or in parallel combinations to obtain a known resultant resistance. For this reason standards of resistance, provided with fixed potential points, are unsuited for obtaining other values by series or parallel combinations.

For the measurement then of low resistance of widely varying range, one needs to be provided with a series of low-resistance

precision standards. A set of precision standards which would be quite complete would consist of 1, 0.1, 0.01, 0.001, 0.0001 ohm, the resistance values being in every case between fixed potential points. The standards would each have, therefore, two-current and two-potential terminals. They are usually mounted in metal cylindrical boxes which can be filled with kerosene or paraffin oil to permit of an accurate determination of their temperatures. They are made usually of manganin wire or sheet, which alloy, consisting of nickel, copper and manganese, has a low temperature coefficient which may be taken roughly as 0.00002 per ohm per degree centigrade. Its thermoelectric force against copper is also very small, which is of importance for many classes of measurements.

The catalogues of Otto Wolff of Berlin and of The Leeds and Northrup Company of Philadelphia, Pa., give very full information upon the best types of low-resistance standards upon the market, as well as upon the apparatus especially adapted to the measurement of low resistance by null methods.

601. Low Resistance Measured with an Ammeter and a Millivoltmeter. — Join the conductor ab, the resistance of which

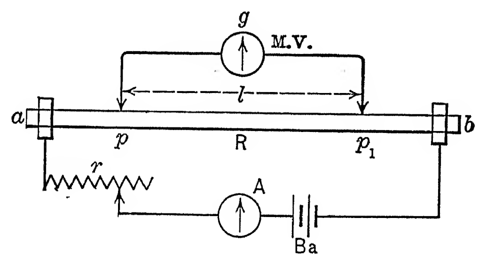

FIG. 601.

is to be determined between two points, in series with a rheostat r, an ammeter A and one or two cells of storage battery Ba (Fig. 601). To the two points p and p_1 between which the resistance is to be determined, join the terminals of a millivoltmeter by means of knife-edges pressed upon the conductor. The length l between these knife-edge contacts, which become potential points, should be accurately measured. The rheostat r should now be adjusted until both the ammeter and the millivoltmeter give suitably high readings on their scales. Then calling V the reading of the milli-

voltmeter, I the reading of the ammeter, g the resistance of the millivoltmeter, and R the resistance to be measured, we have

$$\frac{gR}{g+R}=\frac{V}{I}, \quad \text{or} \quad R=\frac{V}{I-\frac{V}{g}}. \tag{1}$$

If the resistance R is a very low resistance the quantity $\frac{V}{g}$ may be neglected without appreciable error. An example may be found in the determination of the resistance of one meter of No. 0 aluminum wire. The resistance of this at 20° C. would be about 0.00052 ohm. Hence 20 amperes would give a drop of 0.0104 volt or 10.4 millivolts over the wire itself. We would have by the exact formula, if $g = 1$ ohm, $R=\dfrac{0.0104}{20-\dfrac{0.0104}{1}}=0.00052027$ ohm. By neglecting the last term of the denominator an error of about 0.05 of 1 per cent only would be committed. In most measurements of this kind the chief source of error will result from a neglect to accurately determine the temperature of the sample, the resistance of which, if a pure metal, will vary about 0.4 of 1 per cent per degree centigrade. It should be remarked that in this method the resistance is not compared with another resistance taken as a standard, but is measured by two instruments upon the accuracy of the calibration of which the accuracy of the measurement depends.

If a standard of low resistance, of approximately the same magnitude as the resistance to be measured, is available, then the more precise method would be to join the two resistances in series and (still using the ammeter to test the constancy of the current) read with the millivoltmeter the drop, first over the standard and then over the sample. The resistances are then in the same ratio as the drops of potential, provided the current taken by the millivoltmeter is either negligibly small compared with the main current or that the standard resistance is closely the same as the resistance being measured.

602. To Measure the Resistance of Sections of a Closed Circuit; General Method. — The problem is often presented in commercial practice of obtaining the resistance of a portion of a circuit which is closed upon itself and which may contain a source of current, either alternating or direct. If the circuit could be

opened even momentarily the problem could be solved by well-known methods. But if the circuit cannot be opened, the problem is still solvable in more than one way. The following methods have been independently devised* by the writer and carefully tested out in practical cases, and have been found to give such satisfactory results as to warrant a detailed description.

We shall first consider a general method applicable to measuring the resistance of a closed conductor or loop, such as the rim of a cart-wheel, which may be assumed to have a cross-section which varies in an unknown way from one portion of its circumference to another. Referring to Fig. 602, we have the following dispositions of circuits and instruments.

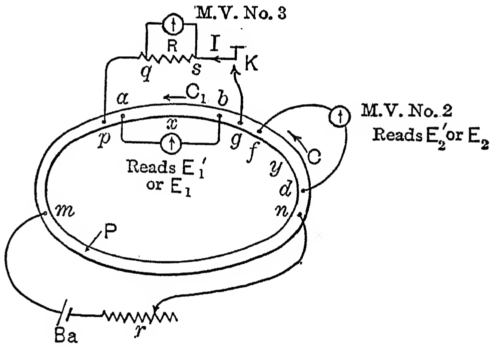

FIG. 602.

P is a closed metallic circuit of medium or very low resistance which *cannot be opened.* It is required to determine the resistance x of a definite length of this closed circuit, as between two points a and b. For this there are required three deflection instruments which deflect proportionally to the current thru them. The constant of these instruments need not be known but, if not known, the value must be the *same* for all three. In the present application of the method there is required one known resistance R provided with potential terminals. This resistance R should be chosen, for the best accuracy, of the same order of magnitude as the resistance x which is to be determined. A cell of storage

* The original development of methods of measuring the resistance of closed circuits and the current in underground mains is due to Dr. Carl Hering. See his paper and author's discussion presented at the 20th Annual Convention of the American Inst. of Elec. Engs., Boston, Mass., June 28, 1912.

battery and a rheostat r to adjust the current from the battery to a suitable value are required, also a key K. The deflection instruments would ordinarily be millivoltmeters, tho three galvanometers having the same constant could be used. The M.V. No. 1 is joined to the potential terminals a, b, between which the resistance is x. The M. V. No. 2 is joined to the potential terminals f, d, between which the resistance is y, and the terminals of the M.V. No. 3 are joined to the potential terminals q, s, between which the resistance is R, which is known. The current terminals of R are joined to the points p, g of the loop and in circuit with R is the key K. The cell of storage battery Ba, which includes in its circuit the rheostat r, is joined to two points, as m and n, of the closed metallic circuit. This supplies to the system the current required for the measurement.

The procedure in making a measurement is as follows:

(a) With the key K open, *read at the same moment* the V.M. No. 1 and call its deflection d_1' and the V.M. No. 2 and call its deflection d_2'.

(b) With the key K closed *read simultaneously* the three deflection instruments. Call the deflection of V.M. No. 1, d_1, of V.M. No. 2, d_2 and of V.M. No. 3, D. Then in case (a)

$$\frac{x}{y} = \frac{d_1'}{d_2'}, \text{ which call } N. \quad \text{Then}$$

$$x = Ny. \tag{1}$$

In case (b), since the deflections D, d_1, and d_2 are proportional respectively to E.M.F.'s V, E_1, and E_2, we have

$$E_1 = K d_1 = C_1 x \tag{2}$$

and

$$E_2 = K d_2 = Cy. \tag{3}$$

Here K is a constant and C is the current thru y, and C_1 is the current thru x. We also have

$$C = C_1 + I,$$

where I is the current thru R. But

$$I = \frac{V}{R} = \frac{KD}{R}, \quad \text{whence,}$$

$$C = C_1 + \frac{KD}{R}. \tag{4}$$

In the relations (1), (2), (3), (4), we have the unknown quantities,

x, y, C, C_1, and hence, there being but four unknowns and four equations, both x and y can be determined.

We finally derive

$$x = \frac{d_2N - d_1}{D}R, \tag{5}$$

and

$$y = \frac{d_2'}{d_1'}x. \tag{6}$$

Equation (5) is obtained as follows:

From Eqs. (3) and (4)

$$\frac{K\,d_2}{y} = C_1 + \frac{K\,D}{R}. \tag{7}$$

From Eqs. (2) and (7)

$$\frac{K\,d_1}{x} = \frac{K\,d_2}{y} - \frac{K\,D}{R},$$

or

$$\frac{d_1}{x} = \frac{d_2}{y} - \frac{D}{R}. \tag{8}$$

Putting in Eq. (8) the value of y from Eq. (1), we obtain

$$\frac{d_1}{x} = \frac{d_2N}{x} - \frac{D}{R}, \tag{9}$$

and from Eq. (9) we find the value of x to be that given in Eq. (5).

The above method possesses four special merits: The circuit of the resistance being measured does not have to be opened; the resistance of no contact enters and hence the contacts at points p, g, m, n, and K need not be made with any special care, while the points a, b, f, d, q, and s are merely potential points and contact at these places may be made with a sharp point or knife-edge pressed against the conductor; the constant of the deflection instruments need not be known, it being only necessary that all three instruments have the same constant; two instruments are read in case (a) simultaneously and three instruments are read simultaneously in case (b), and hence the current in the loop P may be very variable and accurate results still be obtained.

This method was tried by the author, using a brass ring a little over one meter in circumference and of No. 0 B. & S. wire. The ring was placed over an open-core alternating-current electromagnet of very great size. By exciting the alternating-current magnet induced alternating currents were sent thru the ring. It was found that the readings of the three instruments, and hence the

resistance measured, was in no wise affected by the presence of the alternating current induced in the ring, hence the method applies whether the closed loop is or is not carrying an alternating current.

In the above trial the actual readings observed and the results obtained were as follows:

$$d_1 = 20.54, \quad d_2 = 25.18, \quad D = 18.51, \quad R = 0.01 \text{ ohm}.$$

The ratio of d_1' to d_2', or N, was 0.9940. From these readings the value obtained for x was, by Eq. (5),

$$x = \frac{25.18 \times 0.994 - 20.54}{18.51} \times 0.01 = 0.002425 \text{ ohm}.$$

The ring was afterward cut open and the resistance x was determined by an ordinary method and found to be 0.002439 ohm. Hence, the error in the measurement of the closed ring was 0.57 of 1 per cent.

This method has a useful application when applied to the determination of the resistance between two points in a bus-bar while this is carrying direct current.

603. To Measure the Resistance Between Two Points on a Bus-bar. — The arrangement to employ is represented in Fig. 603.

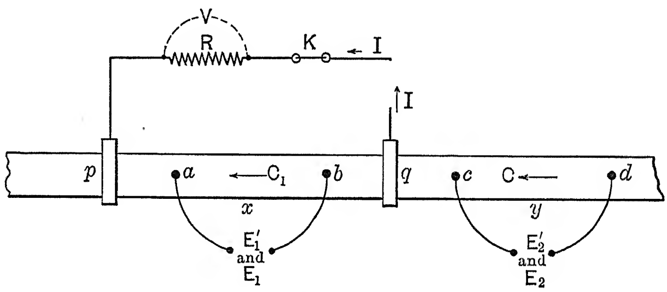

FIG. 603.

The potential points a, b and c, d may be obtained by drilling and tapping small holes in the bus-bar and inserting in these holes small screws to which the terminals of the millivoltmeters may be secured. The terminals of the resistance R may be attached to the bus-bar at p and q by means of iron clamps, as the precision of the method is not affected by contact resistances at these places. The distances between the point a and the clamp p and the point b and the clamp q should be at least three times the width of the

bus-bar. It is also desirable to have the clamps p and q make contact with the bus-bar across its entire width. The purpose of these two precautions is to insure that the stream lines of current are parallel with the bus-bar at the potential points a and b. For the same reason the potential point c should be as far to the right of q as the potential point b is to the left. The distance from c to d should be chosen about equal to the distance from a to b in order to bring the ratio N near unity.

If there is direct current in the bus-bar, supplied by the generator, then there is no necessity of introducing additional current from storage cells, as is required when measuring the resistance of a section of a loop, as described in par. 602.

The standard resistance R should be supplied with potential points and should be not over ten times the resistance of the bus-bar between the clamps p and q. Greater accuracy will be obtained if this resistance is about equal to the resistance of the bus-bar between the clamps. Since the drop of potential over the resistance R is read to give the value of the current I which flows in the branch circuit, one may substitute an ammeter for the resistance R and the millivoltmeter which reads the drop over this resistance. In this case, however, the other two deflection instruments must read, not in arbitrary units, but in volts or millivolts.

The procedure is the same as in the case of the ring, described above. Giving the symbols the meanings designated in Fig. 603, we have, with the key K open,

$$\frac{x}{y} = \frac{E_1'}{E_2'} = N. \tag{1}$$

With the key K closed, we have by readings taken simultaneously by three observers

$$E_1 = C_1 x \tag{2}$$

and

$$E_2 = Cy. \tag{3}$$

We also have the relation,

$$C = C_1 + I = C_1 + \frac{V}{R}. \tag{4}$$

From Eqs. (1), (2), (3) and (4) we deduce, as in the case of the measurement of the resistance of a ring,

$$x = \frac{E_2 N - E_1}{I}, \tag{5}$$

or
$$x = \frac{E_2N - E_1}{V} R. \tag{6}$$

If Eq. (6) is used, E_1, E_2 and V can be multiplied by the same constant a and hence the deflection instruments may be calibrated in arbitrary units, provided the same arbitrary units are used for all three instruments.

604. Measurement of the Current in a Bus-bar. — The purpose to be fulfilled in finding the resistance between two points in a bus-bar is to enable the current in the bus-bar to be measured at any time by reading the drop of potential between the points with a millivoltmeter. A portion of the bus-bar is made in this manner to serve as a shunt for a millivoltmeter, which thus becomes

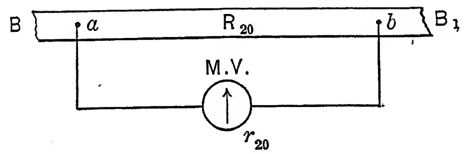

FIG. 60

an ammeter for reading the current in the bus-bar. As bus-bars are made of copper or aluminum which have a large temperature coefficient we have to consider to what extent, if any, their change in resistance with temperature will affect the precision with which the current may be read. Let Fig. 604 represent an arrangement to be employed.

Here, BB_1 is a section of a bus-bar. We shall suppose that the resistance R_{20} has been accurately obtained at 20° C. between the two points a and b by the above method. The millivoltmeter M.V. is joined to the points a and b.

Let
$$r_T = r_{20}(1 + \alpha T) \tag{1}$$

be the resistance of the millivoltmeter at $T°$ C. above 20° C. when r_{20} is its resistance at 20° C. and α is the temperature coefficient of its winding.

Let
$$R_t = R_{20}(1 + \beta t) \tag{2}$$

be the resistance between points a and b of the bus-bar at $t°$ C. above 20° C. when R_{20} is its resistance at 20° C. and β is the temperature coefficient of the material of the bus-bar.

The bus-bar may change in temperature both from changes in

the temperature of the room and from the heating due to the current which it carries. The millivoltmeter M.V. can only change in temperature from changes in the temperature of the room. Hence, in general, the temperature of the millivoltmeter will not be the same as the temperature of the bus-bar.

We wish to determine the nature and magnitude of the errors produced by these temperature changes in reading the current. If I is the current in the bus-bar, the fall of potential from a to b, when the temperature of the bus-bar is t, will be

$$E_t = IR_t. \tag{3}$$

The current thru the millivoltmeter will be

$$C = \frac{E_t}{r_T} = K\,D, \tag{4}$$

where D is the deflection of the millivoltmeter and K is a constant. Hence,

$$E_t = K\,Dr_T. \tag{5}$$

By Eqs. (3) and (5)

$$I = K\,D\frac{r_T}{R_t} = K\,D\frac{r_{20}\,(1 + \alpha T)}{R_{20}\,(1 + \beta t)}. \tag{6}$$

Since the bus-bar and the winding of the millivoltmeter are both of pure metal, as copper or aluminum, the temperature coefficients α and β would be practically the same and may be taken, approximately, 0.004. Eq. (6) can therefore be written

$$I = \frac{KDr_{20}}{R_{20}} \times \frac{1 + 0.004\,T}{1 + 0.004\,t}. \tag{7}$$

The error in the measurement of I is now seen to depend directly upon the amount by which the last term of Eq. (7) departs from unity. In the case of no heating, by the current, of the bus-bar above room temperature (as would be very approximately realized for a loading of the bus-bar of 50 per cent full load or less) $t = T$, and there is no error, whatever the room temperature becomes. Now t can never be less than T but may assume a value $T + \delta T$ where δT represents the temperature of the bus-bar, above the temperature of the air. In this case Eq. (7) becomes

$$I = \frac{KDr_{20}}{R_{20}} \times \frac{1 + 0.004\,T}{1 + 0.004\,T + 0.004\,\delta T}. \tag{8}$$

As a rather extreme case we may take $T = 10°$ C. above 20° C., and $\delta T = 5°$ C. Then

$$\frac{1 + 0.004 \times 10}{1 + 0.004 \times 10 + 0.004 \times 5} = \frac{1.04}{1.06} = 0.981 +.$$

Thus the true value of the current would be, in this case, about 2 per cent less than one would read it upon the millivoltmeter.

The following estimate shows that the fall of potential in a bus-bar is large enough to apply the above method for measuring the current in it.

The resistance of 100 per cent conductivity copper at 20° C. is 67.7×10^{-8} ohm per linear inch per square inch of cross-section. It is good practice to allow 1000 amperes per square inch of cross-section of copper conductor. Then, with 1000 amperes to the square inch of cross-section, the drop of potential per linear inch becomes

$$10^3 \times 67.7 \times 10^{-8} = 0.677 \times 10^{-3} \text{ volt},$$

or 0.677 millivolt per linear inch. If the full scale reading of the millivoltmeter is 20 millivolts, the distance between the potential points, a and b (Fig. 604), would need to be

$$\frac{20}{0.677} = 29.5 + \text{ inches}.$$

This length of bus-bar, to be used for the purpose of a shunt, could be obtained behind almost any switch board, and it is probable that a shunt for the millivoltmeter of this character would serve quite as well and perhaps be superior to the shunts ordinarily used. For, these latter have a very low temperature coefficient and changes in the temperature of the room will increase the resistance of the millivoltmeter without increasing in like degree the resistance of the shunt, and hence there is no automatic compensation, as in the case discussed above, where the bus-bar itself serves as a shunt.

To make the millivoltmeter read directly in amperes requires, of course, that the constant K in Eq. (8) be correctly chosen. As we are at liberty to give any value to the resistance r_{20} it will always be possible to do this.

605. Measurement of the Resistance of Underground Mains. — An important application of the methods described above for measuring the resistance of a portion of a closed circuit is the determination of the resistance between two selected potential

points upon an underground gas or water main. Underground pipes are subject to deterioration from electrolysis, caused by "tramp" currents which get into the pipe line from neighboring electric trolley roads. The electrolysis occurs when current *leaves* the pipes. It becomes important, at times, to be able to quickly and accurately measure the current which flows in some selected section of a pipe line. It is evident that this can be easily accomplished by measuring, with a millivoltmeter, the potential drop between two points on a section of pipe, provided the resistance between these two points has been previously determined. The method given in Fig. 605a, which is a slight modification of those

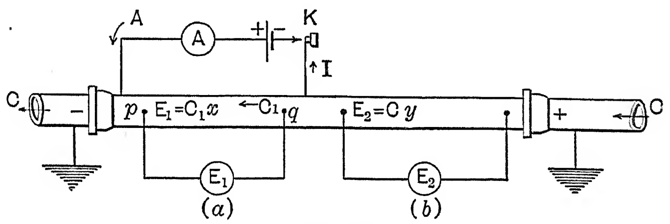

Fig. 605a.

described above, enables this resistance to be measured with considerable precision while the section of pipe is in place in the pipe line.

The measurement is made with two millivoltmeters and an ammeter. One or two cells of storage battery are also required. The cells of storage battery, a key K and the ammeter A are joined in series and connected at two places, as shown in the diagram, to a section of pipe. These connections are best made by drilling 0.25-inch holes, about half way thru the pipewall, and driving brass plugs into them. Heavy copper-wire connections may then be soldered to the brass plugs. The other connections, which serve as potential points, may be made in a similar manner, but smaller holes and plugs will serve. There should be as much separation as possible between a potential point and the place of connection of a current lead, and these should, preferably, be located at the ends of diameters of the pipe which form with each other an angle of 90 degrees. It is well to take one set of readings and calculate the resistance with the polarity of the storage cell in one direction, and then take a second set with the polarity of this cell reversed. By taking the mean of the two

resistances, thus obtained, the error is largely eliminated which results from the flow lines of current from the storage cell not being parallel with the section of pipe between the potential points. This is specially the case when there is considerable current flowing in the pipe from other sources than the storage cell. This error will be small, however, in any case, if the distance between a potential point and the point of connection of a current lead is, say, twice the diameter of the pipe and these terminals are located as above suggested.

Calling I the current thru the ammeter and referring to Fig. 605a for the meaning of the other symbols we have, as in the cases given above, with the key K open

$$\frac{x}{y} = \frac{E_1'}{E_2'} = N, \tag{1}$$

and with the key K closed

$$E_1 = C_1 x, \tag{2}$$

$$E_2 = Cy, \tag{3}$$

and

$$C = C_1 + I, \tag{4}$$

from which we find

$$x = \frac{E_2 N - E_1}{I}. \tag{5}$$

Also

$$y = \frac{E_2 - E_1 \frac{1}{N}}{I}, \tag{6}$$

or

$$y = \frac{x}{N}. \tag{7}$$

In applying the method, one is not in the least troubled by the sudden variations of the current in the pipe which constantly occur, because E_1' and E_2' are read *simultaneously* to obtain the ratio N and then, again, E_1, E_2, and the ammeter A are read simultaneously to obtain the other necessary values. Three observers, reading at the same moment, obtain correct values; for, when the current varies, a variation occurs in all three instruments at the same time, the proper *relation* between the readings of the three instruments being always maintained.

This method was carefully tested by the author upon an actual pipe line with excellent results. The essential features of the test are recorded below:

The diameter of the pipe was 15 inches. Two pipe lengths were uncovered and the connections to the pipe sections were made at distances and in the manner shown in Fig. 605b.

The method embodied the use of two cells of storage battery, which would yield on short circuit, when joined in parallel, from 125 to 150 amperes, also one ammeter reading to 200 amperes and two millivoltmeters giving a full scale deflection with 20 millivolts. In circuit with the ammeter and storage cell a single pole current switch K was used. The following readings were taken:

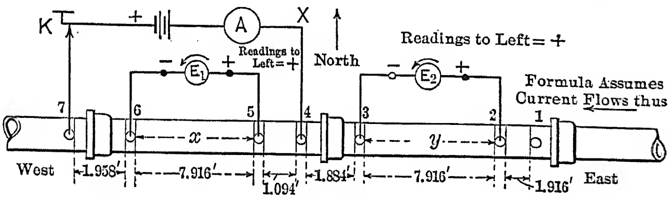

FIG. 605b.

E_1' and E_2' were read simultaneously. The current in the pipe was sufficient for the purpose. The mean of nine readings of E_1' was 7.511 millivolts and the mean of nine readings of E_2' was 7.122 millivolts. Hence the value of the ratio of $\frac{x}{y}$ was

$$N = \frac{7.511}{7.122} = 1.055.$$

There were then taken seventeen sets of readings of E_1, E_2 and I with the positive pole of the battery cell joined to No. 7 terminal and a like number with the negative pole joined to this terminal.

The following table exhibits a few sample readings:

No. 1 current + to No. 7 terminal.				No. 2 current − to No. 7 terminal.			
E_1 M. V.	E_2 M. V.	I amperes	x = ohm for 7.916 ft.	E_1 M. V.	E_2 M. V.	I amperes	x = ohm for 7.916 ft.
−6.8	1.6	116	0.0000731	7.60	0.4	−102	0.0000703
−6.2	1.8	110	0.0000736	7.80	0.8	− 98	0.0000709
−8.3	0.9	127	0.0000728	8.20	1.0	−101	0.0000707

The mean value deduced for x with the current from the storage cell positive to terminal No. 7 was 0.00007315 ohm, and with the current from the storage cell negative to terminal No. 7 was

0.00007077 ohm. The mean of these two results is 0.00007196 ohm for a length of the pipe of 7.916 feet. There were 40 feet of No. 14 wire used, as potential leads, to each millivoltmeter. Calculation showed that to correct for the resistance of these leads the final value of x should be multiplied by 1.088. Doing this and reducing the resistance to a length of one foot of pipe, the final value found was *9.91 microhms per foot*, at 65° F.

The test was defective in that the distances between potential points and points of attachment of current leads were not chosen as great as they should have been and were all made on the top side of the pipe. The " tramp " currents in the pipe were large and very variable at the time of the test. In spite of this the resistance measurement is probably correct within 1.5 or 2 per cent, and should have been better.

It was found in this test that care had to be exercised to give to the readings of the three instruments the proper algebraic sign. By making a diagram, like Fig. 605b, before beginning the test, errors of this character may be avoided, which otherwise might occur and entirely vitiate the results.

606. Comparison of Low Resistances by the Modified Slide-wire Bridge. — The theory of this method has already been given, par. 402, to which reference should be made. As the method requires two settings of a sliding contact and a balance twice obtained, it is inferior to the Kelvin-double-bridge method (§ 609). It would be used generally only when the necessary resistance units, for building up a Kelvin double bridge, are not obtainable.

607. Comparison of Low Resistances by the Carey-Foster-Bridge Method. — The theory and application of the Carey-Foster bridge has been given in par. 403.

When a low resistance is provided with potential points and leads, it is not adapted for use as a standard with which to compare another resistance by the Carey-Foster-bridge method. This method is confined to the measurement of such low resistances as are provided with terminals which may be inserted into mercury cups or clamped in binding posts. Thus the Carey-Foster method is hardly to be considered a method of low-resistance measurement. By the use of connecting leads, however, for the standard resistance and the resistance under comparison, which are alike in size, resistance and temperature coefficient the comparison can

be made without any error being introduced due to lead resistances. The method is one admirably adapted to determinations of temperature coefficients of comparatively short lengths of low-resistance wires. In temperature-coefficient determinations the absolute values of the resistances are of less importance than the variations with temperature of these resistances, and the Carey-Foster method will give these with great precision.

608. Comparison of Low Resistances with a Potentiometer.— Two low resistances which are provided with potential terminals may be compared with considerable precision by using a potentiometer. The two resistances are joined in series and current is passed thru them from a source of constant E.M.F. as a storage cell of large capacity. The fall of potential is read with the potentiometer, in quick succession, first over the one resistance and then over the other. The resistances are in the same ratio as the potential drops, provided the current thru the resistances remains constant during the taking of the two readings.

In using a potentiometer in this way it is unnecessary to make use of a standard cell as the absolute values of the drops in potential are not required. As a good potentiometer will compare E.M.F.'s as low as 0.01 volt, to accuracies of the order of 0.1 of 1 per cent, it is not necessary to pass a very large current thru the two low resistances under comparison, and hence the comparatively small current needed can be maintained very constant for the short time required.

609. The Kelvin Double Bridge; A Network of Nine Conductors. — The most elegant and the most precise methods for comparing any two resistances, which are provided with potential points, are those which employ some form of the Kelvin double bridge. We shall, therefore, discuss fully the theory, the forms, and the applications of this unique arrangement of circuits for the measurement of resistances.

The Kelvin double bridge, due to Lord Kelvin, is a network of nine conductors. This network may be conveniently represented (P and Q, Fig. 609a) by either of the two following diagrams, which are exactly equivalent electrically. The diagrams are drawn so that like letters refer to like branches of the network.

Referring to the diagram (P), we note that the Kelvin net-work becomes a Wheatstone net-work, when d equals zero and also when d equals infinity.

When d equals zero the net-work assumes the form shown at (p), Fig. 609b. Here α and β become two resistances in parallel,

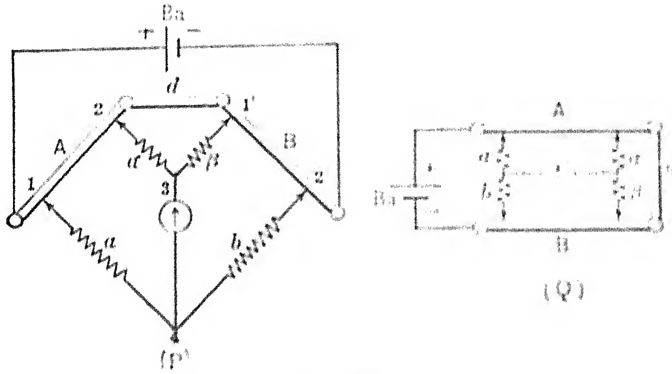

FIG. 609a

which are in circuit with the galvanometer, and hence do not enter into the equation of the bridge, which in this case is

$$Ab - Ba = 0. \tag{1}$$

When d equals infinity the net-work assumes the form shown at (q). Here α and β are thrown, the one in series with A and the

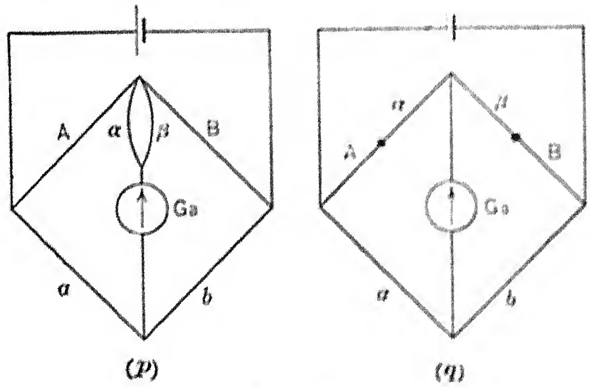

FIG. 609b.

other in series with B. Thus the equation of the bridge in this case becomes,

$$(A + \alpha)b - (B + \beta)a = 0. \tag{2}$$

In the actual employment of the Kelvin double bridge the resist-

ance d can never be made zero, tho it should be made as small as possible.

Referring to Fig. 609a, A and B are two low resistances (provided with potential points at 1, 2, and 1′, 2′) which are to be compared, the resistance of both A and B being that included between potential points. The resistance d, called the "yoke," connects A and B. Besides being made as low as possible it is arranged to be easily removed as, for example, by withdrawing a copper link from two mercury cups. The other four resistances a, b and α, β are relatively high resistances and are called the ratio coils. Because there are two pairs of ratio coils the net-work is called a "double bridge."

610. Theory of the Kelvin Double Bridge. — The relations which must connect the resistances to have the bridge balanced, that is, to have no current flowing thru the galvanometer, may be found as follows: Referring to diagram (P) of Fig. 609a we see that, when there is no current in the galvanometer, the current in the branch a must be the same as the current in the branch b. Call this current i_1. Also the current in the branch α must be the same as the current in the branch β. Call this current i. Also the current in the branch A will equal the current in the branch B. Call this current I. By the law of branched circuits,

$$i = \frac{d}{\alpha + \beta + d} I. \tag{1}$$

Further, the fall of potential from the point 1 to the point 3 must be the same as the fall of potential from the point 1 to the point 4. Otherwise there would be a difference of potential between points 3 and 4 and the galvanometer would deflect. Call this fall of potential E_a. In like manner, the fall of potential from point 3 to point 2′ will be the same as the fall of potential from point 4 to point 2′. Call this fall of potential E_b. Now,

$$E_a = IA + i\alpha = i_1 a, \tag{2}$$

and

$$E_b = IB + i\beta = i_1 b. \tag{3}$$

Hence taking the ratio of Eqs. (2) and (3) we find

$$\frac{a}{b} = \frac{IA + i\alpha}{IB + i\beta}. \tag{4}$$

Substituting in Eq. (4) the value of i given in Eq. (1) we have,

$$\frac{a}{b} = \frac{A(\alpha + \beta + d) + d\alpha}{B(\alpha + \beta + d) + d\beta}. \tag{5}$$

Eq. (5) is readily put in the form,

$$\frac{A}{B} = \frac{a}{b} + \frac{d}{B} \cdot \frac{\beta}{\alpha + \beta + d}\left(\frac{a}{b} - \frac{\alpha}{\beta}\right). \tag{6}$$

In Eq. (6) call,

$$k = \frac{d}{B} \cdot \frac{\beta}{\alpha + \beta + d}\left(\frac{a}{b} - \frac{\alpha}{\beta}\right),$$

and we have

$$\frac{A}{B} = \frac{a}{b} + k. \tag{7}$$

The quantity k is a correction factor which must be either calculated and added to the ratio $\frac{a}{b}$ to give the ratio A to B, or it must be made so small that it may be neglected even for work of the highest precision. In practice the latter course is followed. We have to consider how k may be reduced, and also the magnitude of the errors which will be introduced by neglecting k when it is not absolutely zero.

The correction factor k may be considered as the product of three parts,

$$\frac{d}{B}, \quad \frac{\beta}{\alpha + \beta + d} \quad \text{and} \quad \frac{a}{b} - \frac{\alpha}{\beta}.$$

Each of these parts should be made as small as possible. The yoke d should be made very small in resistance tho this cannot be made absolutely zero. This means that the resistances A and B should be joined together by a heavy copper conductor which makes as good contact as possible with A and B. The smaller the resistance B, the more important it becomes, for keeping the ratio $\frac{d}{B}$ small, to make d as small as possible. If B is over 0.01 ohm, $\frac{d}{B}$ may be made fairly small.

The part $\frac{\beta}{\alpha + \beta + d}$ will always be less than unity. As d is negligible the value of this part cannot be varied, because the relative values of α and β are determined by the relative values of the resistances A and B, being compared. Thus, if α is multi-

plied by any factor, β must be multiplied by the same factor, and the magnitude of the fraction remains unchanged.

The third part $\frac{a}{b} - \frac{\alpha}{\beta}$ is the most important one, for by care in making adjustments it may be made as small as we please. It becomes zero, as does also k, when, accurately, $\frac{a}{b} = \frac{\alpha}{\beta}$.

An estimation of the magnitude of the errors resulting from a neglect of the correction factor k has been made in a pamphlet issued by The Leeds and Northrup Company, of Philadelphia, and is here reproduced in substance.

If a and b, α and β are resistances adjusted to be like each other to 0.01 of 1 per cent and are arranged so that, according to their nominal values $\frac{a}{b} = \frac{\alpha}{\beta}$ then, the greatest value which $\frac{a}{b} - \frac{\alpha}{\beta}$ can have is 0.00020; the probable amount is less than this. This value is based on the assumption that the inequalities of 0.01 of 1 per cent group themselves in such a way as to form the maximum error. Assuming $\frac{d}{B} = 1$ and $\frac{\beta}{\alpha + \beta + d} = 1$ (which is obviously more than it ever will be) the complete formula becomes in this case $\frac{A}{B} = \frac{a}{b} + 0.00020$, which may be written $A = B\left(\frac{a}{b} + 0.00020\right)$. If $\frac{a}{b} = 1$, as will be the case when two resistances of equal value are under comparison, the maximum error in this case will be 0.02 of 1 per cent. If $\frac{a}{b} = 10$, as will be the case when the known resistance is ten times as large as the unknown, the maximum error will be 0.002 of 1 per cent. If $\frac{a}{b} = 0.1$, as will be the case when the known resistance is 0.1 of the unknown, the maximum error will be 0.2 of 1 per cent. These considerations show that for ordinary cases where the connecting resistance can be kept smaller than the unknown and the unknown resistance is equal to or smaller than the known, that no attention need be paid to the correction term provided the ratio coils are like each other to 0.01 of 1 per cent.

There may be cases in which it is not possible to meet all these conditions.

In these cases the following procedure is usually possible. Referring again to Fig. 609a it is evident that if the connector d is removed, the combination forms an ordinary Wheatstone bridge in which for a balance we must have $\frac{a}{b} = \frac{A + \alpha}{B + \beta}$. If the galvanometer does not show a balance it may be brought about by shunting α or β. When $\frac{a}{b}$ is made exactly equal to $\frac{A + \alpha}{B + \beta}$ then indeed $\frac{a}{b}$ will be very nearly equal to $\frac{\alpha}{\beta}$ because A and B will always be very low resistances compared with α and β and, furthermore, they are in the same ratio as α and β. Consequently when $\frac{a}{b}$ is made equal to $\frac{A + \alpha}{B + \beta}$ it may generally be assumed without error that $\frac{a}{b} = \frac{\alpha}{\beta}$.

611. Sensibility Which Can be Obtained With the Kelvin Double Bridge. — We shall now examine the percentage variation in one of the low resistances which can be detected when

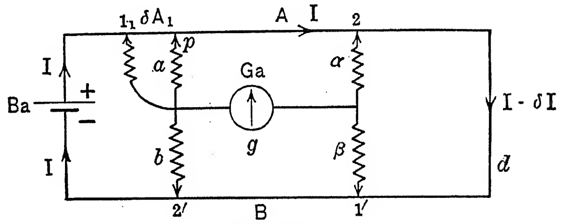

FIG. 611.

using a galvanometer purchasable upon the market. An inquiry involving a rigid solution of the problem would require a complicated application of Kirchoff's laws. But this is unnecessary as a very closely approximate solution may be obtained quite simply as follows:

Represent the bridge as in Fig. 611. First, let the bridge be balanced when contact p is at the point 1 on A. Let the current in A be I. It will then be $I - \delta I$ in d. But δI will be a very small quantity as compared with I, and, further, the value of I will be very little affected by small displacements of the contact p upon A. We may, therefore, without sensible error consider the current at all times to be I in the branches A, d, B, and in the battery circuit. Now displace p to point 1_1 thus increasing A by

a small quantity δA. The fall of potential from 1 to 2 was IA before the displacement, and this fall of potential was required to balance the bridge. The fall of potential from 1_1 to 2 is $IA + I\delta A$. Hence, increasing the fall of potential by the amount $I\delta A$ is equivalent to introducing in circuit with A a small E.M.F. This E.M.F. sends a current thru the galvanometer. This current must pass thru the two resistances a and α in series with the galvanometer, and thru the galvanometer resistance g shunted by the resistances b, B and β in series.

We may assume that this E.M.F. sends no current thru d or the battery because we have considered that I remains constant. Also the resistances A and B do not need to be considered as they are very small compared with the others. For the current which flows thru a, due to the E.M.F. $I\delta A$, we thus obtain,

$$C = \frac{I\delta A}{R} \tag{1}$$

where $$R = a + \alpha + \frac{g\,(b + \beta)}{g + b + \beta}.$$

The current thru the galvanometer will be

$$C_g = C\frac{b + \beta}{g + b + \beta}. \tag{2}$$

From Eqs. (1) and (2) we find

$$C_g = \frac{I\delta A\,(b + \beta)}{(a + \alpha)\,(g + b + \beta) + g\,(b + \beta)}. \tag{3}$$

If $d_0 = KC_g$, where d_0 denotes the deflection of the galvanometer in scale divisions when the scale is 1000 scale divisions from the mirror, and K is the galvanometer constant, we have

$$\frac{d_0}{KI} = \frac{\delta A\,(b + \beta)}{(a + \alpha)\,(g + b + \beta) + g\,(b + \beta)}. \tag{4}$$

In ordinary practice we would have $a = \alpha$ and $b = \beta$.
Give these values to α and β. We then find, in solving Eq. (4) for δA, dividing it by A, and multiplying it by 100 that,

$$100\frac{\delta A}{A} = \frac{100\,d_0}{KIA}\left[2\,a + g\left(1 + \frac{a}{b}\right)\right]. \tag{5}$$

Eq. (5) expresses the percentage variation in A which will produce a deflection d_0 in the galvanometer, the battery current being I amperes. The author has verified Eq. (5) experimentally.

We proceed to apply Eq. (5) to calculations of the percentage precision with which certain low resistances may be compared by the Kelvin-double-bridge method. First take the following case:

Let $d_0 = 10^{-1}$ of a division as the smallest departure from zero which can be detected with certainty.

Let $A = B = 10^{-3}$ ohm.

Let $I = 1$ ampere.

Let $a = 10^2$ ohms, then $b = 10^2$ ohms.

Let $g = 180$ ohms.

Let $K = 1.21 \times 10^8$.*

Substituting these values in Eq. (5) we obtain

$$\frac{\delta A}{A} 100 = \frac{10^{-1} \times 10^2}{1.21 \times 10^8 \times 1 \times 10^{-3}} \times (200 + 180 \times 2)$$
$$= \frac{1}{1.21 \times 10^4} \times 560 = 0.046 + \text{per cent.}$$

Second, take the case in which $A = 10^{-3}$ ohm, and $B = 10^{-2}$ ohm. Then if $a = 10^2$ ohms, we shall have $b = 10^3$ ohms.

If all the other quantities are the same as in the first case, we find

$$\frac{\delta A}{A} 100 = \frac{1}{1.21 \times 10^4} \times 398 = 0.0328 + \text{per cent.}$$

Third, take the case in which $A = 10^{-2}$ ohm and $B = 10^{-3}$ ohm. Then if $a = 10^3$ ohms, we shall have $b = 10^2$ ohms, and if all the other quantities remain the same

$$\frac{\delta A}{A} 100 = \frac{1}{1.21 \times 10^5} \times 3980 = 0.0328 + \text{per cent.}$$

We thus note from the second and third cases that the precision is the same whether we make $A = 0.001$ ohm and $B = 0.01$ ohm or $A = 0.01$ ohm and $B = 0.001$ ohm. In the above examples $I = 1$ ampere. But the per cent displacement $\frac{\delta A}{A} 100$ diminishes

* The values of g and K are those belonging to a galvanometer made by the Leeds and Northrup Co., of Philadelphia, known as their No. 2280, narrow-coil galvanometer. This galvanometer has a complete period of 1.8 seconds. Its figure of merit is

$$F = \frac{S_m}{T^2 \sqrt{R}} = \frac{121.8}{1.8^2 \sqrt{180}} = 2.8 \text{ D'Arsons.}$$

See "The Comparison of Galvanometers and a New Type of Flat-coil Galvanometer," by E. F. Northrup, *Jour. of the Franklin Inst.*, Oct., 1910. See also par. 1504, and the table.

directly as I increases, that is, the possible sensibility in the measurement increases as I increases. A resistance as low as 0.01 ohm may generally be made to carry a current as high as 20 or more amperes without undue heating. Hence, by using a current of 20 amperes the sensibility in the above three cases would be increased 20 times, that is, $100 \frac{\delta A}{A}$ would be 0.0023 of 1 per cent in the first case and 0.00164 of 1 per cent in the other two cases. These results show that resistances of the order of 0.001 ohm may be compared by the Kelvin double bridge with much the same order of accuracy, as resistances of the order of 1000 ohms may be compared by the Wheatstone bridge.

612. Methods of Applying the Kelvin-Double-Bridge Principle.—From the formula of the Kelvin double bridge $A = \frac{a}{b} B$ when $\frac{a}{b} = \frac{\alpha}{\beta}$ by construction, it is seen that, if B is the standard resistance, a balance may be obtained either by varying B or by varying the ratio $\frac{a}{b}$, provided the ratio $\frac{\alpha}{\beta}$ is varied also in the same way. A combination of these two methods for securing a balance may likewise be applied. In this case the ratios $\frac{a}{b}$ and $\frac{\alpha}{\beta}$ would be given values such as 10, 1, 0.1, etc., and the standard B would then be varied in infinitesimal steps to secure an exact balance.

The method whereby B is maintained fixed and the ratios $\frac{a}{b}$ and $\frac{\alpha}{\beta}$ are similarly varied to secure a balance is that adopted by the instrument maker, Otto Wolff of Berlin. For accomplishing this end a special box of ratio coils is provided. This box has four double dials which give the values of a and of α in steps of 0.1 ohm and two sets of blocks with plugs with which the values of b and β are set at 25, 50, 100, or 25 + 50, 25 + 100, 50 + 100, or 25 + 50 + 100. The general plan of the connections is given below in Fig. 612a.

Here the dials a_{100}, a_{10}, a_1, $a_{0.1}$ and α_{100}, α_{10}, α_1, $\alpha_{0.1}$ are varied at the same time by turning a single handle for each pair of dials. The dial a_{100} reads in ten steps of 100 ohms each, the dial a_{10} in ten steps of 10 ohms each, the dial a_1 in ten steps of 1 ohm each and the dial $a_{0.1}$ in ten steps of 0.1 ohm each.

The resistances b and β have their values varied by plugging between brass blocks, between which there are resistance coils. The value obtained for b and β is that of the coils between the blocks left unplugged. The points 1 and 2 are joined to the potential points of the resistance to be measured and the points

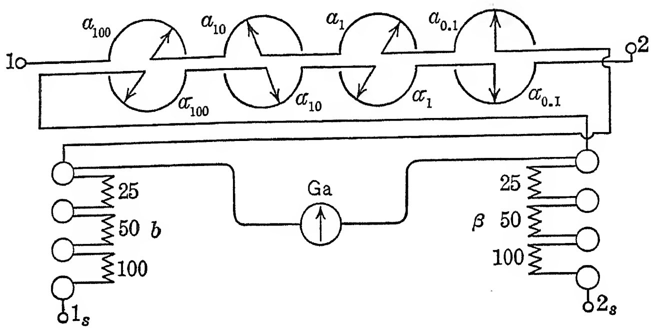

FIG. 612a.

1_s and 2_s are joined to the potential points of the standard. To go with a box, such as the above, fixed standards of low resistance, furnished with potential points, are required. A typical set of standards, such as are furnished by Otto Wolff, would be:

Nominal value	Load capacity (approximate)
Ohm	Watts
1	1
0.1	5
0.01	500
0.001	500
0.001	1000
0.0001	1000
0.0001	2500
0.00001	2500

The above standard resistances of the lower values are arranged to be water cooled. When used as standards for measuring current by reading the fall of potential with a potentiometer between their potential points they may be much more heavily loaded than when used as precision standards for comparison resistances. As they are made of manganin their temperature coefficient is small, being of the order 0.00002 per ohm per degree centigrade.

The method adopted, by leading American instrument makers, for the application of the Kelvin double bridge is the mixed method. In this the ratios $\frac{a}{b}$ and $\frac{\alpha}{\beta}$ are set to various values, and the positions of two contacts, made, one with a plug and one by means of a knife-edge sliding upon a bar of manganin, are varied until a balance of the bridge is obtained.

This method is diagrammatically shown in Fig. 612b.

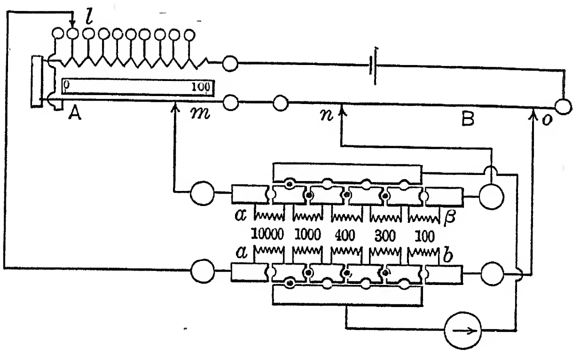

FIG. 612b.

A comparison of Fig. 612b and Fig. 609a will make the relation of the parts clear. A is a bar of manganin which, in one construction, has a resistance of 0.001 ohm between the points 0 and 100 of a vernier scale which is placed alongside the bar. It will carry 150 amperes. Upon this bar slides a knife-edge contact. The left end of the bar joins to a heavy strip of manganin. At points separated by a resistance exactly equal to that of the bar, namely 0.001 ohm in the case cited, are brought off 9 potential leads to brass blocks. An extra potential lead is brought off from the bar opposite the 0 of the scale. The two contacts l and m correspond to the two contacts 1 and 2 of Fig. 609a. The contact l is made with the block, to which a potential lead is joined, by means of a plug. Altogether a resistance of 0.01 ohm, in infinitesimal steps, may be obtained between the contacts l and m. B represents the unknown resistance to be measured between the potential points n and 0. The two sets of ratio coils, which in one construction are mounted together in a separate box, are shown at a, b and α, β. Many different ratios may be obtained by

suitably plugging this box. For example, the ratio value given in the diagram is

$$\frac{b}{a} = \frac{\beta}{\alpha} = \frac{100}{10000}. \quad \text{Hence, as } B = \frac{b}{a}A,$$

we would obtain a balance, if $B = 0.00001831$, when $A = 0.001831$, which value may be read from the setting of the plug at l and by the vernier of the scale set at m.

Excellent apparatus for the above adaptation of the Kelvin double bridge to low-resistance measurements has been placed upon the market by The Leeds and Northrup Company of Philadelphia, Pa. In their highest grade apparatus, constructed according to the above plan and used with the set of ratio coils, shown in Fig. 612b, resistances may be compared in the range of from 1 to 10^{-5} ohm, and, with reduced precision, considerably lower. The precision of the measurement is a maximum when the standard and resistance under comparison are about equal. In this case 0.02 of 1 per cent or better may be expected.

The utilization of the Kelvin-double-bridge principle is not confined to the employment of apparatus (often of high cost) specially constructed for the purpose. One standard of low resistance provided with potential terminals must be available. The rest of the apparatus may be assembled from boxes of resistance coils, such as are found in most well-equipped laboratories. What may be accomplished with such facilities is best shown by giving a brief description of some measurements, made upon a bar of magnesium, as an exercise for students.

In all that has been said above no mention has been made of the importance of maintaining the temperature of the resistance being measured at a known value at the time of the measurement. When standard resistances made of manganin are to be compared, comparatively rough measurements of temperature will suffice; but, in the measurement of the resistance of a pure metal, the temperature must be very carefully determined, hence the precautions taken in this regard in the sample measurement given in par. 614.

613. Plan of Procedure for Making and Recording a Measurement. — It is important when one comes to plan, execute, and record an electrical measurement to follow a systematic procedure. As the result of the author's experience he has found the following outline to embody the most satisfactory plan to follow in recording

an electrical measurement: (Following this plan is an illustration of a record of a sample measurement to determine the resistivity of magnesium.)

Date of experiment or measurement and names of observers.

Object of the experiment or measurement.

The precision sought.

The sample to be measured; described.

The method of measurement; described or reference given to a description.

Theory of method; explain or give book references. Give formulæ to be used in deduction of results.

The apparatus; briefly described. Give instrument numbers, etc.

The electrical connections used; make complete and clear diagrams.

Procedure. State clearly the way in which the observations were made.

The data obtained. Give data clearly in tabular form; use great care not to omit some one fact essential to the deductions.

Deduction of results. Make deductions concise and clear and plot curves when several observations are taken of more than one quantity if one quantity is a function of another.

Reliability and precision of the measurement. Very important to state clearly the probable precision attained and the basis upon which the probability of the precision is judged to rest.

614. Sample of a Low-resistance Measurement; Resistivity of Magnesium. — The **object of the measurement** was to determine the resistivity and temperature coefficient of a bar of magnesium metal, in the temperature range of from 20° C. to 155° C.

The **precision sought** was 0.2 of 1 per cent in the final result.

The **sample selected** for the measurement was a bar of magnesium which had been accurately shaped in a milling machine to give it a rectangular cross-section. Its dimensions were determined with a micrometer caliper and a comparator.

Length of bar over all = 35.60 cms.

Length of bar between potential points = 21.883 cms.

Breadth of bar = 1.412_7 cms.

Thickness of bar = 1.444_8 cms.

Cross-section of bar = 2.041_1 cms.^{2}

Weight of entire bar = 78.760 grams.

Density of bar = 1.723_2.

The potential points were located by drilling two holes in the bar each about 6.8 cms. from an end of the bar. No. 24 brass wire pins were driven into these holes and to these the potential leads were soldered.

The current terminals were fastened to each end of the bar by means of small brass clamps having jaws like a vise. Heavy copper leads were soldered to these brass clamps. As the jaws of the clamps gripped the bar upon opposite sides, the current entered the bar so that the stream lines of current very soon became parallel with the length of the bar and were assumed to be almost perfectly so at the potential points.

A chemical analysis of the sample was made by Mr. H. E. Rankin of Princeton University. He found that the magnesium was 100 per cent pure within the limits of error of his analysis.

The **method of measurement** was the Kelvin-double-bridge method. For the **theory** of the method see par. 610.

The **apparatus** used consisted of the following: A manganin standard resistance (very accurate) of 0.00125 ohm which would carry 200 amperes without undue heating. For the proportional arms, two 100-ohm coils for the a and α resistances, kept fixed, and for the b and β resistances two plug-decade resistance boxes of 10,000 ohms capacity and variable in steps of 1 ohm. The resistance in these boxes was varied to secure a balance. The source of current was three small storage cells joined in parallel, and a rheostat held the current at between 10 and 20 amperes. A key was used in the battery circuit. The galvanometer was a Leeds and Northrup H Form Galvanometer of the suspended-coil D'Arsonval type. It was undamped upon open circuit. Its approximate constants were: Resistance of coil 550 ohms.; complete period 7 seconds; megohm sensibility 1 scale division deflection on a scale 1000 scale divisions from its mirror with 1 volt and 290 megohms in circuit.

Since it was necessary to regulate and measure accurately the temperature of the bar, a tin trough was provided in which the bar could be placed and kept underneath oil. Paraffin oil (instead of kerosene to avoid fire risk) was used. In the bottom of the trough was a solenoid of cotton-insulated german-silver wire thru which current from the 110-volt mains could be passed for the purpose of heating the oil. The temperatures were read with two accurate mercury thermometers.

The **connections** employed are given in the diagram below, Fig. 614.

The **procedure** was first to heat the oil to about 156° C. and to vary the ratio coils b and β until a rough balance was secured. The current was then cut off from the heating coil and the oil was vigorously stirred with a wooden paddle. When the temperature was uniform thruout the tank an accurate balance of the bridge was obtained by varying b and β together. As the

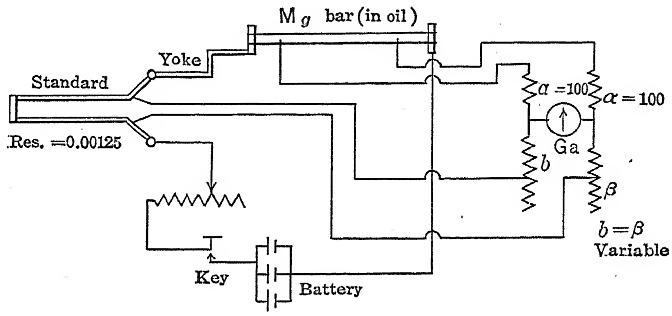

FIG. 614.

galvanometer was sensitive to changes in the resistances b, β of less than one ohm and as these resistances could not be varied in smaller steps the simple plan was adopted of setting the values of b and β so that, for the bridge to balance exactly, the oil must cool down a degree or two. Then, at the moment the galvanometer showed no deflection when the key was closed, a reading was taken of the temperature of the oil. This reading of temperature and the setting of b, ($b = \beta$), were recorded. The oil was then allowed to cool a few degrees further and a new reading was taken with a new setting of b and β. Altogether 37 readings were taken while the oil cooled from 154.8° to 21.4° C. As the temperature of the oil approaches room temperature the cooling proceeds very slowly and the last few readings were taken at widely separate intervals. The process might have been hastened by pouring into the can artificially cooled oil.

After the run was finished careful measurements were made of the resistances a and α including the lead wires to redetermine their resistances. They were found to be each 100 ohms within 0.02 of 1 per cent and were called exactly 100 ohms. The follow-

ing table exhibits the **data obtained** together with some obvious deductions.

Observers *A*, *B*, *C*, Nov. 13, 14, and 20.

Temp. degs. C.°	$b = \beta$ ohms	Resistivity ohms $\times 10^{-6}$	Conductivity mhos $\times 10^6$	Remarks
154.8	1677	6.9522	0.14384	Cooling started here, 3.07 P.M.
147.3	1710	6.8174	0.14657	
139.0	1740	6.7004	0.14924	
135.8	1760	6.6243	0.15096	
132.7	1780	6.5499	0.15268	
126.8	1810	6.4413	0.15525	3.17 P.M.
119.9	1840	6.3362	0.15782	
118.9	1850	6.3020	0.15868	
115.1	1870	6.2346	0.16040	
109.8	1900	6.1361	0.16247	
106.1	1920	6.0723	0.16468	
102.8	1940	6.0097	0.16640	
99.2	1960	5.9483	0.16812	
95.7	1980	5.8881	0.16983	
92.0	2000	5.8294	0.17155	
88.7	2020	5.7716	0.17326	3.27 P.M.
85.6	2040	5.7150	0.17498	
82.5	2060	5.6595	0.17669	
79.0	2080	5.6056	0.17841	3.45 P.M.
76.0	2100	5.5518	0.18013	
73.0	2120	5.4994	0.18184	
70.1	2140	5.4480	0.18355	
67.3	2160	5.3851	0.18569	
64.6	2180	5.3480	0.18699	
61.7	2200	5.2994	0.18870	4.00 P.M.
59.0	2220	5.2518	0.19041	
55.5	2250	5.1817	0.19299	
52.6	2270	5.1359	0.19470	4.20 P.M.
49.6	2290	5.0911	0.19642	
47.9	2310	5.0471	0.19814	
45.0	2330	5.0037	0.19985	
43.0	2350	4.9611	0.20157	4.40 P.M.
40.5	2370	4.9192	0.20328	
38.1	2390	4.8781	0.20500	5.00 P.M.
27.7	2479	4.7030	0.21263	6.00 P.M.
21.4	2537	4.5954	0.21761	11 A.M. (next day).
21.9	2532	4.6046	0.21718	2.30 P.M. (next day).

In column (1) are recorded the temperatures of the oil taken each time after the oil had been vigorously stirred. In column (2) are given the values of b and β. In column (3) are given the resistances in micro-ohms per centimeter-cube. In column (4) are given the reciprocal values of column (3), namely, the conductivity in mega-mhos. In column (5) are made some remarks relating especially to the time of taking the readings.

Various **deductions** might be made from the above data. It

is of special interest to note that the resistivity increases very nearly directly as the temperature. We are therefore justified in finding the value of the temperature coefficient from the formula,

$$\alpha = \frac{R_T - R_t}{R_t T - R_T t} = 0.00418_9$$

when we use the temperatures $t = 21.4°$ and $T = 154.8°$ and the resistivities corresponding to these temperatures. Applying this coefficient we find:

Resistivity of magnesium at 20° C. $= 4.56_9 \times 10^{-6}$ ohm.

The **probable precision** of the above results cannot be ascertained from this single determination, for unrecognized systematic errors may have entered. As the measurements were made, however, with much care and all the resistances were known to be accurate to better than 0.05 of 1 per cent the presumption is strong that the accuracy of 0.2 of 1 per cent aimed for, has been obtained. It would, however, be unscientific to claim this precision as having been certainly obtained. To give the determination a very high probability of precision, within a specified per cent, it would be necessary to repeat the measurements with the dimensions of the specimen changed and with the use of different ratio coils, standard, and thermometer, if any of these were under suspicion. As a matter of fact, this same sample was measured about a year previously at which time it had not been milled down and its dimensions were greater. A different standard, ratio coils and thermometer were also employed. The value then found, for the resistivity at 20° C., was $4.56_4 \times 10^{-6}$ ohm.

This differs from the above result by about 0.11 of 1 per cent. Thus it may be concluded with a high degree of probability that the value last obtained is correct within 0.2 of 1 per cent, the precision sought.

To make the exhibition of the results of the measurements complete, curves giving the relations of resistivity and conductivity to temperature were plotted upon a sheet of cross-section paper of large size.

In any case where a number of observations are made of two quantities, where one is a function of the other, the most probable relations between the quantities are best exhibited by means of a curve. The curves drawn belonging to the above measurement are not given here because of lack of space.

CHAPTER VII.

THE DETERMINATION OF ELECTRICAL CONDUCTIVITY.

700. Standards of Conductivity; Their Relation. Useful Formulæ. — The determination of electrical conductivity has become commercially important because the money value of conductors, as conveyers of electrical energy, is directly proportional, other things being equal, to the conductivity of their material. There is, however, a certain confusion of ideas respecting the precise meaning of conductivity, which arises from the use of three different standards for this property. It is therefore important to preface our description of the methods of determining conductivity by a statement of the definitions of the standards employed and by showing the mathematical relations which connect them.

Conductance and resistance are terms which apply to the electrical properties of an electrical circuit.

$$\text{Conductance} = \frac{1}{\text{resistance}}.$$

Conductivity and resistivity are terms which apply to specific electrical properties of a conductor.

$$\text{Conductivity} = \frac{1}{\text{resistivity}}.$$

If ρ denotes resistivity, namely, the resistance between opposite faces of a centimeter-cube of the material, and S denotes the cross-section (supposed uniform) of the conductor, and R is the resistance of a length l, then

$$\rho = \frac{S}{l} R. \qquad (1)$$

Hence if σ is the conductivity, as above defined,

$$\frac{1}{\rho} = \sigma = \frac{l}{RS}. \qquad (2)$$

Now the resistance R cannot be considered constant as it varies with the temperature. Hence if R_0 is the resistance of the con-

ductor at the temperature $0°$ C. and θ is a coefficient by which R_0 must be multiplied to obtain the resistance at some other temperature t of the conductor we have

$$R_t = R_0\theta$$

whence,

$$\frac{1}{\rho_t} = \sigma_t = \frac{l}{R_0\theta S}. \tag{3}$$

According to Eq. (3) conductivity is a specific property of the material and is a function of the temperature. As resistivity is expressed in ohms, and as conductivity, according to Eq. (3) is the reciprocal of resistivity, it is expressed in the unit called the mho, cubic centimeter. (Ohm written backwards.)

It becomes convenient, however, to treat of the property of a conductor, whereby its quality as a conveyer of electrical energy is considered, in comparison with the property of another conductor taken as a standard in this respect. We shall then find, that, in the same way as specific gravity is the relation of the density of a substance in its actual state to the density of water under specified conditions, so conductivity may be considered to be the relation of the conductance of one conductor in its actual state to that of another conductor of like length and cross-section or like length and mass, which has been selected as the standard.

This point of view leads to two other definitions of conductivity, according as the sample selected for the comparison is reduced to a conductor of like length and cross-section or to a conductor of like length and mass, with the standard. The advantage to be gained, in defining conductivity as the ratio of two conductances, results from the fact that two conductors, which have the same temperature coefficients, may be compared in respect to their conductances without determining the temperature of either. It is then only necessary to be assured first, that the temperature coefficients are sufficiently near alike for making the comparison and second, that when the comparison is made both conductors, sample and standard, have the same temperature.

When the conductor selected for a standard is specified in a particular way with respect to resistance at a particular temperature, length and cross-section, or length and mass, it is defined as having unit conductivity without regard to temperature. If then another conductor be reduced to like dimensions, or like length and mass, and be compared with this standard in respect to con-

ductance (the temperature of standard and sample being merely the same at the time of the comparison) it will be found to have a conductance regardless of temperature which is a certain per cent of that of the standard. This is called the per cent conductivity of the sample.

Unfortunately two standards of conductivity are in common use and this has led to more or less confusion. These two standards are called Matthiessen's standards and were recommended by the Standard Wiring Table Committee, Jan. 17, 1893. They are thus defined:

Matthiessen's standard with respect to diameter is: a copper wire of circular and uniform cross-section, 1 meter long and 1 millimeter in diameter, which has a resistance at 0° C. of 0.0203 international ohm.

This definition applies also to a wire of uniform, but not circular cross-section when the cross-section is $\frac{\pi}{4}\ mm^2$.

Matthiessen's standard with respect to mass is: a copper wire of uniform cross-section, 1 meter long, which has a mass of 1 gram and a resistance at 0° C. of 0.14173 international ohm.

The connecting link between these two standards is the *density of copper*. For the two standards to be equivalent, that is, to be alike in specific resistance when at the same temperature, the copper of the meter-gram standard must have a particular density. To show this, call A the meter-millimeter standard, and B the meter-gram standard, and,

let l = the length of standard A,
S = the cross-section of standard A,
D = the diameter of standard A,
ρ_a = the resistivity of the copper of standard A,
δ = the density of the copper of standard B,
ρ_b = the resistivity of the copper of standard B,
L = the length of standard B,
s = the cross-section of standard B,
m = the mass of standard B,
r_a be the resistance at 0° C. of standard A, and
r_b be the resistance at 0° C. of standard B.

Then we have

$$\rho_a = \frac{Sr_a}{l}, \tag{4}$$

or
$$\rho_a = \frac{\pi D^2 r_a}{4l}, \tag{5}$$

and
$$\rho_b = \frac{s r_b}{L}. \tag{6}$$

Since
$$s = \frac{m}{L\delta},$$

we have, also,
$$\rho_b = \frac{m r_b}{L^2 \delta}. \tag{7}$$

If the standards A and B are to be alike in specific resistance we must have $\rho_b = \rho_a$, or
$$\frac{m r_b}{L^2 \delta} = \frac{\pi D^2 r_a}{4l}, \tag{8}$$

or we must have
$$\delta = \frac{4 l m r_b}{\pi D^2 L^2 r_a}. \tag{9}$$

Now in Eq. (9) we have, by the definitions of standards A and B,

$$l = L = 100 \text{ cms.},$$
$$m = 1,$$
$$r_b = 0.14173 \text{ ohm},$$
$$r_a = 0.0203 \text{ ohm},$$
$$D = 0.1 \text{ cm.},$$
$$\pi = 3.1416,$$

whence,
$$\delta = \frac{4 \times 100 \times 0.14173}{3.1416 \times 0.1^2 \times \overline{100}^2 \times 0.0203} = 8.8895 +,$$

which equals 8.89 within 0.0056 of 1 per cent.

It may be noted that the meter-gram standard, when in the form of a wire of circular cross-section of density 8.89, is 0.3785 mm. in diameter.

The value of the meter-millimeter standard when at 0° C., when expressed in mhos is readily obtained from Eq. (5). Thus, calling σ_0 its conductivity in mhos, we have
$$\sigma_0 = \frac{1}{\rho_a} = \frac{4l}{\pi D^2 r_a} = \frac{4 \times 100}{3.1416 \times 0.1^2 \times 0.0203} = 0.6273 \times 10^6. \tag{10}$$

Likewise the conductivity σ_0' of the meter-gram standard, when at 0° C. and of density 8.89, is obtained from Eq. (7) and is
$$\sigma_0' = \frac{1}{\rho_b} = \frac{L^2 \delta}{m r_b} = \frac{\overline{100}^2 \times 8.89}{1 \times 0.14173} = 0.6273 \times 10^6, \tag{11}$$

which value equals the other, as it should, when the density is taken as 8.89.

The value of either standard in mhos at other temperatures than 0° C. may be found if we know the temperature coefficient of the copper of which the standard is made.

The Standard Wiring Table Committee in defining the Matthiessen's standards did not specify the coefficient which the copper standards should have. But calling $\theta = 1 + \alpha t$, where α is the ordinary temperature coefficient of resistance, the value of Matthiessen's standards at any temperature expressed in mhos becomes

$$\sigma = \frac{0.6273 \times 10^6}{\theta}. \tag{12}$$

We shall now give some further useful relations between the general and the specific properties of conductors. It is first required to find an expression for the conductivity of any homogeneous conductor of uniform, circular cross-section when referred to Matthiessen's meter-millimeter standard.

The ohmic resistance r_T of any conductor at temperature T and temperature coefficient θ', where this is defined as the ratio of the resistance r_T at temperature T to the resistance r_0 at temperature 0° C., is

$$r_T = \frac{4\,L\theta'}{\pi\,D^2}\,k_0. \tag{13}$$

Here L is the length and D the diameter of the conductor, and k_0 is a constant which will depend for its value both upon the nature of the material and upon the units chosen for the length and diameter of the conductor. From Eq. (13)

$$\sigma_0 = \frac{1}{k_0} = \frac{4\,L\theta'}{\pi\,D^2 r_T}, \tag{14}$$

where σ_0 now expresses the conductance of the conductor at 0° C. Similarly, we would have for the conductance at 0° C., of any other conductor of circular cross-section,

$$\sigma_0' = \frac{1}{k_0'} = \frac{4\,l\theta}{\pi\,d^2 R_t}. \tag{15}$$

Here l is the length and d the diameter of the conductor. R_t is the resistance of the length l at t degrees and $\theta = \frac{R_t}{R_0}$, its temperature coefficient. Now, by definition, conductivity, is the ratio of the conductance of one conductor to the conductance of another

conductor taken as the standard, when the comparison is reduced to a comparison between conductors of like length and diameter at the same temperature.

Calling c conductivity (the unit of conductivity not as yet being defined), we have

$$c = \frac{\sigma_0'}{\sigma_0} = \frac{l\theta}{d^2R_t}\frac{D^2r_T}{L\theta'}. \tag{16}$$

If c is to be expressed in terms of Matthiessen's meter-millimeter standard, and is called C_s, then we shall have σ_0 of standard value and $c = C_s$, when,

$\theta' = 1$, $r_T = r_0 = 0.0203$, and $D = 1$ millimeter and $L =$ 1 meter. Thus

$$C_s = \frac{l\theta r_0}{d^2R_t}. \tag{17}$$

Eq. (17) enables the conductivity by Matthiessen's meter-millimeter standard to be calculated from a resistance measurement of a wire of circular cross-section, of temperature coefficient, $\theta = \frac{R_t}{R_0}$, of diameter d millimeters, of length l meters and of measured resistance at $t°$ C. of R_t ohms for l meters.

The expression for the conductivity of any conductor, when referred to Matthiessen's meter-gram standard, may be derived as follows:

The mass of a length l of any conductor, having a uniform cross-section S is $m = k'lS$, where k' is a constant. The ohmic resistance at $t°$ C. of any conductor is

$$R_t = \frac{k_0 l\theta}{S}$$

where k_0 is a constant which depends both upon the units of length and cross-section chosen and upon the nature of the material, and $\theta = \frac{R_t}{R_0}$ is the temperature coefficient. Placing the value of S found from the former expression in the latter gives

$$R_t = \frac{k'k_0l^2\theta}{m}. \tag{18}$$

If we choose $l = 1$, $m = 1$, and $t = 0°$ C., then $\theta = 1$, and $R_t = R_0 = k'k_0$,

and

$$R_t = \frac{R_0l^2\theta}{m},$$

or
$$C = \frac{1}{R_0} = \frac{l^2\theta}{mR_t} \tag{19}$$
is the conductance at 0° C. of a conductor of length l, mass m and resistance R_t, at t° C.

Similarly for any other conductor
$$C' = \frac{1}{r_0'} = \frac{L^2\theta'}{Mr_t'} \tag{20}$$
is the conductance at 0° C. of a conductor of length L, mass M and resistance r_t' at t° C.

A conductivity being the ratio of two conductances, we have
$$\frac{C}{C'} = \frac{l^2\theta}{mR_t}\,\frac{Mr_t'}{L^2\theta'}. \tag{21}$$
Now C' is a unit conductance, according to Matthiessen's meter-gram standard, when $L = 1$ meter, $M = 1$ gram, $r_t' = r_0' =$ 0.14173 ohm, and $t = 0$° C. in which case $\theta' = 1$. Using these values for L, M, r_t' and θ' in Eq. (21) we thus derive, as the expression for the conductivity in terms of the meter-gram standard,
$$C_w = \frac{l^2\theta r_0'}{mR_t}. \tag{22}$$
It should be noted that in Eq. (22) neither diameters nor densities appear. If the resistance of m grams of wire of length l meters is known at t° C., then by Eq. (22) we may calculate its conductivity in terms of Matthiessen's meter-gram standard, provided $\theta = \frac{R_t}{R_0}$ is known.

To connect expressions (17) and (22) write
$$m = \frac{\pi\, d^2 l\delta}{4},$$
where δ is the density of the material which is assumed to be in the form of a conductor of circular and uniform cross-section having a diameter d, and a length l.

Putting this value of m in Eq. (22) gives
$$C_w = \frac{4\, l\theta r_0'}{\pi\, d^2 R_t\, \delta} \tag{23}$$
and taking the ratio of Eq. (23) to Eq. (17) gives
$$\frac{C_w}{C_s} = \frac{4\, r_0'}{\pi r_0\, \delta}. \tag{24}$$
Giving in Eq. (24) r_0' and r_0 their values according to Matthiessen's

meter-gram and meter-millimeter standards, which are 0.14173 and 0.0203 respectively, we have the expression,

$$\frac{C_w}{C_s} = \frac{8.89}{\delta}. \tag{25}$$

The relation given in Eq. (25) enables the conductivity of a metallic conductor, when found by any method and expressed in terms of one of Matthiessen's standards, to be expressed in terms of the other, provided the density of the material is known. If the material of the conductor has a density the same as the standard, namely 8.89, then its conductivity will be expressed by the same number whether it be referred to the meter-millimeter or to the meter-gram standard. This will be the case for the accuracy required in engineering, when expressing the conductivities of samples of copper. This being so, a statement of the particular standard to which the conductivity of the copper is referred is often omitted. When the material, however, is iron or aluminum having a density different from that of copper, it is essential to state to which standard its conductivity is referred. The author believes that there would be a gain in simplicity and an avoidance of considerable confusion if one standard, the meter-gram standard, were exclusively employed in all engineering practice.

We shall illustrate by numerical examples the uses of Eqs. (17), (22), and (25) and then describe the methods in use for determining conductivities.

Example 1. — **Conductivity of Magnesium.**

By Eq. (17) conductivity by the meter-millimeter standard is

$$C_s = \frac{l\theta\, 0.0203}{d^2 R_t}.$$

Referring to the data in par. 614 for magnesium we find $R_t = R_{20} = 4.569 \times 10^{-6}$ ohm for the resistance between opposite faces of a centimeter cube. Now $\theta = 1 + \alpha\, 20 = 1 + 0.004189 \times 20 = 1.0838$. If A is the cross-section of the sample, which in this case is 1 sq. cm. or 100 sq. mm., we have

$$A = 100 = \frac{\pi\, d^2}{4}, \quad \text{hence} \quad d^2 = \frac{400}{\pi},$$

also $l = 0.01$ meter. Using these values in the general formula above we obtain

$$C_s = \frac{0.01 \times 1.0838 \times 0.0203 \times 3.1416}{400 \times 4.569 \times 10^{-6}} = 0.3782 \text{ conductivity},$$

or, as this quantity is usually stated, 37.82 per cent conductivity according to Matthiessen's meter-millimeter standard. By Eq. (25), the conductivity by Matthiessen's meter-gram standard is

$$C_w = \frac{8.89}{\delta} C_s.$$

Since the density found for magnesium is $\delta = 1.7232$ we find

$$C_w = \frac{8.89}{1.7232} \times 0.3782 = 1.951 \text{ conductivity},$$

or 195.1 per cent conductivity according to Matthiessen's meter-gram standard. Thus for corresponding cross-sections magnesium is 0.3782 times as good a conductor as copper and for corresponding masses it is 1.951 times as good a conductor.

Example 2. — **Conductivity of Aluminum.**

See, for the example, Appendix II, 5, Example 2.

Example 3.

See, for the example, Appendix II, 5, Example 3.

701. The Measurement of Conductivity. — The commercial measurement of conductivity, in terms of one of Matthiessen's Standards, involves the comparison of the resistance of a low-resistance wire having a large temperature coefficient with the resistance of another low-resistance wire having approximately the same temperature coefficient. If the standard and sample wires are maintained at the same temperature and both have nearly the same temperature coefficient, the comparison can be made quite accurately without a knowledge of the actual temperature. The method and apparatus best adapted commercially to a rapid and accurate determination of conductivity were both invented by Mr. Wm. Hoopes and the apparatus is generally known as the "Hoopes Bridge."

702. The Hoopes Bridge for Conductivity Determinations; Described. — The essential feature of the method consists in an adaptation of the Kelvin-double-bridge principle to a bridge fitted with a standard conductor and special scales from which the per cent conductivity of the sample, inserted in the bridge, is read directly, without calculation. This method may be explained as follows:

From the Kelvin-double-bridge arrangement, exhibited in Fig. 702a, it is evident that when the ratio $\frac{r}{p} = \frac{r_1}{p_1}$, we have the resistance

X between the potential points c and d given by the relation,

$$X = \frac{r}{p} P, \qquad (1)$$

where P is the resistance included between the potential points a and b. In practice $\frac{r}{p}$ is made equal to unity. Both r and p are chosen equal to 300 ohms, in order that the small resistances at the knife-edge contacts used for potential points, shall be very

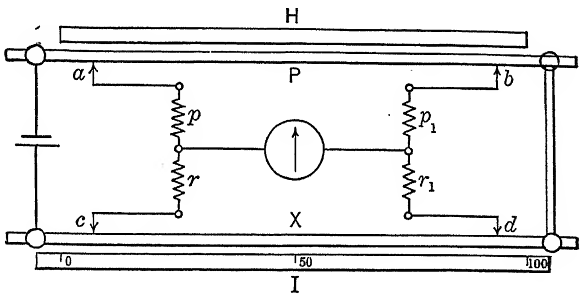

Fig. 702a.

small in comparison with the resistance of the ratio coils and hence negligible. A galvanometer of about 300 ohms resistance and 200 megohms sensibility is used with the bridge. When the galvanometer indicates that the bridge is balanced

$$X = P. \qquad (2)$$

Referring to Fig. 702a, suppose that X is a rod of copper having a cross-section S. Suppose the scale I to have 100 equal divisions of arbitrary length between the points 0 and 100. The resistance and its reciprocal, the conductance, of the rod X will depend upon the purity and the physical condition in regard to hardness, temperature, etc., of the copper of which it is made.

Now suppose that P is also a rod of copper having a uniform, but not necessarily known, cross-section. Suppose the points a and b are separated until they include a length of the rod P which will have the same resistance as the resistance of the rod X of cross-section S, of 100 per cent conductivity, and a length equal to 100 divisions of scale I. This being done, the bridge will be balanced. Now, if no change is made in P or in the position of the contacts a and b, and we substitute for X another rod X' of the same cross-section but of less conductivity, it will then be

necessary, in order to balance the bridge, to move the contact d towards the zero mark of scale I. Suppose the second rod X' has twice the resistance of X, or half the conductivity, then d will have to be moved for obtaining a balance half way down the scale I, that is, to the 50-division mark. Hence, in this case the 50-division mark indicates that rod X' has 50 per cent conductivity according to a standard based on cross-section.

Again, suppose we substitute for the rod X a rod X_1, which has 100 per cent conductivity, but twice the cross-section of rod X. In order to obtain a balance it will be necessary now to make the distance between a and b one-half what it was before. Then set b at the middle point of scale H. If now we substitute for X_1 a rod X_1' having the same dimensions, but of lower conductivity than X_1, a balance will be obtained by moving d toward the zero mark of scale I, and the reading of the scale will, as in the former case, give the per cent conductivity of X_1', which has twice the cross-section of the original rod X. Thus we may find a series of positions on the scale H corresponding to rods to be tested of various diameters or cross-sections.

Having a rod to test of a particular diameter, the slider b is set to a division on scale H corresponding to that particular diameter, then the reading on scale I gives directly, a balance being obtained, the per cent conductivity of the sample being tested. The Kelvin double bridge, fitted with the Hoopes' scales, as above described, gives the conductivity in terms of the meter-millimeter standard. As this bridge is actually constructed by its makers at the present time, the standard scale H, Fig. 702a, is laid off in gram weights instead of diameters, and the bridge readings are then given on the scale I in per cent conductivities based upon Matthiessen's meter-gram standard. The bridge gives by a direct reading the value designated in Eq. (22), par. 700, by the symbol C_w, multiplied by 100.

In Fig. 702b are given views of Hoopes' conductivity bridge out of and in its inclosing metal case. In practice it is always used in its case, which is made of metal to insure equality of temperature thruout its interior. The bridge measures the conductivity of wires from No. 0000 B. & S., to No. 18 B. & S. gauge.

It may be supplied with different standards to take care of wires of different sizes and various kinds. One standard covers a range of three wire-sizes, B. & S. gauge. The standard wires

are made of the same material as the samples to be measured to insure an equality between the temperature coefficients of standard and samples.

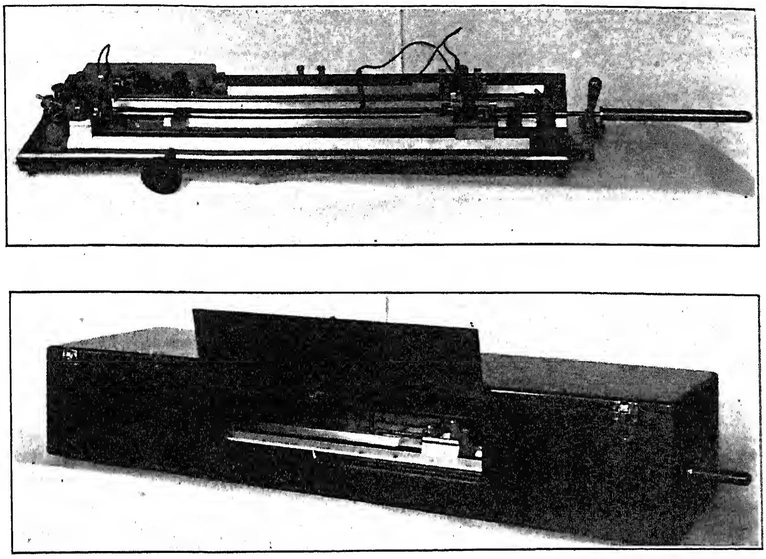

Fig. 702b.

703. The Hoopes Bridge; Operations Required for Using. — "The actual operation of making a measurement is so simple that it scarcely requires explanation. The sample to be tested is placed in a cutting-off machine and cut to the standard length. It is then weighed to within an accuracy of 0.05 of 1 per cent, and inserted in the bridge under the clamps and drawn out straight, but not stretched, by a gripping tool provided for the purpose.

The contact on the standard is placed at a setting corresponding to the weight in grams of the sample. The box is then closed and a few minutes allowed to elapse to give the sample time to acquire the temperature of the standard. The sliding contact is then moved back and forth, until a balance is obtained. The metal lid covering the glass in front of the box is then raised and the scale reading taken. This reading is the required per cent conductivity of the sample tested. If many samples are to be

tested they may all be cut off at one time, weighed, tagged and left in the rack in the box while measurements are being made. It will then require but two or three minutes for the particular sample under test to acquire the temperature of the standard. Ten samples an hour can easily be measured in this way. A larger number of samples may be measured per hour after the operator has gotten the measurement down to a system. As high as 150 samples per day may be tested with this bridge."

704. **Precautions to Observe in Using Hoopes Bridge.** — "The Hoopes bridge gives accurate results only when the temperatures of the standard wire and the sample are the same. A difference of 1° C. will affect the results about 0.4 of 1 per cent. A difference of temperature may be caused:

(a) By taking a reading too soon after placing a sample in the box, the sample not having had time to acquire the temperature of the standard.

(b) By using too large a current. This will cause an unequal heating of the sample and the standard when the two are not of the same dimensions.

(c) By changing standards. When a new sample wire is placed in the box it acquires very rapidly the temperature of its surroundings. When, however, a new standard bar is placed in the box it takes a very long time for the mass of rubber upon which this bar is mounted to take the same temperature as the rubber bar on which the sample is placed. As the standard wire rests upon the rubber of the standard bar it will follow it in its temperature changes. Hence when checking up two standards by comparing one with the other, it is important to make sure that the standard bar and the sample bar come to the same temperature. This may require that a standard bar newly put in the box be left in position a long time before a reading is taken.

As small a current should always be used as will give the necessary sensibility.

For No. 15 standard wire use about 4 amperes.
For No. 12 standard wire use about 4 to 5 amperes.
For No. 9 standard wire use about 8 to 10 amperes.
For No. 6 standard wire use about 11 to 12 amperes.
For No. 3 standard wire use about 15 amperes.
For No. 0 standard wire use about 18 to 20 amperes.
For No. 000 standard wire use about 50 to 100 amperes.

The standards are checked between themselves by the makers and are presumably correct. If the bridge fails to give accurate results the failure probably should be laid not to the standards but should be sought in

(a) unequal temperatures,
(b) too much current,
(c) bad contacts,
(d) inaccurate weighing,
(e) wrong setting on the standard scale, or
(f) wrong cutting off of a sample."

More detailed descriptions of this bridge and its accessories have been given by the makers, The Leeds and Northrup Company of Philadelphia, Pa., to whom the above description is due.

705. **Other Methods of Measuring Conductivity.** — As the Hoopes' conductivity bridge, described above, is intended for use where a large number of samples are to be measured, and as this rather costly piece of apparatus is not always to be had other methods are often employed. The method described in par. 614 under the heading, " Resistivity of Magnesium," yields data from which the conductivity is readily calculated by the Eqs. (17) and (22), par. 700. As Eq. (17) involves R_t, a resistance at a temperature t, and $\theta = \frac{R_t}{R_0}$, a coefficient which must be obtained by measurement when not known, any method which will determine these quantities with precision can be used for determining conductivity. If the conductivity is obtained in terms of Matthiessen's meter-millimeter standard, then by the relation

$$C_w = \frac{8.89}{\delta} C_s \text{ (Eq. 25, § 700)},$$

the conductivity in terms of Matthiessen's meter-gram standard may be calculated provided the density δ of the material is known or determined. However, as conductivity determinations are of great importance commercially, instrument makers have devised special apparatus other than the Hoopes bridge for this purpose, which, tho no new principles are involved, are adapted to yield good results with speed and precision.

706. **Equipment for Conductivity Determination; The Standard Resistance Variable.** — The following description of

an equipment is taken in part from literature printed by The Leeds and Northrup Company as it contains certain useful information of a practical nature.

The equipment recommended includes a Kelvin double bridge of the type described in par. 612, Fig. 612b. In the apparatus, as made by The Leeds and Northrup Company, the ratio coils giving ratios 0.1, 1 and 10 are permanently mounted on a base board with a variable low-resistance standard. The low-resistance standard itself has a total resistance of 0.1 ohm which may be varied between potential points by infinitesimal steps. Settings of one potential point in steps of 0.01 ohm are made by means of a plug inserted between blocks and a metal bar, and settings of the other potential point are made by a knife-edge contact which slides over a rod of manganin. This has a resistance of 0.01 ohm between the 0 and the 100 mark of a scale which lies alongside the rod.

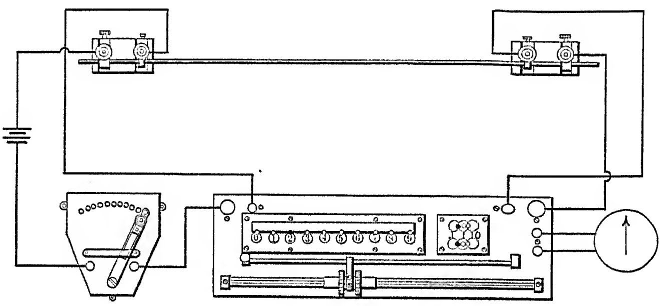

Fig. 706.

The entire equipment when fitted up for speed and convenience consists of the following:

The sliding-contact Kelvin double bridge, just described.

A galvanometer which has a sensibility of from 100 to 200 megohms, and a complete period of from 3 to 5 seconds and a resistance of about 200 to 500 ohms.

Means for cutting off the sample wires or rods to a standard length. (A special cutting-off machine is sold for this purpose.)

Suitable scales for weighing the samples.

Special clamps for holding the samples, and which at the same

time serve to lead the current in and out of the sample and to connect knife-edge potential points to the sample.

A rheostat switch by which the main circuit may be entirely opened or closed thru a resistance which can be diminished gradually.

Two cells of a storage battery of from 20 to 30 amperes-capacity each.

An accurate mercury thermometer reading from 0° C. to 60° C. and graduated in tenths of a degree.

A suitable tank of metal to hold kerosene oil and long enough to contain the sample.

The above equipment should be assembled as shown in Fig. 706 (the tank not being shown in the figure).

707. Method of Using Variable Resistance Standard for Conductivity Determinations. — To make a conductivity determination one should proceed as follows: The sample wire is cut off to some definite length, a length of 38 inches being recommended. The wire is first weighed to within an accuracy of 0.05 of 1 per cent. It is then fastened in the two clamps so as to bring the two potential points a known distance apart. If the two clamps are fastened rigidly to a marble or wood base the distance between the potential points will always be the same and need only be measured once. With a wire 38 inches long it may be taken 28 inches. For the best precision the wire and clamps should be placed under kerosene oil in the tank. The temperature of the oil is now carefully read, while it is being stirred, and at the same time the resistance of the sample between its potential points is determined. This last operation is effected by first adjusting the galvanometer reading to the zero of the scale with the main circuit open. This circuit is then closed by means of the rheostat key thru a high resistance and the bridge is roughly balanced by adjusting the one potential point of the bridge to within 0.01 ohm with the plug, and the other potential point with the sliding contact. The rheostat key is then moved so as to cut out more resistance, and a still closer adjustment for a balance of the bridge is made with the sliding contact. Finally the current thru the bridge is still further increased and an exact adjustment of the bridge is made. The current, however, should never be made so large as to perceptibly heat the wire. If the current is not heating the wire perceptibly the bridge will balance with the sliding contact

at the same point when the current employed is reduced to half the value used in obtaining the setting. The resistance of the sample at the temperature $t°$ C. is now

$$R_t = \text{bridge setting} \times \text{ratio plugged.}$$

Suppose the plug contact is set at 0.07 ohm and the sliding contact at 0.0058, then the bridge reading is 0.0758 ohm, and if the ratio plugged is 0.1 the resistance is $R_t = 0.00758$ ohm.

If the sample is copper or aluminum and its temperature coefficient is known, its conductivity may be calculated from the data obtained from the above observations. If its temperature coefficient is not known it must be determined by again measuring the resistance of the wire at some other temperature. The temperature may be changed by refilling the tank with oil which is colder or warmer than the oil used in the first measurement. For precision the oil in the second case should differ at least 15° C. from the oil in the first case. Let $R_t = R_0\,(1 + \alpha t)$ be the resistance when the oil is at temperature t, and let $R_t = R_0\,(1 + \alpha t_1)$ be its resistance when at temperature t_1, then eliminating R_0 from the two relations above, we have

$$\alpha = \frac{R_t - R_{t_1}}{R_{t_1}t - R_t t_1}.$$

Then as

$$\theta = \frac{R_t}{R_0} = 1 + \alpha t$$

(see § 700), this quantity is determined.

708. Method of Calculating Conductivity from Resistance Data. — We can now calculate the conductivity in terms of the meter-gram standard by Eq. (22), par. 700. This equation may be used as it stands, but it may be simplified for purposes of calculation when the samples used are copper and these are always cut off to a given length and the potential points are always set the same distance apart.

It has recently been shown by researches made at the Bureau of Standards at Washington that the coefficient θ for copper increases in direct proportion to the conductivity in the range from 0° to 100° C. For the commercial determination of the conductivity of copper, however, we may assume that this coefficient has at 20° C.

the value 1.0797.* Putting in Eq. (22), par. 700, this value of θ and the value 0.14173 for r_0' we have

$$C_w = \frac{l^2\, 0.14173 \times 1.07968}{mR_{20}}. \qquad (1)$$

Here R_{20} = the measured resistance in ohms of the sample between the potential points which distance is to be taken in meters. If l is this distance, which has been chosen 28 inches, we have $l = 28 \times 0.0254 = 0.7112$ meter. If m is the weight in grams of the 38 inches of sample cut off, then $\frac{28}{38}\, m$ is the weight in grams of the length l between the potential points. Hence,

$$C_w = \frac{\overline{0.7112}^2 \times 0.14173 \times 1.0797 \times 38}{28\, mR_{20}} = \frac{0.10504}{mR_{20}}. \qquad (2)$$

Instead of using Eq. (2) as it stands we may simplify the calculation by the use of logarithms.

Call $$K = 0.10504,$$

then,

$$\log C_w = \log K - (\log m + \log R_{20}). \qquad (3)$$

For example: suppose the sample is a No. 10 B. & S. gauge copper wire and we find $m = 45.163$ grams, and that the resistance at 22° C. is 0.00238 ohm. We must first calculate the resistance for 20° C. This can be done by applying the known temperature coefficient for copper, which may be taken 0.003984, or we can obtain the required value with sufficient exactness by subtracting from the value of the resistance at 22° C. $2 \times 0.004 = 0.008$ of this value. Thus $R_{20} = 0.00238 - 0.00238 \times 0.008 = 0.002361$ ohm. (The more exact value is 0.002363 ohm.)

We then find by Eq. (3)

$$\text{Log}\, C_w = \log 0.10504 - (\log 45.163 + \log 0.002363),$$

or

$$\text{Log}\, C_w = 9.0213547 - 10 - (1.6547828 + 7.3734637 - 10),$$

or $C_w = 0.9842$ or $C_w = 98.42$ per cent conductivity according to Matthiessen's meter-gram standard.

In the equation $C_w = \dfrac{K}{mR}$ the constant K will assume different values when R is taken at different temperatures. When R is taken at 20° C., the value of K is that in Eq. (2). Where many measurements are to be made a table should be constructed which

* See later value given in circular No. 31, "Copper Wire Tables," issued April 1, 1912 by the Bureau of Standards.

will give the values which K assumes when R is taken at different temperatures. It will then not be necessary always to reduce the resistance to its value at 20° C.

709. Conductivity Determinations with Fixed Resistance Standard and Variable Ratios. — For measuring the resistance of the sample a fixed standard low resistance, used in connection with variable ratio coils (see § 612) may be employed. The procedure would then be exactly the same as in the case described above, except that the bridge would be balanced by varying ratio coils instead of a low resistance. The fixed standard resistance should be, preferably, of approximately the same resistance as the sample.

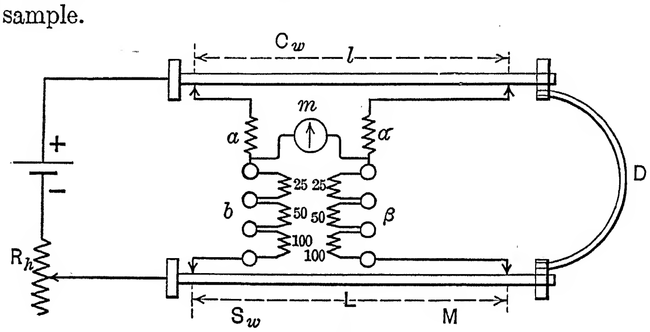

Fig. 709.

A set of variable ratio coils, as made by Otto Wolff of Berlin and described in par. 612, may be used for conductivity determinations very advantageously as follows: The method of using the ratio coils is planned to avoid the necessity of measuring the temperature of the sample by employing a standard made of the same material as the sample when making the comparison. The method is then similar in principle to the Hoopes bridge method described in par. 702.

A standard rod of the same material as the sample to be measured is prepared. This is provided with two potential terminals (set at or soldered) a fixed distance apart. This standard is connected with the variable ratio coils and sample to form a Kelvin bridge as shown in Fig. 709.

The sample and standard should be laid close to one another, and perhaps be covered so they will assume the same temperature. The connection D should be given as low a resistance as practicable. The rheostat key R_h may be used with additional

convenience. When a balance of the bridge has been obtained, the conductivity by the meter-gram standard is readily calculated from the following data and relations.

Let M = weight of the standard between its potential points.

Let L = distance between potential points of standard.

Let S_w = conductivity of the standard.

Also let m, l, and C_w be the corresponding quantities for the sample to be measured.

Then by Eq. (22) par. 700, we have

$$C_w = \frac{l^2\theta r_0'}{mR_t}, \tag{1}$$

and

$$S_w = \frac{L^2\theta r_0'}{MR_t'}. \tag{2}$$

If standard and sample are of the same material and are at the same temperature we have

$$\frac{C_w}{S_w} = \frac{Ml^2}{mL^2}\,\frac{R_t'}{R_t}.$$

Now, by the equation of the Kelvin bridge,

$$\frac{R_t'}{R_t} = \frac{b}{a},$$

and we have,

$$C_w = \frac{Ml^2}{mL^2}\,\frac{b}{a}S_w. \tag{3}$$

It will often be possible to choose l of such a length that the product $Ml^2 = mL^2$, in which case $C_w = \frac{b}{a}S_w$. Or at least one can make $l = L$, in which case $C_w = \frac{b}{a}\,\frac{M}{m}S_w$.

This method is to be recommended when one possesses a set of Otto Wolff variable ratio coils. If the standard is accurate it will give very accurate results and the calculations are very simple. In conclusion it should be remarked, that any method which will measure with accuracy the resistance of a short rod or wire at two temperatures will give the necessary data from which the conductivity may be deduced. When the density of the material is known, the conductivity may be expressed; in mho, cubic centimeter, units, or as a per cent conductivity of either Matthiessen's meter-millimeter or meter-gram standard. For scientific purposes the first is to be preferred and for commercial purposes the last.

CHAPTER VIII.

THE MEASUREMENT OF HIGH RESISTANCE.

800. **High Resistance Specified and Described.** — If a Wheatstone bridge has ratio arms which will give a ratio of 10,000 to 1 and a rheostat which reads to 10,000 ohms, then a resistance of 10^8 ohms or 100 megohms may be measured, theoretically. But the insulation of the bridge would need to be very high and the ratio coils very accurately adjusted to assure even moderately precise results. When a resistance, therefore, exceeds 10 megohms it can be more conveniently and more accurately measured by some one of the methods which have been devised specially for the purpose. We shall treat all resistances as "high resistances" which exceed 10 megohms and give the methods best adapted for their measurement.

Practically all metallic resistances are excluded from the class "high resistances," because it is rare to find a single metallic resistance unit which exceeds a megohm.

The methods to be described are adapted to the measurement of high resistances which may be considered under two general classes: First, resistances which are not associated with an appreciable electrostatic capacity and which obey exactly or approximately Ohm's law. Such high resistances are: the insulation resistance of electrical apparatus, the resistivity of insulating materials, the insulation over a surface of insulating material, or any high resistance where the effect of capacity need not be taken into account in making the measurement. Under this class, also, may be considered the special methods which are required for determining the insulation resistance of a wiring system while the power is on.

Second, high resistances which are associated with more or less electrostatic capacity, the presence of which causes the resistance to act as if it did not obey Ohm's law, at least in the first few moments after the current is made or broken. This class includes the resistance of condensers and the insulation resistance of long cables. For the determination of such resistances special methods

of procedure and specifications must be adopted to obtain concordant results which shall have a precise significance.

While the accurate measurement of ordinary resistances is generally made by some null or balance method, high resistances, on the other hand, are generally measured by some type of deflection method. Two general methods, of which there are many modifications in detail, are in use; one in which the high resistance is measured in terms of the deflections of a voltmeter, galvanometer, electrometer or like instrument, and one in which the high resistance is determined by the time required for the charge given to a condenser to leak partly away thru the resistance being measured. A high resistance of a few hundred megohms may also be measured by a balance method employing the principle of the Wheatstone bridge, when one has a sensitive galvanometer and a standard resistance of 0.1 megohm or more. We proceed to a description of these various ways of measuring a high resistance and the precautions which should be observed.

801. Wheatstone-Bridge Method of Measuring a Resistance of from 10 to 1000 Megohms.—For this measurement there will

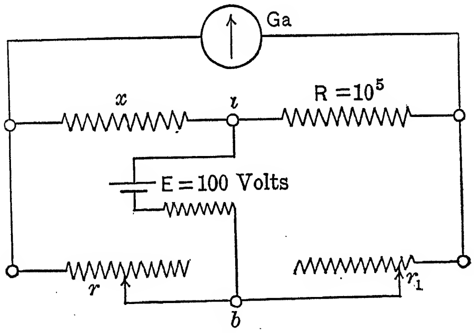

FIG. 801.

be required a standard 0.1 megohm and two accurate adjustable rheostats, each of 10,000 ohms total resistance; a D'Arsonval galvanometer of high resistance with a sensibility of 10^9 megohms or 10^{-9} ampere; and a source of direct E.M.F. of about 100 volts. Make the Wheatstone-bridge connections as shown in Fig. 801. Then if either r or r_1 be varied until the galvanometer shows no deflection

$$x = \frac{r}{r_1} R. \tag{1}$$

Suppose the standard R is 10^5 ohms, and r is plugged at its extreme value 10^4 ohms and r_1 is plugged at 1 ohm, then

$$x = \frac{10^4}{1} 10^5 = 10^9 \text{ ohms, or } 1000 \text{ megohms}$$

is the greatest resistance which can be measured with this disposition of resistances. If R is a megohm then ten times this resistance may be measured, provided the galvanometer is sufficiently sensitive.

Suppose the galvanometer will deflect one scale division with 10^{-9} ampere, or one tenth of a scale division with 10^{-10} ampere, and we wish to measure x to within 1 part in 1000, when $x = 10^9$ ohms and $R = 10^5$ ohms. We have to inquire what E.M.F. E must be applied to the bridge at the points a and b. The approximate value of this E.M.F. is easily found as follows: First, suppose the bridge is balanced when the unknown resistance has the value x, then very approximately, the current i which will flow thru x is, $i = \frac{E}{x}$.

The fall of potential over x with the bridge balanced is

$$E = ix, \tag{2}$$

and, if x receives an increment δx this fall of potential is

$$E + \delta E = ix + i\,\delta x. \tag{3}$$

Hence $\delta E = i\,\delta x$, or

$$\delta E = \frac{E}{x}\,\delta x. \tag{4}$$

Now δE is the E.M.F. effective to send a current δi thru the galvanometer and this is, very approximately, $\delta i = \frac{\delta E}{x}$. Hence $\delta E = x\,\delta i$ which value of δE put in Eq. (4) gives

$$E = x^2 \frac{\delta i}{\delta x}. \tag{5}$$

Since $x = 10^9$ ohms, $\delta i = 10^{-10}$ ampere and $\delta x = 10^6$ ohms, for an accuracy of 1 part in 1000, we obtain

$$E = 10^{18} \times \frac{10^{-10}}{10^6} = 100 \text{ volts.}$$

One hundred volts then is the necessary E.M.F. for measuring 1000 megohms, by the bridge method, to an accuracy of 0.1 of 1 per cent with a galvanometer which will give one division deflection with 10^{-9} ampere.

The chief precaution to observe in applying this method is to make sure that the galvanometer and the 0.1 megohm are perfectly insulated from earth. This is easily accomplished by setting the apparatus upon plates of glass or hard rubber and running the connections thru the air, supporting the wires on glass or hard rubber.

It is well also to include in the battery circuit a resistance of not less than 1000 ohms to protect the galvanometer from injury, should the resistance being measured break down.

802. Use of a Capacity in Connection with a Wheatstone Bridge for High-Resistance Measurements. — This method,

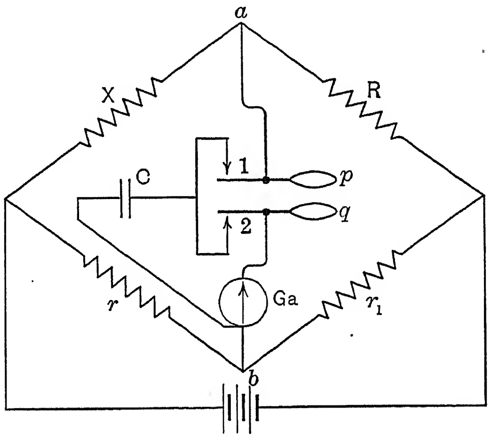

FIG. 802.

which is taken from "Measurement of Electrical Resistance," by W. A. Price, is added for the sake of completeness. The author thinks, however, there would not be much occasion for its employment in view of the better methods which are available.

The diagram, Fig. 802, is almost self-explanatory.

Here a condenser C is charged to the difference of potential of the points a, b by moving the levers of the key so lever q is insulated and lever p makes connection with point 1, and then the condenser is discharged thru the galvanometer Ga by moving the levers so p is insulated and q makes connection with point 2.

The resistance r or r_1 is adjusted until, when the condenser is connected for discharge, there is no deflection. When this adjust-

ment is effected, a and b must be at the same potential and the ordinary Wheatstone-bridge formula holds, giving

$$X = \frac{r}{r_1} R. \tag{1}$$

The advantage which the method is supposed to possess is one of greater sensitiveness, which results from the fact that C has time to become fully charged by the slow leak of current thru X, and this charge is then able to expend its energy suddenly upon the galvanometer, producing a deflection far greater than would be obtained by the same degree of unbalance of the bridge and with the galvanometer joined directly to the points a and b. In using the method the condenser C should be chosen of as great capacity as possible and the galvanometer should be of very high resistance.

803. Major Cardew's Electrometer Method of Measuring a High Resistance. — In this method, proposed by Major Cardew,

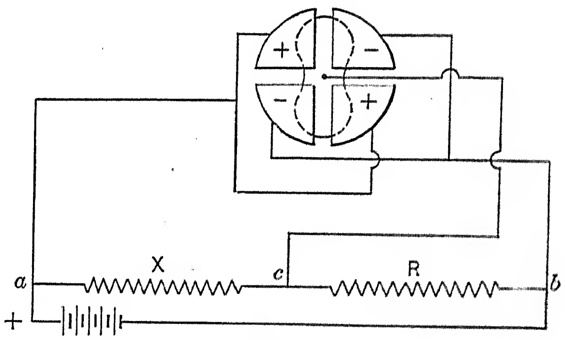

FIG. 803.

(Fig. 803) the standard R which must be made variable and the high resistance X to be measured are joined in series and their free ends a and b are connected to the quadrants of an electrometer. The vane is joined to the point of junction c of the two resistances. The resistance R is varied until the electrometer shows no deflection. Then $X = R$. In using this method it would not be necessary to produce an exact balance. The deflection of the electrometer is noted when R is too small, and calling R the resistance and d the deflection we then increase R by an amount δR and again note the deflection d' which should be in the opposite

direction. If for small deflections we assume the deflections to be proportional to the potential applied to the vane, we have

$$X = R + \frac{d}{d + d'}\delta R.$$

804. The Measurement of High Resistances, Unassociated with an Appreciable Capacity; Deflection Methods. — The measurement of the specific resistances of insulating materials, the insulation resistance of electrical apparatus, etc., is not a measurement which usually demands high precision. The resistance of insulating materials is subject to considerable fluctuation from temperature changes and other causes and hence the less precise, but more convenient and sensitive deflection methods are to be preferred to the null methods which are so superior in the case of medium and low resistances.

In describing these methods we shall reserve for separate paragraphs the methods of measuring the insulation of cables and condensers, as the presence of an appreciable capacity must considerably modify the procedure.

805. The Galvanometer and Accessory Apparatus for High-Resistance Measurement. — The instrument most used and best adapted to very high-resistance measurements by a deflection method is the galvanometer. In connection with the galvanometer a standard resistance is required. This standard may be either a megohm or a one-tenth megohm. Because of the expense of the former the latter is now almost universally employed. To increase the range of measurement the galvanometer is generally used with a high-resistance shunt which is made variable. This shunt serves the same purpose in high-resistance measurements as the variable ratio arms of a Wheatstone bridge in the measurement of medium resistances. Special types of highly insulated keys and insulating posts and plates complete the accessories required. We proceed to give the theory and uses of galvanometer shunts.

806. Galvanometer Shunts. — There are two types of galvanometer shunts, the ordinary and the universal or Ayrton shunt.

In the use of the ordinary shunt the resistance of the galvanometer must be known. In the diagram, Fig. 806a, let S be the resistance of the shunt and g the resistance of the galvanometer. The main current C will divide thru the shunt and galvanometer

in the inverse ratio of their resistances. If C_s is the current in the shunt and C_g the current in the galvanometer,

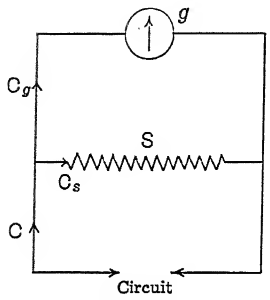

FIG. 806a.

$$\frac{C_g}{C_s} = \frac{S}{g}, \quad \text{and} \quad C_g + C_s = C,$$

whence,

$$C_g = \frac{S}{g + S} C \tag{1}$$

is the current thru the galvanometer, and

$$C_s = \frac{g}{g + S} C \tag{2}$$

is the current thru the shunt.

To obtain the value of the main current from the current thru the galvanometer, we have from Eq. (1)

$$C = \frac{g + S}{S} C_g = MC_g. \tag{3}$$

The quantity $M = \frac{g + S}{S}$ is called the multiplying power of the shunt, namely, it is the quantity by which the galvanometer current must be multiplied to obtain the main current.

If we wish to make the current in the galvanometer $\frac{1}{M}$ of the main current, that is, to reduce the sensibility of the galvanometer to $\frac{1}{M}$ we must make $S = \frac{g}{M - 1}$.

For putting this value of S in Eq. (1) we have

$$C_g = \frac{\frac{g}{M - 1}}{g + \frac{g}{M - 1}} C = \frac{1}{M} C. \tag{4}$$

The introduction of the shunt, however, changes the resistance of the circuit. After the galvanometer is shunted its resistance will be

$$g' = \frac{Sg}{S + g} = \frac{g}{M}. \tag{5}$$

If it is necessary to keep the resistance of the circuit constant,

when a shunt is added to the galvanometer, there must be introduced into the circuit a resistance which is

$$r' = g - g' = g - \frac{g}{M} = \frac{M-1}{M} g. \tag{6}$$

It is customary to make shunt boxes so that M may be given such values as 1, 10, 100, 1000, and 10,000. We should then have

$$M_1 = 1 \qquad S = \infty$$

$$M_2 = 10 \qquad S = \frac{1}{9} g$$

$$M_3 = 100 \qquad S = \frac{1}{99} g$$

$$M_4 = 1000 \qquad S = \frac{1}{999} g$$

$$M_5 = 10{,}000 \qquad S = \frac{1}{9999} g.$$

Some shunt boxes are provided also with means for adding the proper resistance in series with the circuit to maintain the resistance of the circuit constant when different values are given to the shunt. One arrangement used is shown in Fig. 806b.

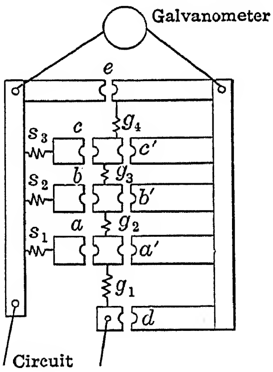

FIG. 806b.

Here the shunts are the coils S_1, S_2, S_3 and the compensating resistances the coils g_1, g_2, g_3, and g_4. A plug inserted at d puts the circuit directly to the galvanometer without a shunt. A plug inserted at b and b', for example, shunts the galvanometer with the shunt S_2 and puts into the circuit the compensating resistances $g_1 + g_2$, and similarly for plugs inserted at a, a' or c, c'. A plug at e short circuits the galvanometer and puts into the circuit the compensating resistance $g_1 + g_2 + g_3 + g_4$, which sum equals the resistance of the galvanometer alone.

This type of shunt should be wound with wire of the same temperature coefficient as the wire with which the galvanometer is wound. Practically all galvanometers are wound with copper

wire and this changes in resistance about 4 per cent for every 10° C. change in temperature. Unless the coils in the shunt have the same temperature coefficient and are maintained at the same temperature as the coil in the galvanometer (a matter hard to realize in practice) unallowable errors may result from the employment of this type of shunt. Furthermore every shunt must be adapted to the particular galvanometer with which it is to be used. These disadvantages are overcome in the universal or Ayrton shunt, which is the kind now almost universally in use. The theory and use of the Ayrton shunt is as follows:

807. The Ayrton or Universal Shunt. — Fig. 807a shows the disposition of the circuits employed. *a, b, c, d, e,* are resistance coils of manganin or other low-temperature-coefficient wire. These are joined in series and the galvanometer terminals are permanently connected to the terminals of the series. One terminal of the main circuit is permanently joined at one end, as at

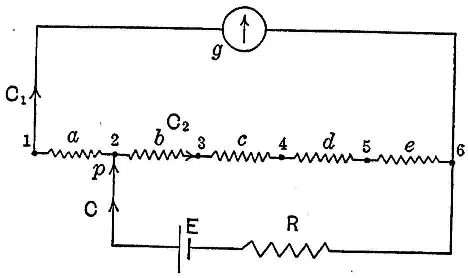

FIG. 807a.

the point 6, and an arrangement is provided by which the other terminal p may be moved to any of the points 1, 2, 3, 4, 5, 6. R represents the total resistance and E the E.M.F. included in the main circuit. The resistance of the galvanometer alone is g.

It was shown, Eq. (3), par. 806, that the multiplying power of any shunt is $M = \frac{g + S}{S}$, where S is the total resistance of the shunt and g is the resistance of the galvanometer. We can now construct for the different resistances in the galvanometer circuit and for the different resistances which shunt the galvanometer, when p is moved from the point 1 to 2 to 3, etc., the following table:

p on point	Res. in gal. circuit	Value of shunt, S	Multiplying power of shunt
1	g	$a+b+c+d+e$	$M_1=\frac{g+a+b+c+d+e}{a+b+c+d+e}$
2	$g+a$	$b+c+d+e$	$M_2=\frac{g+a+b+c+d+e}{b+c+d+e}$
3	$g+a+b$	$c+d+e$	$M_3=\frac{g+a+b+c+d+e}{c+d+e}$
4	$g+a+b+c$	$d+e$	$M_4=\frac{g+a+b+c+d+e}{d+e}$
5	$g+a+b+c+d$	e	$M_5=\frac{g+a+b+c+d+e}{e}$
6	$g+a+b+c+d+e$	0	$M_6=$Infinity.

It now appears from the 4th column of this table that the relative value of the multiplying power of the shunt for any two positions of the contact p is independent of the resistance of the galvanometer. Thus, calling $a + b + c + d + e = r$

$$\frac{M_1}{M_5} = \frac{e}{r}, \qquad \frac{M_1}{M_2} = \frac{b + c + d + e}{r},$$

$$\frac{M_1}{M_4} = \frac{d + e}{r}, \qquad \frac{M_1}{M_1} = 1.$$

$$\frac{M_1}{M_3} = \frac{c + d + e}{r},$$

It follows that, if the resistances are chosen so $e = 0.0001\,r$, $d + e = 0.001\,r$, $c + d + e = 0.01\,r$ and $b + c + d + e = 0.1\,r$, the sensibility possessed by the galvanometer (when shunted with the resistance r) will become 0.1, 0.01, 0.001, or 0.0001 as great according as the contact p rests on point 2, 3, 4, or 5. This result is obtained theoretically with a galvanometer of any resistance and with any value given to the total resistance r. The question then arises: what considerations govern the value which should be given to r? It will be observed, if the resistance r is made very high as compared with the resistance of the galvanometer, that, with the contact on point 3 or 4, the galvanometer has thrown in series with it a very considerable resistance which will reduce greatly the current C in the main circuit (Fig. 807a) unless the resistance R in this main circuit is also very high. On the other hand, if the resistance r is made very small, as compared with g, the galvanometer being permanently shunted with a low resistance has its intrinsic sensibility much reduced. Also,

if this is a D'Arsonval galvanometer it will be overdamped when r is small, and the coil will move sluggishly. Experience and practice show that r should be chosen approximately ten times the average resistance of the galvanometers which are to be used with the shunt.

In considering the principle of the Ayrton shunt it should be carefully noted, that, while the shunt reduces the *sensibility* of any galvanometer in a definite way it does not in general reduce the *current* thru the galvanometer in the same definite way. Thus, if we call C' the main current when p (Fig. 807a) is on point 1, the galvanometer current will be

$$C_g' = \frac{r}{g+r} C' = \frac{C'}{M_1}.$$

If the contact is now moved to some other point as 3, and we call the main current which is then flowing C''' the galvanometer current will be

$$C_g''' = \frac{c+d+e}{g+r} C''' = \frac{C'''}{M_3}.$$

The ratio of the galvanometer currents in these two cases is

$$\frac{C_g'''}{C_g'} = \frac{c+d+e}{r} \frac{C'''}{C'} = \frac{M_1}{M_3} \frac{C'''}{C'}.$$

Only when the external resistance R is very large, so that the effective resistance of the entire circuit remains practically constant for the different positions of the shunt contact, will the current C''' be the same as the current C'. In this case only will the ratio of the galvanometer currents for any two positions of the shunt contact be in the inverse ratio of the multiplying powers of the shunt for these two positions.

If by any device the main current C is maintained exactly constant, then the shunt will exactly cut down the galvanometer current in the same way it cuts down the sensibility. In measuring insulation resistances, R is usually very large and the Ayrton shunt may be used not only as a device to reduce galvanometer sensibility but also to reduce in like manner the galvanometer current by known and fixed amounts. It is in this latter way and for this purpose that the Ayrton shunt is chiefly used and it is therefore necessary to investigate the magnitude of the errors introduced, under different conditions of use, when it is assumed

that the galvanometer current is cut down in the same proportion as the galvanometer sensibility.

Referring to Fig. 807a, the current thru the galvanometer is

$$C_g = \frac{1}{M} C, \tag{1}$$

where M is the multiplying power of the shunt (which takes values M_1, M_2, etc., according as p is on point 1, 2, etc.).

Call R_s the shunted value of the galvanometer resistance and R the resistance in the main circuit. Then if E is the E.M.F. of the source,

$$C = \frac{E}{R + R_s},$$

and

$$C_g = \frac{E}{M(R + R_s)}. \tag{2}$$

Let the contact be upon a point p such that $M = M_p$ and $R_s = R_s'$, then the galvanometer current will be

$$C_g' = \frac{E}{M_p} \frac{1}{R + R_s'}. \tag{3}$$

Now move the contact to a point q such that $M = M_q$ and $R_s = R_s''$; then the galvanometer current will be

$$C_g'' = \frac{E}{M_q} \frac{1}{R + R_s''}. \tag{4}$$

By taking the ratio of Eq. (4) to Eq. (3) we find

$$\frac{C_g''}{C_g'} = \frac{M_p}{M_q} \frac{R + R_s'}{R + R_s''}. \tag{5}$$

Eq. (5) shows that, in so far as $\dfrac{R + R_s'}{R + R_s''}$ differs from unity the ratio of the galvanometer currents for any two positions of the shunt differs from the inverse ratio of the multiplying powers of the shunt for the two positions.

Since

$$\frac{R + R_s'}{R + R_s''} = \frac{1 + \dfrac{R_s'}{R}}{1 + \dfrac{R_s''}{R}},$$

we note that when R is very large the fraction is practically unity, that is, the current in the main circuit is practically constant, while the current in the galvanometer is changed in the same ratio but inversely as the multiplying power of the shunt is changed.

The following typical case will serve to show the magnitude of the errors which actually result from assuming that the current in the galvanometer is altered in the ratio $\frac{M_p}{M_q}$. Reference is here made to Fig. 807b.

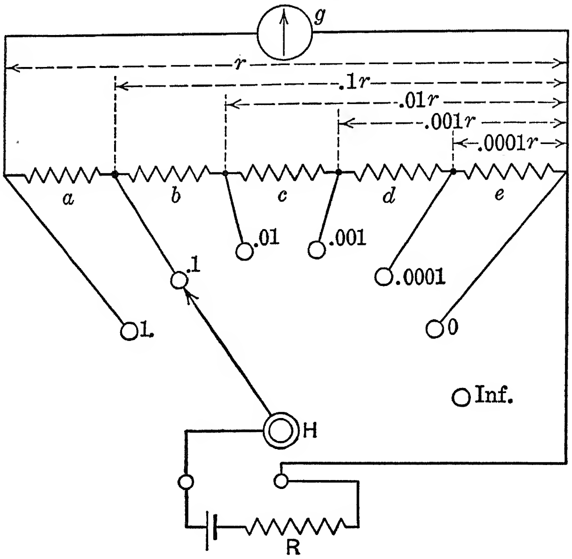

FIG. 807b.

By means of the handle H the contact can be moved to positions 1, 0.1, 0.01, 0.001, 0.0001, 0, Inf. In the case selected the shunt is intended for use with galvanometers having from 100 to 500 ohms resistance. We shall take

$$g = 350 \text{ ohms},$$
$$r = 3000 \text{ ohms},$$

and first assume that the external resistance is

$$R = 10^5 \text{ ohms}.$$

Then

$$a + b + c + d + e = r = 3000 \text{ ohms and } M_1 = \frac{g + r}{r},$$
$$b + c + d + e = 0.1\,r = 300 \text{ ohms and } M_2 = 10\,M_1,$$
$$c + d + e = 0.01\,r = 30 \text{ ohms and } M_3 = 100\,M_1,$$
$$d + e = 0.001\,r = 3 \text{ ohms and } M_4 = 1000\,M_1,$$
$$e = 0.0001\,r = 0.3 \text{ ohm and } M_5 = 10{,}000\,M_1.$$

The shunted galvanometer resistance then takes the following values:

$$R_s^{\mathrm{I}} = \frac{gr}{g+r} = \frac{350 \times 3000}{3350} = 313 \text{ ohms.}$$

$$R_s^{\mathrm{II}} = \frac{(g+a)(b+c+d+e)}{g+r} = \frac{(350+2700) \times 300}{3350} = 303.0,$$

$$R_s^{\mathrm{III}} = \frac{(g+a+b)(c+d+e)}{g+r} = \frac{(350+2700+270) \times 30}{3350} = 29.73.$$

$$R_s^{\mathrm{IV}} = \frac{(g+a+b+c)(d+e)}{g+r} = \frac{(350+2700+270+27) \times 3}{3350} = 2.997,$$

$$R_s^{\mathrm{V}} = \frac{(g+a+b+c+d)e}{g+r} = \frac{(350+2700+270+27+2.7) \times 0.3}{3350} = 0.3.$$

If we now put these numerical values in expressions of the form given in Eq. (5), we obtain the following values for the current in the galvanometer with the shunt in positions 1, 2, 3, 4, 5:

For position	Galvanometer current
1	C_g^{I}
2	$C_g^{\mathrm{II}} = \frac{C_g^{\mathrm{I}}}{10} \times \frac{10^5+313}{10^5+303}$
3	$C_g^{\mathrm{III}} = \frac{C_g^{\mathrm{I}}}{100} \times \frac{10^5+313}{10^5+30}$
4	$C_g^{\mathrm{IV}} = \frac{C_g^{\mathrm{I}}}{1000} \times \frac{10^5+313}{10^5+3}$
5	$C_g^{\mathrm{V}} = \frac{C_g^{\mathrm{I}}}{10000} \times \frac{10^5+313}{10^5+0.3}$

In this case it is seen that the last terms differ very little from unity. Thus the largest departure is for the position 5, where

$$\frac{10^5 + 313}{10^5 + 0.3} = 1.00313.$$

With an external resistance as great as 100,000 ohms and a shunt of total resistance 3000 ohms it is legitimate to assume, for most work, that the shunt cuts down the galvanometer current in the same way as it cuts down the galvanometer sensibility, that is, inversely as the multiplying power of the shunt. If, on the other hand, R is made as low as 1000 ohms the error for position 5 of the shunt would be as much as 31.3 per cent.

We can now find from the galvanometer deflection, with the shunt set in any position, the value of the main current C as follows: By the principle embodied in Eq. (1), par. 807, the current thru the galvanometer is equal to the main current divided by the multiplying power of the shunt, or, in general,

$$C_g = \frac{C}{M} = \frac{S}{g+S} C.$$

808. Galvanometer Constant, Obtained by Using an Ayrton Shunt. — To determine the constant of the galvanometer using an Ayrton shunt to which the galvanometer is permanently attached (as in Fig. 807b) we may proceed as follows:

Let $C' = M_1 C_g'$ be the main current with the shunt in position 1,
$C'' = M_2 C_g''$ be the main current with the shunt in position 2,
$C''' = M_3 C_g''$ be the main current with the shunt in position 3, with similar expressions for the other shunt positions.

If the galvanometer current is $C_g' = K_1 d_1$ for the shunt in position 1, where K_1 is a constant and d_1 the galvanometer deflection, we have

$$C' = M_1 K_1 d_1 = \left(\frac{g+r}{r} K_1\right) d_1 = K d_1,$$

where K is another constant. Similarly

$$C'' = M_2 K_1 d_2 = 10\, M_1 K_1 d_2 = 10 \left(\frac{g+r}{r} K_1\right) d_2 = 10\, K d_2,$$

$$C''' = 100\, K d_3, \text{ etc.}$$

Also,

$$C' = \frac{E}{R + R_s'}, \quad C'' = \frac{E}{R + R_s''}, \quad C''' = \frac{E}{R + R_s'''} \text{ etc.}$$

where R_s', R_s'', R_s''', etc., are the shunted galvanometer resistances with the contact on positions 1, 2, 3, etc. We therefore obtain

$$\frac{E}{R + R_s'} = K d_1,$$

or

$$K = \frac{E}{R + R_s'} \frac{1}{d_1},$$

or

$$K = 0.1 \frac{E}{R + R_s''} \frac{1}{d_2}, \quad \text{or} \quad K = 0.01 \frac{E}{R + R_s'''} \frac{1}{d_3} \text{ etc.}$$

Or, in general,

$$K = N \frac{E}{R + R_s} \frac{1}{d}. \tag{1}$$

Where N refers to the numbers 1, 0.1, 0.01, etc., stamped upon the

shunt for the shunt position used, R_s is the shunted galvanometer resistance for that position and d is the galvanometer deflection. In determining the constant K it is customary to take R so large, usually 10^5 ohms, that R_s is negligible in comparison. For insulation testing this approximation may be permitted and we have, with sufficient precision for many purposes,

$$K = \frac{NE}{Rd}. \tag{2}$$

809. Insulation Measurements with a Galvanometer and an Ayrton Shunt. — When the insulation resistance to be measured

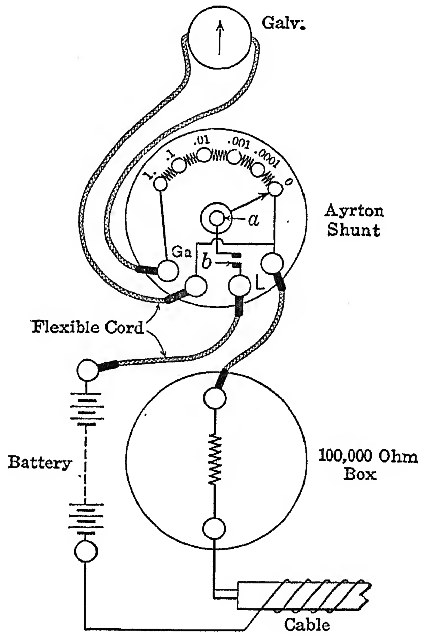

FIG. 809.

has a comparatively small capacity and dielectric absorption the procedure is very simple and is carried out in practice as follows:

A galvanometer (usually a D'Arsonval instrument), a standard one-hundred-thousand-ohm box, an Ayrton shunt, and the resistance to be measured are joined as in Fig. 809.

In a convenient type of construction, the battery key b is combined with the Ayrton shunt for compactness and also for convenience in manipulation. The handle a controls the key b

and is mounted so that it projects up thru the handle which operates the shunt. By depressing a, the contact b is closed. This is arranged so that it can be locked in the closed position. If the shunt and the D'Arsonval galvanometer are properly related the latter will be just aperiodic.

The shunt may be held in one hand while its handle and key is manipulated with the other hand. Both the shunt and one-hundred-thousand-ohm box should be highly insulated and are, for this purpose, often constructed entirely of hard rubber.

To make a measurement of insulation resistance, of a short length of cable for example, the procedure would be as follows: A battery of 50 or 100 dry cells is used. These should be first joined directly to the one-hundred-thousand-ohm box instead of to the cable as indicated in the figure. The *constant* of the galvanometer may now be obtained. This is not the same constant as that given by Eq. (2) in par. 808, which is the true galvanometer constant. It is an arbitrary constant defined by the relation $D = \frac{GN}{0.1}$. Here G is the constant sought, N the shunt setting 0.1, 0.01, 0.001, or 0.0001. The 0.1 is the one hundred thousand ohms expressed in megohms, and D is the galvanometer deflection which is obtained with the particular battery used for the measurement. Thus we have

$$G = \frac{D}{10N}. \tag{1}$$

In obtaining this constant, the shunt is first set at zero. The battery circuit is then closed by depressing the battery key b and the shunt is moved, first to $N = 0.0001$, and the deflection noted. If this is less than 25 small scale-divisions, the shunt is moved to $N = 0.001$, and, if still less than 25 scale-divisions, to $N = 0.01$. The final deflection, which should not be less than 25 divisions, is noted and called D. The shunt setting also being noted, the constant is given by expression (1) above.

The constant G having been thus determined the insulation resistance may now be measured by the following procedure: First connect the battery to the cable or resistance to be measured, as shown in Fig. 809. Reset the shunt to position zero. The one-hundred-thousand-ohm box, or 0.1 megohm, may be left in circuit or it may be short-circuited. In the former case the resistance measured will include this and will be 0.1 megohm too large.

Close the battery key b. If the cable has any considerable capacity and dielectric absorption, sufficient time must be allowed for the cable to become fully charged. No definite time for this can be specified and this matter will receive further treatment later on. But assuming electrification is complete, move the shunt successively to positions 0.0001, 0.001, 0.01, etc., until the deflection obtained is as large as possible and yet remains upon the galvanometer scale. Note this deflection calling it d, and also the shunt position used and call it N_1. Then in the same way that we obtained

$$D = G\frac{N}{0.1} \text{ we find } d = G\frac{N_1}{I},$$

where I is the insulation resistance sought expressed in megohms, or

$$I = \frac{GN_1}{d}. \tag{2}$$

The above procedure assumes that the insulation resistance of the lead wires and the one-hundred-thousand-ohm box is infinite. This may not be the case, however, and the matter must be tested by disconnecting the wire from the core of the cable and noting if there is any deflection, the shunt setting remaining the same. If there is a small deflection d_1 it must be subtracted from d, and then the final expression for the true insulation resistance becomes

$$I_t = \frac{GN_1}{d - d_1}. \tag{3}$$

I_t is the insulation resistance of the entire cable. If its total length is L feet, then its insulation resistance per mile will be

$$I_m = \frac{GN_1L}{(d - d_1)\,5280} \tag{4}$$

The following example will illustrate the use of formulas (1) and (4):

The galvanometer deflection, in obtaining its constant, was $D = 83$ millimeter-divisions. The shunt setting was $N = 0.0001$. Hence the constant was

$$G = \frac{83}{10 \times 0.0001} = 83{,}000.$$

This galvanometer was then used to determine the insulation resistance per mile of a cable 3200 feet long. The deflection due to the cable and lead wires was $d = 53$ divisions, and the shunt

position used was $N_1 = 1$. The deflection due to the lead wires alone was $d_1 = 3$ divisions. Hence by Eq. (4)

$$I_m = \frac{83{,}000 \times 1 \times 3200}{(53 - 3) \times 5280} = 1006 \text{ megohms per mile.}$$

810. Measurement of High Resistances by Leakage Methods. — When a high resistance is practically free from capacity it may be quite accurately and easily measured by observing the time which is required for a certain portion of the charge in a condenser to leak thru the high resistance. If the high resistance is associated with an appreciable capacity, as is the case with the insulation resistance of a cable, the methods of leakage may still be applied but require certain modifications. The leakage methods which follow apply when the capacity associated with the resistance is negligible.

811. Theory of Leakage of Condensers. — Assume, as a first approximation, that the resistance from one terminal to the other of the condenser itself is infinite.

Call C the capacity of the condenser. First, let the condenser be charged to a potential V_1. Second, join the terminals of the condenser with a high resistance R and maintain this connection for a time T until the potential of the condenser falls to a value V_1. Then disconnect the resistance and measure the potential V_1. The following relations will now hold:

$$q = Cv,$$

where q is the quantity of electricity in the condenser at any instant when its potential is v.

$$i = -\frac{dq}{dt} = -C\frac{dv}{dt}$$

is the current which leaves the condenser when the potential changes with the time. With the resistance R as the only resistance in circuit with the condenser this current must also equal, by Ohm's law, $\frac{v}{R}$. Hence,

$$\frac{v}{R} = -C\frac{dv}{dt}, \tag{1}$$

or

$$\int_0^T dt = -RC\int_{V_1}^{V_2}\frac{dv}{v} \tag{2}$$

from which we obtain, by integration,

$$T = -RC\,(\log_e V_2 - \log_e V_1),$$

or
$$T = RC \log_e \frac{V_1}{V_2}. \tag{3}$$

In Eq. (3) C is in farads, R in ohms and T in seconds. Since V_1 and V_2 occur as a ratio they may be taken in any units. Expressing C in microfarads and changing to common logarithms, we have

$$T = 2.3026 \times 10^{-6} RC_f \log_{10} \frac{V_1}{V_2}. \tag{4}$$

Eq. (4) is a useful relation between time, resistance and capacity, which enables any one of these three quantities to be calculated when the other two have been determined.

If we express the resistance in megohms and call it R_m, we have, within 0.018 of 1 per cent,

$$R_m = \frac{T}{C_f\, 2.303 \log_{10} \dfrac{V_1}{V_2}}. \tag{5}$$

812. High Resistance Measured by Leakage; Method I. — In this, the simplest application of the method of leakage, the apparatus required is a ballistic D'Arsonval galvanometer, of moderate sensibility and proportional scale, a mica condenser (preferably of 1 microfarad and adjustable in steps of 0.05 microfarad) and suitable highly insulated keys. The connections may be made as in Fig. 812a.

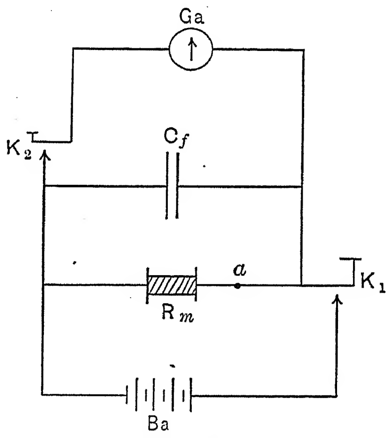

FIG. 812a.

In the figure, R_m is the resistance in megohms to be measured, C_f the mica condenser and Ga the ballistic galvanometer. The E.M.F. of the battery Ba should be chosen such that with the galvanometer and condenser C_f, which are used, a deflection to near the end of the galvanometer scale will be obtained when the condenser is charged and then immediately discharged thru the galvanometer.

To measure the resistance R_m the key K_1 is first closed and then opened, and immediately thereafter the key K_2 is closed which discharges the condenser C_f, charged to the full potential

of the battery, thru the ballistic galvanometer. The deflection d_1 is carefully noted. If the galvanometer is proportional in its readings to the quantity of the electricity discharged thru it, we shall have the deflection of the galvanometer,

$$d_1 = KQ_1 = KC_fV_1 = kV_1,$$

where Q_1 is the quantity of electricity discharged, K and k are constants, and V_1 is the potential of the battery.

The key K_2 is left open and the key K_1 is again closed. K_1 is now opened for a known time T seconds. While K_1 and K_2 are open the condenser C_f is losing its charge by leakage thru the high resistance R_m. At the end of T seconds K_2 is closed and the electricity which remains in the condenser is discharged thru the galvanometer giving a deflection $d_2 = KQ_2 = KC_fV_2 = kV_2$.

The resistance in megohms is then given, as in Eq. (5), par. 811, by the relation

$$R_m = \frac{T}{C_f\, 2.3 \log_{10} \dfrac{d_1}{d_2}}. \tag{1}$$

The result expressed by Eq. (1) assumes that the insulation resistance of the condenser itself is perfect. If the condenser leaks, as will generally be the case, a separate experiment must be made to determine its insulation resistance. To do this, disconnect the resistance R_m at a, and proceed in exactly the same manner as above. Let R_m' be the resistance of the condenser in megohms which is found, and call R_m'' the resistance of the condenser and specimen when in parallel. Then

$$\frac{1}{R_m''} = \frac{1}{R_m} + \frac{1}{R_m'},$$

from which

$$R_m = \frac{R''_m R'_m}{R' - R''_m}. \tag{2}$$

It is to be noted, that the choice of the time T and the capacity C_f in Eq. (1) is arbitrary, and the question arises, How should these quantities be selected in order that the precision of the measurement shall be as great as possible?

We may consider that the conditions of the measurement are made as favorable to precision as possible, when matters are so arranged that the proportionate error in the time being measured is equal to the proportionate error in the deflection being read.

Now the throw deflection, after the leakage has taken place, is proportional to the E.M.F. to which the condenser is then charged, namely to V_2. We may write, then, as the condition for maximum precision,

$$\frac{\delta T}{T} = -\frac{\delta V_2}{V_2}. \tag{3}$$

The negative sign is here used because as the time increases, the potential decreases. If we express the time, as in par. 811, by the relation

$$T = -RC \log_e V_2 + RC \log_e V_1$$

and call V_1 constant, we obtain

$$\delta T = -RC\frac{\delta V_2}{V_2}, \text{ or } -\frac{\delta V_2}{V_2} = \frac{\delta T}{RC}. \tag{4}$$

From Eqs. (3) and (4) we thus obtain

$$T = RC. \tag{5}$$

Putting $T = RC$ in Eq. (3), par. 811, we obtain

$$1 = \log_e \frac{V_1}{V_2}, \text{ or } V_2 = \frac{V_1}{e}.$$

Since the throw deflections may generally be taken as proportional to the potentials V_1 and V_2 we have, if d_1 and d_2 are these deflections,

$$d_2 = \frac{d_1}{e} = \frac{d_1}{2.7}. \tag{6}$$

(For functions of e see Appendix II, 1.)

Further, if, in Eq. (3), par. 811, we write $V_2 = \frac{V_1}{e}$, we have

$$T = RC \log_e e, \text{ or, as } \log_e e = 1,$$
$$T = RC.$$

Thus, if we make the second deflection $\frac{1}{e}$ of the first, we have $R = \frac{T}{C}$ and the calculation is simplified. If time is expressed in seconds, resistance in megohms and capacity in microfarads, we have

$$T = R_m C_f. \tag{7}$$

Eq. (7) implies that, for highest precision, the time in seconds that the condenser is permitted to leak has been so chosen

that it is equal to the product of the resistance being measured expressed in megohms, times the capacity used, expressed in microfarads. To use relation (7) we should first choose C_f of any convenient value and then by a preliminary trial find roughly the value of R_m. After this the measurement should be repeated allowing the condenser to leak a time $T = R_mC_f$. If this time is inconveniently long or short then the capacity C_f may be reduced or increased when the time of leak, for best precision, will change in the same proportion.

In ordinary practice it is not very essential to closely regard the above rule for highest precision, and the conditions for precision will be sufficiently met if the condenser is allowed to leak a time such that V_2 is about one-half or one-third of V_1. If the second deflection d_2 *is* made $\frac{1}{e} = \frac{1}{2.7}$, or 37 per cent of the first deflection d_1, the best conditions will be exactly realized.

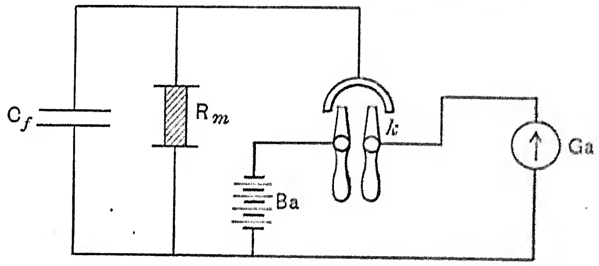

FIG. 812b.

The above procedure and arrangement of circuits for the measurement of resistance by leakage may be modified in details in a variety of ways. For example, for the keys K_1 and K_2, one may substitute a Rymer-Jones key when the connections would be as shown in Fig. 812b.

In this arrangement the first operation is to throw the handles of the Rymer-Jones key k to the right so as to charge the condenser to the difference of potential of the battery Ba. As soon as it is charged, the left-hand handle of the key is thrown to the left, when the condenser begins to leak thru the resistance R_m. When this has continued for T seconds, the right-hand handle is thrown to the left which discharges the remaining electricity thru the galvanometer. The high resistance is now calculated in the way given above.

813. High Resistance Measured by Leakage; Method II. — An electrometer of the quadrant type may be substituted for a galvanometer. In this case the connections may be made as in Fig. 813.

If the electrometer does not give deflections proportional to the potential applied to its quadrants it is necessary to make a preliminary calibration of its scale, plotting in a curve potentials against deflections. In this arrangement the needle is maintained charged by the dry pile or other source of high potential *b*.

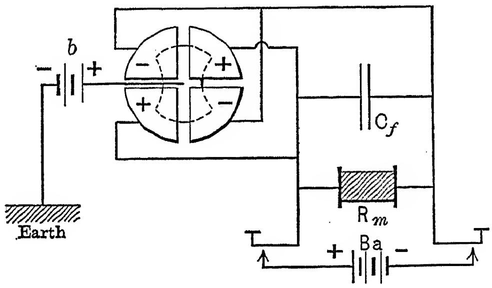

FIG. 813.

The specimen R_m is connected in parallel with the mica condenser C_f the opposite sides of which are joined to the two pairs of quadrants of the electrometer. By means of the battery *Ba* the condenser and the electrometer quadrants are charged to the same potential V_1. The battery *Ba* is then disconnected and the time accurately noted for the potential to fall from V_1 to V_2, the values of V_1 and V_2 being obtained from the observed deflections which will give the potentials from the previously obtained curve.

With the excellent galvanometers, having nearly proportional scales, which are now available, one would hardly select for this test an electrometer in preference to a galvanometer.

In the above class of measurements it generally happens that the specimen R_m being measured has not only resistance but more or less capacity. This would be the case if the sample were a few meters of rubber-covered wire of which the insulation resistance per unit of length of the covering is to be determined.

When such is the case, for precision, the capacity of the sample must be determined by a preliminary experiment. As this

capacity will be in shunt to the condenser C_f, used in the test, it must be added to the capacity of C_f in estimating its value. This capacity of the sample may usually be determined with sufficient exactness by charging it to a high potential and immediately thereafter discharging it thru a ballistic galvanometer. The throw deflection obtained when compared with the throw deflection which may be obtained by discharging a known capacity thru the galvanometer, will give the value of the unknown capacity.

If the capacity of the sample resistance is very considerable it will then not be necessary to use any auxiliary condenser, it being only necessary to note the time of leak of the charge of the capacity associated with the high resistance under measurement. Such a case arises in the measurement by leakage of the insulation resistance of a submarine cable.

814. Insulation Resistance of a Celluloid Condenser Obtained by the Method of Leakage. — We now give a description of an actual determination, by the method of leakage, of the insulation resistance of a celluloid condenser. It will serve to illustrate what has been said above respecting the determination of high resistance by leakage and will bring out some of the peculiarities of celluloid and the difficulty of precisely defining what constitutes the true ohmic resistance of a material of this nature.

The determination was made, under the author's direction, by Mr. W. Eves and Mr. R. T. Roche, in January, 1912.

The **object of the measurement** was to determine the specific resistance of pure translucent celluloid at one temperature.

The **precision sought** was not very high because it was assumed that a compound material of this nature, of which the chemical constitution was unknown, is not of such a definite character as to justify the expenditure of time which would be required to obtain a highly accurate determination. It was decided that a determination of the specific resistance in which the maximum error should not exceed 10 per cent would meet the requirements.

The **sample selected** consisted of sheets of translucent celluloid furnished by the Eastman Kodak Company and presumably of the same quality as that used for Kodak films. These sheets were 0.0050 cm thick. The sheets were made up with tin foil into a tightly compressed condenser. The tin foil was 0.0020 cm thick and the effective capacity area of each sheet of tin foil was 54.1 sq. cms. The over-all thickness of the condenser was 0.9 cm.

Consequently there were 127 sheets of foil and 128 sheets of celluloid.

The **method of measurement** was that of leakage. As the condenser (the specific insulation of which was to be determined) had sufficient capacity itself for the test it was unnecessary to use another condenser in parallel with it while the leak occurred; hence the condenser C_f in Fig. 812a, now represents the capacity of the celluloid condenser itself, while the resistance R_m represents the resistance thru the celluloid sheets of the condenser.

The celluloid condenser was connected in the circuits represented in Fig. 814.

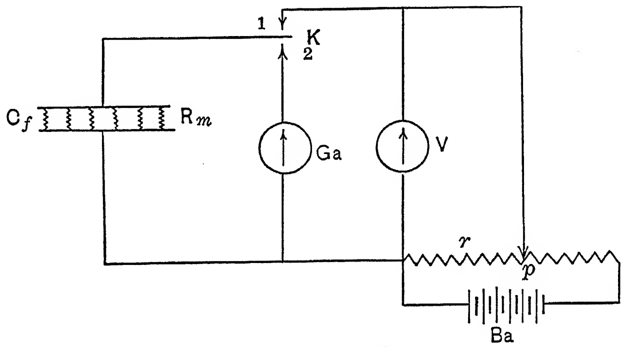

Fig. 814.

Here C_f is the condenser of celluloid. R_m is the internal resistance of its sheets in parallel. *Ga* is a D'Arsonval galvanometer. V is a standard voltmeter. *Ba* is a storage battery of a few cells, and r is a slide rheostat for adjusting the voltage applied to the condenser by moving the slider p. K is a highly insulated key which in position 1 charges the condenser to the potential indicated by V, in the middle position insulates it, and in position 2 discharges it thru the galvanometer.

The condenser was charged for a known time T' seconds. It was then instantly discharged by throwing K from 1 to 2 and the deflection d_1 noted. The condenser was then recharged for the same charging time, and insulated for such a time T seconds that, on discharging, the deflection d_2 indicated that only about 37 per cent of the original quantity of charge remained. This operation was repeated for times of charge varying from 1 to 5 seconds. The values of the insulation resistance of the condenser

were then obtained according to the theory given in par. 812, the formula used being

$$R_m = \frac{T}{C_f\, 2.3 \log_{10} \dfrac{d_1}{d_2}} \text{ megohms.}$$

In order to obtain the capacity of the celluloid condenser, for use in the above formula, a standard mica condenser of known capacity $C_s = 0.44$ M.F. was used. This condenser was charged and then immediately discharged thru the D'Arsonval galvanometer giving a deflection d_m. The celluloid condenser was next charged to the same potential for a time T' seconds and immediately discharged thru the same galvanometer giving a deflection d_c. The capacities being in the same ratio as the two deflections, the capacity of the celluloid condenser was $C_f = \dfrac{d_c}{d_m} C_s$, which is the value of the capacity used in the above formula.

The data obtained are given in the following table:

T' seconds	T seconds	d_1 / d_2	C_f microfarads	R_m megohms
1	0	$d_1 = 162$	2.308	2.01
1	5	$d_2 = 55$		
2	0	$d_1 = 190$	2.707	3.85
2	12	$d_2 = 60$		
3	0	$d_1 = 216$	3.078	4.09
3	14	$d_2 = 71$		
4	0	$d_1 = 236$	3.362	4.36
4	16	$d_2 = 79$		
5	0	$d_1 = 252$	3.590	4.57
5	18	$d_2 = 84$		

Note.—Charging voltage $V = 0.6$ volt. Temp. $= 16.5°$ C.
Scale 600 mm from mirror.

$$K = \frac{\text{microcoulombs}}{\text{galv. deflection in mm}} = 8.55 \times 10^{-3} = \text{galv. constant.}$$

A separate test was made to determine the proportionality of the galvanometer deflections and these were found to be proportional to the quantity of electricity discharged to better than 1 per cent.

The deduction of the results gives the relation which was found to exist between the specific insulation resistance of celluloid at 16.5° C. and the time that the insulation is submitted to the impressed difference of potential.

It appears that the resistance of celluloid is not constant but gradually increases more or less assymtotically with the time that a given E.M.F. is applied to it, within the limits of time, from 1 to 5 seconds, used in the measurement.

The data for the resistivity are deduced from the dimensions of the condenser and the above observations as follows:

$R = \frac{l}{S} \rho$ = resistance of 1 sheet of celluloid of effective area S and thickness l.

$\frac{R}{n} = R' = \frac{l}{nS} \rho$ = resistance of n sheets of celluloid joined in parallel, where R' is the resistance measured and given in megohms in the table above. Hence, $\rho = \frac{nS}{l} R'$. There were 127 sheets of tin foil which served as electrodes to 126 sheets of celluloid. S = 54.1 sq. cms and l = 0.005 cm. Taking the values of R' from the 5th column of the above table, expressed in ohms, for 1, 2, 3, 4 and 5 seconds charge we obtain, as the specific resistance of celluloid,

$$\rho_1 = 2.69 \times 10^{12} \text{ ohms}, \qquad \rho_4 = 5.84 \times 10^{12} \text{ ohms},$$
$$\rho_2 = 5.06 \times 10^{12} \text{ ohms}, \qquad \rho_5 = 6.12 \times 10^{12} \text{ ohms}.$$
$$\rho_3 = 5.48 \times 10^{12} \text{ ohms},$$

The specific resistance is seen to vary as the time of the application of the E.M.F. in the manner indicated.

The **reliability and precision** of the above results cannot be estimated with any great exactness because only one set of measurements was made and only one method was used. In this case, however, it is very probable that all the standards used were accurate and it is certain that the observations were made with care, and the calculations were checked by two computations. The method is, however, open to objections and both the quantity measured and the capacity value used in the formula are indefinite. Thus, since the resistance of the dielectric of the condenser was found to vary with the time of applied potential, Is one to take the time of charge or the time of charge plus the time allowed for the leakage as the true time that the dielectric is submitted to a potential? Probably an unknown intermediate value for the time of applied potential would be more correct than either.

This determination has been given here in full, as it serves to

illustrate the method of measurement by leakage, to indicate the procedure recommended by the author in recording a physical measurement, and because it serves to show the difficulties sometimes involved in obtaining a precise result in an actual case, tho the method and formula would lead one to expect that a high precision is easily obtainable.

815. High Resistance Measured by Leakage; Method III. — The following method of leakage for measuring either short intervals of time of electric contact,* or high resistance is a special application, devised and used by the author, of Lord Kelvin's null method of mixtures:

The method consists in charging two condensers with potentials of opposite sign, then permitting one of the condensers to leak for a known interval thru the high resistance to be measured, then mixing the charge which remains with the charge in the other condenser and discharging what still remains thru a galvanometer. The interval of leak may be adjusted until a zero deflection of the galvanometer is secured. From the data so obtained the value of the high resistance is deduced.

For explaining the practical carrying out of this method reference is made to Fig. 815a.

Here R is the high resistance to be measured. C_1 and C_2 are two, preferably equal, mica condensers of from 0.2 to 0.5 microfarads each. r_1 and r_2 are two resistances, the former being variable. They would ordinarily lie between 1000 and 10,000 ohms. S is a rocking switch which when thrown to the left makes contacts in the mercury cups 1 and 2, and when thrown to the right makes contacts in the mercury cups 3 and 4. The mercury stands higher in cup 4 than in cup 3 so contact will occur in 4 slightly earlier than in 3. Ba is a source of E.M.F., and Ga a D'Arsonval or other type of galvanometer. K is a highly insulated single contact key (not essential to the method).

If we assume $C_1 = C_2$ and R is without appreciable capacity the measurement would then be made as follows: Let r_1 equal approximately 2.7 r_2. With K open or closed throw S to the left giving C_1 and C_2 charges which will have the ratio 2.7 to 1. Return S to the insulate position and with K closed note the time T

* Consult, "On the Duration of Electrical Contact Between Impacting Spheres," by A. E. Kennelly and E. F. Northrup, *Journal of the Franklin Institute*, July, 1911.

during which C_1 will be losing its charge thru R. At the end of the interval T throw switch S to the right. This will mix the charge that remains in C_1 with the charge in C_2 and an instant thereafter discharge what remains thru the galvanometer. If the interval T is so chosen that the potential of C_1 sinks to the potential of C_2 the galvanometer will show no deflection when S

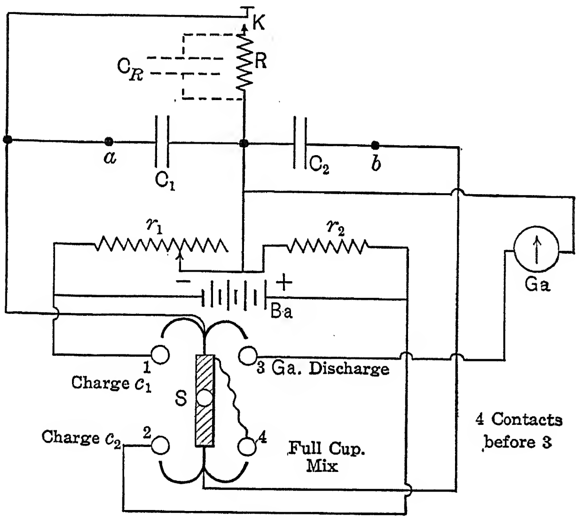

FIG. 815a.

is thrown to the right. The galvanometer will, however, probably deflect either to the right or to the left with the interval T which is first chosen. The amount and direction of the deflection will give information to enable a value of T, for a zero deflection, to be chosen closer upon a second trial. Three, or at most four trials should be sufficient to find the value of T to give a zero deflection. Thus, the change in the deflection will be very nearly proportional to the change in the time of leak, or

$$d_1 - d_2 = K\,(T_1 - T_2).$$

Hence,

$$d_1 - 0 = K\,(T_1 - T).$$

From these two relations we find

$$T = \frac{d_1T_2 - d_2T_1}{d_1 - d_2} \tag{1}$$

as the true value for the time of leak which will make the deflection zero. This relation may be verified by a third trial.

When this adjustment of the time is completed the value of the resistance being measured is, in megohms, when C_1 is in microfarads,

$$R_m = \frac{T}{2.303\, C_1 \log_{10} \frac{r_1}{r_2}}. \tag{2}$$

If the resistance R has a capacity C_R, as indicated by dotted lines in Fig. 815a, it will be necessary to determine this capacity. It is not, however, necessary to determine its value separately but only in conjunction with the capacity C_1.

Let $C_p = C_1 + C_R$. With K closed throw S to the left and after some definite interval throw S quickly to the right. Repeat, adjusting the resistances r_1 and r_2 until there is no deflection of the galvanometer. Then, by the theory of the method of mixtures, if r_1' and r_2' are the values of r_1 and r_2 which give a balance, we have

$$C_p = C_2 \frac{r_1'}{r_2'}.$$

We may now proceed to determine the resistance R as above but the value of R_m will now be given by the relation

$$R_m = \frac{T}{2.303\, C_2 \log_{10} \frac{C_p r_1}{C_2 r_2}}. \tag{3}$$

The series of values and the connections of a variable standard mica condenser are usually such that one box of capacities may be made to serve for the two condensers C_1 and C_2.

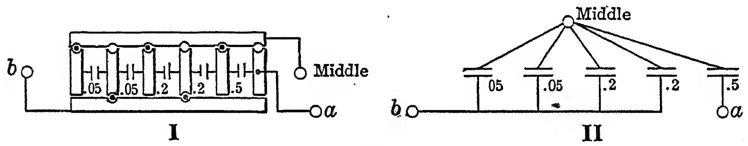

Fig. 815b.

A usual subdivision of a one-microfarad standard condenser and the manner of using it in a circuit like that of Fig. 815a is given in I and II, Fig. 815b, where the lettering of the two diagrams corresponds. This last method of measuring a high resistance by leakage has advantages over the others described as follows:

The method is a zero method and hence independent of the

proportionality of a galvanometer. A ballistic galvanometer is not required, as any type of galvanometer of moderate sensibility will serve as the indicating instrument. The sensibility of the method may be made anything desired by simply increasing or decreasing the E.M.F. of the source Ba (Fig. 815a). The final value of R_m is given in terms of quantities all of which can be very precisely determined. Thus $\frac{C_p}{C_2}$ is given as the ratio of two resistances, and likewise the ratio $\frac{r_1}{r_2}$ can be far more precisely determined than the ratio of two deflections. Lastly, if the two mica condensers C_1 and C_2 are precisely alike and mounted in the same box any diminution of the charge of one due to its own leakage can be assumed as practically the same as the diminution of the charge of the other due to its own leakage, hence the insulation resistance of the two condensers does not require consideration. But if there is a difference in the insulation resistance of each it is readily determined.

The method permits the easy determination of the capacity C_p without any change in the circuits. It may be objected that the method requires several adjustments of the interval of leak to secure a balance and hence an undue expenditure of time. In practice this objection hardly holds because an exact balance is not required as the precise time that would be required for an exact balance may be obtained by a simple calculation from the small deflection which finally remains after two time-adjustments have been made.

The above method was applied to the measurement of the specific resistance at room temperature of some samples of red fiber. The samples were in sheets 0.35 cm thick. In the middle and upon opposite sides of a sheet, square pieces of tin foil (much smaller than the sheets) were fastened. In one case the tin-foil sheets were fastened down with thin glue and in the second case they were simply pressed down with weights. The tin-foil sheets were 6.28 cms $\times$ 10 cms. The results were

$\rho_1 = 13.7 \times 10^3$ megohms between opposite faces of a centimeter cube when the tin foil was glued down, and

$\rho_2 = 616 \times 10^3$ megohms when the tin foil was merely pressed on the fiber. (Compare Appendix IV, 5.)

To make these determinations the following values were used for

the quantities (their meaning being given in Fig. 815a):

$$r_1 = 3672 \text{ ohms}, \ r_2 = 3000 \text{ ohms}.$$
$$C_1 = C_2 = 0.5 \text{ microfarad}.$$

With the electrodes glued on, the time of leak for a balance within − 4 scale-divisions was 8 seconds, and with the electrodes pressed on, it was 300 seconds for a balance within + 10 scale-divisions. With corrections in the time made for the small deflections obtained, the above values for the resistivity of fiber, expressed in megohms, were obtained. The ratio of about 45 to 1 between the resistivity obtained with electrodes pressed upon the fiber and electrodes glued upon the fiber shows how the apparent resistivity of a substance like fiber is affected by apparently slight changes in the conditions under which the test is conducted.

CHAPTER IX.

INSULATION RESISTANCE OF CABLES.

900. Introductory Note. — In the manufacture and installation of long cables, especially those of the submarine type, the insulation resistance per mile or per kilometer of the cables is tested. This, however, is only one of several tests which are made upon such cables. The phenomena observed in testing the insulation resistance are complicated and not well understood and a full treatment of the subject would not only be too extended for this work but it belongs rather to the subject of fault location and the study of the electrical properties of dielectrics. We shall, therefore, confine our treatment of insulation-resistance measurements of long cables to a brief outline of the standard methods employed and merely state the phenomena observed, requesting the reader to refer to special publications upon the subject for a fuller treatment, and for the explanations of the phenomena, which have been attempted.

901. Formula for Calculating the Insulation Resistance of a Cable. — If the specific resistance ρ of the insulation of a cable is known, the insulation resistance per kilometer of the cable may be calculated as follows:

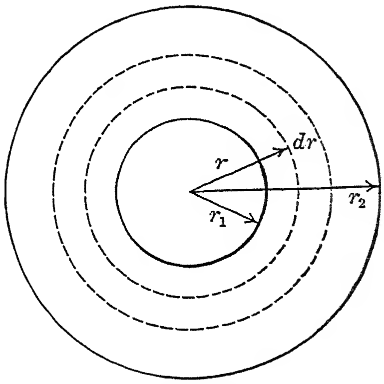

FIG. 901.

In Fig. 901, which represents the cross-section of an insulated cable, let r_1 = radius of metal core, r_2 = radius of outside surface of insulation, r = radial distance from center of core to any point p within the insulation, dr = the radial depth of an infinitely thin annulus of insulation and l = the length of the cable.

Then the resistance of the annulus is

$$dR = \frac{dr\rho}{2\pi r l}. \tag{1}$$

Integrating between the limits r_2 and r_1 we have

$$R = \frac{\rho}{2\pi l}\int_{r_1}^{r_2}\frac{dr}{r} = \frac{\rho}{2\pi l}\log_e\frac{r_2}{r_1}. \tag{2}$$

Expressing in common logarithms and combining the constants we obtain

$$R = \frac{0.3664\,\rho}{l}\log_{10}\frac{r_2}{r_1}. \tag{3}$$

Example.—Assume $\rho = 4.5 \times 10^{14}$. Take $l = 1$ kilometer $= 10^5$ cms, $r_1 = 0.13$ cm and $r_2 = 0.53$ cm. These constants apply, approximately, to a No. 10 B. & S. wire insulated with rubber $\frac{5}{32}$ inch thick.

Then $R = \dfrac{0.37 \times 4.5 \times 10^{14}}{10^5}\log_{10}\dfrac{0.53}{0.13} = 1.016 \times 10^9$ ohms per kilometer or, approximately, 1000 megohms per kilometer

902. Theorem upon the General Relation Between Capacity and Resistance.

(1) From a unit charge of electricity in a medium of constant specific inductive capacity K, $\dfrac{4\pi}{K}$ lines of electrostatic force issue. This is well established but may be easily proved as follows: If two point charges e and e' are located at a distance r from each other in space, the specific inductive capacity of which is K, then the force in dynes between these charges is, by the law of Coulomb,

$$F = \frac{ee'}{Kr^2}. \tag{1}$$

Now by definition the force at any point in space is equal to the rate of fall of the potential in the direction of the line of force at that point. If we take the potential to decrease as the distance n measured along the line of force increases, we have

$$F = -\frac{dv}{dn} = \frac{ee'}{Kr^2}. \tag{2}$$

Or if ds is an element of surface taken normal to the lines of force at any point in the space,

$$F\cdot ds = -\frac{dv}{dn}ds = \frac{ee'}{Kr^2}ds. \tag{3}$$

If we imagine one of the charges surrounded by a sphere of radius r and we integrate over the surface of this sphere, the area of which is $4\pi r^2$, we have

$$F\int\int ds = F\,4\pi r^2 = -\int\int\frac{dv}{dn}\,ds = \frac{ee'}{Kr^2}\int\int ds = \frac{ee'\,4\pi}{K}. \qquad (4)$$

Let the charge $e = e' = 1$, then

$$-\int\int\frac{dv}{dn}\,ds = \frac{4\pi}{K}. \qquad (5)$$

Eq. (5) states that the total induction, namely, the total number of lines of force which issue from a unit charge in space of specific inductive capacity K, is $\frac{4\pi}{K}$. If the charge is e then $\frac{4\pi e}{K}$ lines of force issue from it.

(2) If σ is the surface density of electricity upon any surface, namely, the number of electrostatic units of charge per unit area, then for the unit of area we have the electric force or induction,

$$F = -\frac{dv}{dn} = \frac{4\pi\sigma}{K}. \qquad (6)$$

This states that the electric force or fall of potential from the surface in the direction of the field is $\frac{4\pi}{K}$ times the surface density of the electricity.

From Eq. (6) we obtain

$$\sigma = -\frac{K}{4\pi}\frac{dv}{dn}. \qquad (7)$$

(3) Now let Fig. 902 represent an element of space. Let a current of electricity enter the element at the face $dx\,dy$, and normal to this face. If I_d is the current density, then the current which enters the face of the element will be

$$di = I_d\,dx\,dy. \qquad (8)$$

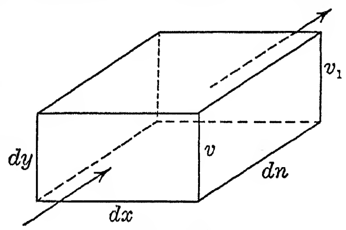

FIG. 902.

Suppose the difference of potential between the two ends of the element is $v - v_1$, and that dn is the length of the element. Then,

$$di = \frac{v - v_1}{dR},$$

where dR is the resistance in ohms of the element parallel to dn.

But $$dR = \frac{dn}{dx\,dy}\rho,$$

hence,

$$di = I_d\,dx\,dy = \frac{v - v_1}{dn\,\rho}\,dx\,dy,$$

or

$$I_d = \frac{v - v_1}{dn\,\rho} = -\frac{dv}{dn}\frac{1}{\rho} = -K'\frac{dv}{dn}, \tag{9}$$

where $\frac{1}{\rho} = K'$, the conductivity of the medium (assumed constant).

If we compare Eq. (7) with Eq. (9) we see that $\frac{K}{4\pi}$ in the electrostatic case corresponds with K' in the electromagnetic case. Thus any formula for lines of electrostatic induction becomes a formula for lines of current flow when we make $4\pi\sigma = I_d$ and change K into K'. Or one can say, by comparing Eq (7) and Eq. (9), that

$$\rho I_d = \frac{4\pi}{K}\sigma. \tag{10}$$

Eq. (10) leads to the important conclusion that, *any formula which gives the electrostatic induction thru a surface will express the value of the current density multiplied by the specific resistance of the medium when the medium of specific inductive capacity K is replaced by a medium of specific resistance ρ.*

(4) As an example of the above take the general case of the relation between electrostatic capacity and ohmic resistance. In general the capacity of any two electrodes will be the total charge upon the surface of one of them divided by the difference of potential between them. Or if σ is the surface density at any point of one electrode and ds is an element of the surface of one electrode, their electrostatic capacity is

$$C_{st} = \frac{\iint \sigma\,ds}{v - v_1}. \tag{11}$$

Now multiply both sides of (11) by $\frac{4\pi\sigma}{\rho K}$, and we have

$$\frac{4\pi C_{st}}{\rho K} = \frac{\iint \frac{4\pi}{\rho K}\sigma\,ds}{v - v_1} = \frac{\iint I_d\,ds}{v - v_1} = G \tag{12}$$

the conductance from one electrode to the other. In making this statement we conceive the medium of which the specific inductive capacity is K to be replaced by a medium of which the specific resistance is ρ, or we conceive the same medium to have this specific resistance. If we write $G = \frac{1}{R}$, where R is the resistance between the two electrodes, then

$$\frac{4\pi C_{st}}{\rho K} = \frac{1}{R}, \quad \text{or} \quad RC_{st} = \frac{\rho K}{4\pi}. \tag{13}$$

Eq. (13) shows that in every circuit where the electrostatic lines and the lines of current flow pass thru the same medium the product of the resistance and the capacity of the circuit is constant and equal to $\frac{\rho K}{4\pi}$.

A very simple case will make this physically clear. Suppose we have two parallel plates, each of area A, and separated by a distance d. Suppose that between these plates there is a slab of dielectric (much larger than the plates) and that the specific inductive capacity of this slab is K, and its specific resistance is ρ. Then the resistance between the plates is

$$R = \frac{\rho\, d}{A}. \tag{14}$$

The capacity of the plates is

$$C_{st} = \frac{AK}{4\pi d}, \tag{15}$$

and we have

$$RC_{st} = \frac{\rho\, d}{A}\,\frac{AK}{4\pi d} = \frac{\rho K}{4\pi}.$$

Namely, we cannot increase the electrostatic capacity of the plates by increasing the area without decreasing the resistance between them by a like amount, nor can we diminish the distance between them, thereby increasing their capacity, without equally diminishing the resistance between them.

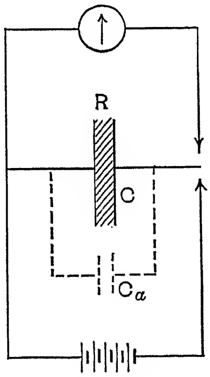

FIG. 903.

903. Application of Theorem to the Measurement of a High Resistance by Leakage. — Let it be required to measure the resistance of any condenser, as C, Fig. 903, by the method of leakage,

and assume first that the only capacity used is that of the condenser itself. By Eq. (3), par. 811, the time of leak is

$$T = RC \log_e \frac{V_1}{V_2}. \tag{1}$$

Now we have shown, Eq. (6), par. 812, that for the best results, the capacity, which has been charged to the potential V_1, should be allowed to leak until this potential has fallen to $\frac{V_1}{e}$. Hence, in this case,

$$T = RC \log_e e \quad \text{or, as } \log_e e = 1,$$
$$T = RC. \tag{2}$$

Since the product RC will remain constant however we change the thickness or area of the dielectric, it follows that the time of leak required for the potential to fall to $\frac{1}{e}$ of its value is the same for a condenser of any form, size or number of plates, provided the quality of the dielectric remains the same. We may express the value of this time as follows: In Eq. (2) C is expressed in farads and R in ohms. If we express C in electrostatic units, we have

$$T = \frac{RC_{st}}{9 \times 10^{11}}. \tag{3}$$

But by Eq. (13), par. 902,

$$RC_{st} = \frac{\rho K}{4\pi};$$

hence,

$$T = \frac{\rho K}{4\pi 9 \times 10^{11}} = 0.885 \times 10^{-13} \rho K. \tag{4}$$

An ordinary case would be one in which $\rho = 4.5 \times 10^{14}$ and $K = 5$, which values placed in Eq. (3) give

$$T = 199 \text{ seconds or about } 3\tfrac{1}{3} \text{ minutes.}$$

The conclusion to draw from this result is, that (in measuring the insulation resistance of a cable where the capacity of the cable is the only capacity which leaks) it will require the same time for the charge to leak away so $\frac{1}{e}$ of the original charge remains whatever dimensions are given to the cable either in length or in cross-section. By using a high charging potential the charge will give

a sensitive galvanometer a full-scale deflection, even tho the cable is very short. Hence, there will be no gain in this case in using a long length of cable over a short one.

We may, however, use an auxiliary capacity C_a as indicated in dotted line in Fig. 903. Then the total capacity which leaks will be

$$C_t = C_a + C_{st} = C_a + \frac{\rho K}{4\pi R},$$

the capacity in this relation being expressed in electrostatic units. By Eq. (3) we shall then have,

$$T_t = \frac{RC_a}{9 \times 10^{11}} + \frac{\rho K}{4\pi 9 \times 10^{11}}, \tag{5}$$

or

$$T_t = RC_{mf}\, 10^{-6} + \rho K\, 0.885 \times 10^{-13} \tag{6}$$

where C_{mf} is the capacity of the auxiliary condenser expressed in microfarads.

Eq. (6) shows that, if C_{mf} is taken large the time of leak is increased, but as C_{mf} may be an accurate high-resistance mica condenser the capacity of which can be accurately determined, while the capacity of the resistance under measurement may not be determinable with exactness, it may be very advantageous for precision to use an auxiliary condenser. Furthermore, it will not in this case be necessary to submit the resistance under measurement to the high potential which might otherwise be required to secure the necessary galvanometer deflections.

904. Insulation Resistance of a Long Cable by Deflection Methods. — When the insulation resistance of a long cable, such as a section of a submarine cable, is to be tested, it is more customary to use a direct deflection method than any other. A systematic plan of procedure is usually followed, which may be described as follows:

The apparatus used generally consists of a sensitive galvanometer of high resistance, a galvanometer shunt, a short-circuit key for the galvanometer, and a Rymer-Jones key, also a standard high resistance for calibrating the galvanometer. The resistances commonly measured in practice may range from 1 to 40,000 megohms. The value of the resistance is interpreted in terms of galvanometer deflections.

In making the test three operations are performed.

1st. The negative pole of the battery is joined to the cable core, the galvanometer being in circuit after the first rush of current is over. The positive pole of the battery is put to earth. The cable being in a tank of water, its outside sheath is earthed.

2d. The cable is allowed to discharge itself for a certain time, being disconnected from the battery. The deflections produced by the discharge current, after the first rush of current is over, are noted.

3d. The positive pole of the battery is joined to the cable core and the negative pole to the earth, and after the first rush of current is over the galvanometer deflections are again noted.

In these three operations a record of the time, as well as the galvanometer deflections, is carefully kept.

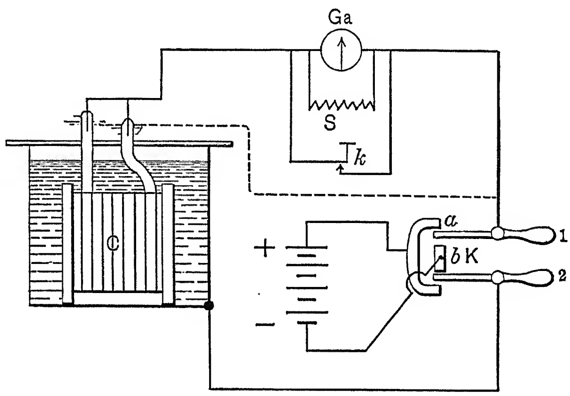

FIG. 904a.

For carrying out these three operations the apparatus may be connected as given in Fig. 904a. *K* is the Rymer-Jones key, and it is to be noted that the levers of the key cannot be placed in any position which will short-circuit the battery. *S* is the galvanometer shunt, which may or may not be required, depending upon the relation which the galvanometer sensibility bears to the resistance being measured and to the E.M.F. of the battery employed. *k* is the short-circuit key. The cable *C* is usually coiled up in a tank of water.

At the point where the wire from the galvanometer is attached to the cable core, care must be exercised to arrange matters, as far as possible, so no current will leak from the core over the out-

side surface of the insulation to the water in the tank. A leakage path of this kind will be a resistance in parallel with the resistance being measured and its unrecognized existence may lead to entirely incorrect results. Two methods are employed to prevent this leak. The first is to carefully pare the insulation at the end of the cable into a conical shape and then, being careful not to touch, moisten or otherwise contaminate the surface, plunge the end of the cable into hot paraffin wax. This will give the surface a high insulation. The other method is to employ a guard wire. This wire is shown in Fig. 904a in dotted line. Two or three turns of a fine wire are taken about the conical end of the insulation and then carried to the terminal of the galvanometer which is not attached to the cable core.

When the cable core is being charged negatively or positively there will be no leak over the surface of the insulation which will pass thru the galvanometer, because the conical surface of the insulation is maintained by the guard wire at practically the same potential as the core. In the second test, where the cable is allowed to discharge itself for a certain time, being disconnected from the battery, the guard wire will only serve to increase the rate of leak and hence is of doubtful advantage for this part of the test.

It is seen from the connections that with the levers 1 and 2 on a and b, respectively, the core of the cable is put to the positive pole and the tank to the negative pole of the battery. Putting levers 1 and 2 on b and a, respectively, puts the negative pole to the core and the positive pole to the tank. Putting both levers on b connects the cable core and tank thru the galvanometer without the battery being in circuit.

In the first operation, testing with negative current, the short-circuit key is kept closed while levers 1 and 2 are thrown to b and a until the first rush of current is over, this rush of current being due to the rapid charging of the capacity of the cable.

This rush of current is usually over, even in very long cables, in a few seconds. After about five seconds, the short-circuit key is opened. The galvanometer will now deflect to a maximum deflection, more or less rapidly, depending upon its natural period, and when it has obtained this maximum deflection, which will at once begin to decline, a record should be made of the deflection and the time, counting the time zero when the negative pole of

the battery is put to the core. The deflection of the galvanometer will steadily decrease, rapidly at first and less rapidly as the time advances, gradually reaching a minimum deflection after several minutes. It reaches its minimum or asymptotic value in perhaps 30 minutes. The test may be continued with advantage for this period, reading the deflections every minute.

The second operation is now begun. The key k is first closed and the levers of K are thrown to b. A few seconds thereafter k is opened again and the deflections of the galvanometer are now noted as in the first operation, again counting the time zero when the levers of K are changed. These deflections will be large at first and then gradually die away as in the first operation. This is due to the fact that the electricity, which has been stored in the dielectric, is given out, part of it with a rush, as a condenser discharges itself, while, thereafter, the remainder discharges slowly and at a decreasing rate.

This second part of the test may be continued for five minutes, the deflections being read every minute.

The third operation is performed exactly like the first except that the positive pole of the battery is put to the cable core. When the short-circuit key is opened it will be noted that the direction of the deflection is opposite to that obtained when charging the cable negatively and the same as that obtained upon discharging the cable. This part of the test may be continued for five minutes, the deflections being recorded every minute. It is called testing with a positive current and concludes the observations. The following curves, Fig. 904b, drawn from data given in "Kempe's Handbook of Electrical Testing," will exhibit the general character of the phenomena.

Curve A exhibits the decline of the deflection with the time, the electrification being negative, in the interval 1 minute to 15 minutes. The course of the curve in the interval 0 to 1 minute is not known. Beyond 15 minutes it may be studied and is found to assume an asymptotic value, as suggested by the dotted line.

Curve B exhibits the deflections, where the core and sheath of the cable are joined, or the cable is "earthed," in the interval 1 minute to 5 minutes. The course of the curve between 0 and 1 minute is not known. The deflection after the fifth minute is known to continue to decline gradually to a zero value, as suggested by the dotted line. In some instances it may take many

hours for the cable to become quite neutral; that is, for the deflection to become sensibly zero. In a cable of 10 or 15 miles length the deflection practically reaches zero in the course of 30 minutes.

Curve C exhibits the deflections, which gradually decline with the time, when the core of the cable is put to the positive pole of the battery, immediately upon the removal of the earth connection. The course of this curve is like that of curve A, and the same remark holds, that between 0 and 1 minute the curve is not accurately known, at least not from galvanometric observations.

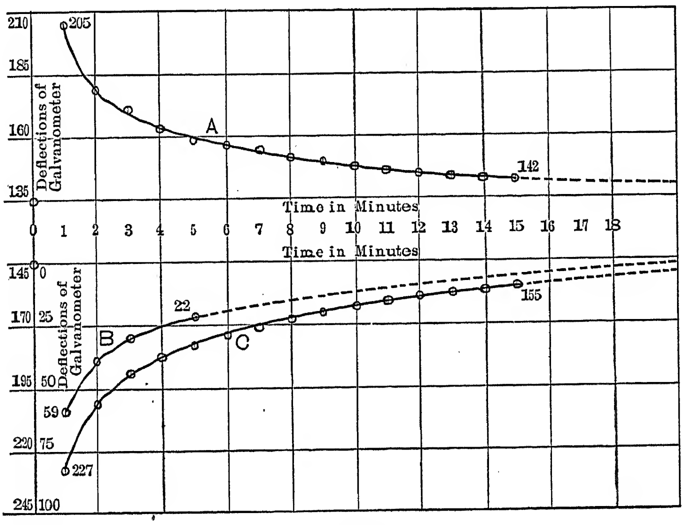

FIG. 904b.

In cables without defects or "faults" in their insulation the course of the curves will be regular and of the general character shown in Fig. 904b. Unless both ends of the cable core are joined to the galvanometer, as indicated in Fig. 904a, irregularities in the curves may be produced by inductive effects which would result, on shipboard, by the rolling of the ship, or, in factories, by induction from neighboring currents. With both ends of the core joined to the galvanometer these effects are avoided. Defective insulation of the lead wires may also produce irregularities in the curves, so one must not too hastily conclude that a cable is defective when the curves are seen to be irregular.

The quality of the insulation may be judged from the curves which in a perfect cable exhibit the following relations: If the current in the earth-reading (curve B) at the end of the first minute be added to the current at the end of the negative electrification (curve A) the sum should equal the current for the negative electrification at the end of 1 minute. Or in the case shown by the curves, $59 + 142 = 201$, which is not very different from 205.

Again, if the *last* negative electrification-reading be added to the recorded earth-reading at any period, the sum should equal the negative electrification-reading at the end of the same period. Thus the last electrification-reading is 142, the earth-reading at the end of the fourth minute is 25, and the negative electrification-reading at the end of the fourth minute is 164. We have $25 + 142 = 167$, which is approximately the correct sum.

Again, if from the deflection at the end of the first minute of positive electrification (curve C) we take the last earth-reading, it should equal the deflection at the end of the first minute of negative electrification. Thus, $227 - 22 = 205$.

The accuracy of these relations, the general smoothness of the curves, and the rate at which the deflections decline give the necessary information, which enables those accustomed to the requirements to judge of the perfection of the insulation of the cable.

The tests necessary for merely determining the specific resistance of the dielectric of the cable are not, of course, as elaborate as those given above, and in ordinary practice, when land cables or marine cables of moderate length are tested, only the first operation of testing with negative electrification is undertaken. The constant of the galvanometer and the method of calculating the specific resistance have already been fully considered. It should be noted here, however, that as the resistance of the insulation apparently, or really, increases with the time of electrification, one should always give, in stating the value of the resistance or the resistivity of the insulation, the time of electrification used in obtaining the value given. It is also found that the resistivity is greatly affected by changes of temperature and care should therefore be used to state the temperature which corresponds to the value given. Thus, in stating the resistivity of gutta percha, we should say: resistivity $= 4.5 \times 10^{14}$ at temperature 20° C.

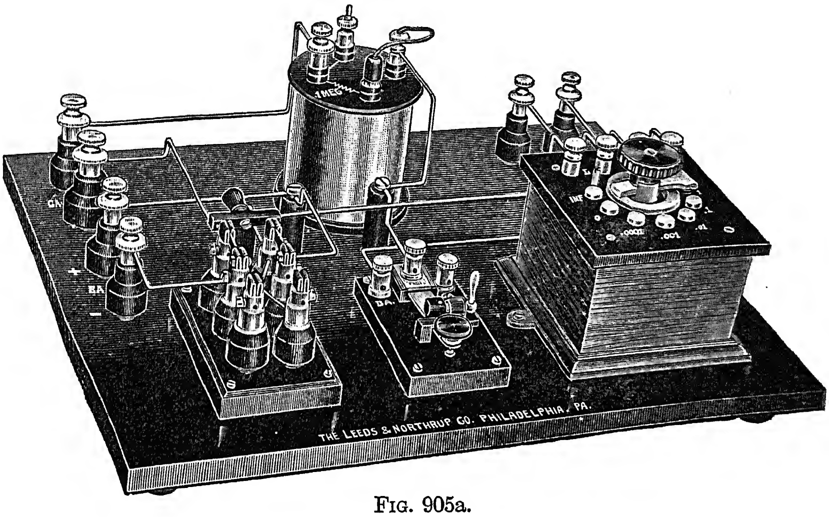

Fig. 905a.

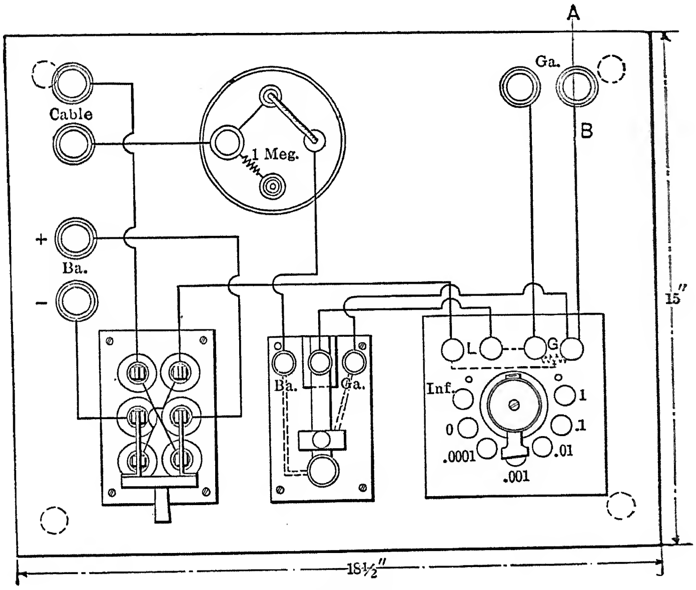

Fig. 905b.

and was obtained by deflection method with electrification of 1 minute.

905. Factory Testing Set for Insulation Measurements. — Wherever insulation measurements are to be made frequently, as in factories where cables are manufactured, it is necessary, or at least very desirable, to provide a full equipment designed expressly for the purpose. Such an equipment, called a "factory cable-testing set" made by The Leeds and Northrup Company of Philadelphia, Pa., has been upon the market for several years. This set is illustrated in Fig. 905a. Fig. 905b gives a top view of the set which shows the connections.

The galvanometer, and lamp and scale are not shown in the illustration. The chief features of this set are: All the instruments, except the galvanometer, and lamp and scale, are mounted upon a hard rubber plate to give high insulation between earth and the instruments. The wire connections are carried from the tops of hard rubber posts, petticoat-insulated. Except where the wires rest upon the posts, they are air-insulated. The switches and keys are arranged for convenient manipulation.

The instruments of the outfit consist of a D'Arsonval galvanometer of high resistance and about 1200 megohms sensibility; a lamp and scale, the latter 1 meter long; a standard resistance of 0.1 megohm mounted in a cylindrical case; an Ayrton universal shunt which gives multiplying powers of 1, 10, 100, 1000, 10,000 and infinity. There is a highly insulated double-pole, double-throw switch, and a key which has a lock-down device by which it may be permanently closed. The methods of using this set for insulation measurements do not differ in principle from those already described, while detailed instructions for its use will be furnished by the makers of the set.

CHAPTER X.

RESISTANCE AS DETERMINED WITH ALTERNATING CURRENT.

1000. **Remarks upon Resistance when Determined with Alternating Current.** — We call attention to the sense in which the term resistance is used in this work. It is a quantity which, in general, may be determined by direct currents and is defined as the fall of potential in any portion of a circuit, divided by the direct current which flows in that portion of the circuit. In metallic conductors, Ohm's law is obeyed, when regard is had to the temperature of the conductor.

When, however, the current in a circuit is not steady, or alternates, there will be a fall of potential in any portion of the circuit which is the product of a certain quantity R_{ac} and that component of the current which is in phase with the electromotive force, or more properly the fall of potential. The quantity R_{ac} is often a different quantity than that which is defined as ohmic resistance. It represents, in addition to a true ohmic resistance, anything which causes energy losses of whatever character occuring in the portion of the circuit considered. These energy losses may be due to hysteresis of iron in the circuit, to an electric absorption of dielectrics, to power wasted by currents and electrostatic potentials (resulting in currents external to the circuit) induced in neighboring circuits, to electric radiation and to other causes not to be classed with ohmic resistance. In one sense this quantity R_{ac} is a resistance, but it varies greatly with changes in frequency, current density, etc., and is not to be considered as a constant quantity or as a true ohmic resistance. From a broad viewpoint, a work of this character, which is intended to give a full treatment of the methods of measuring resistance, should include, also, all useful methods of measuring alternating-current resistance. The subject is, however, so extensive that we should transgress our purposes and limitations if we considered them here in full. We shall, however, describe one

method,* devised by the author, which will, in general, enable an alternating-current resistance to be determined with sufficient precision to meet commercial requirements. We proceed to a description of the method.

1001. To Measure an Alternating-Current Resistance; Apparatus Required. — For carrying out this method in a precise manner the apparatus required is a frequency meter to measure the frequency of the current used (which must be known, as the quantity being measured will vary with frequency), an alternating-current ammeter to give roughly the value of the current (for the alternating-current resistance will also, in general, depend upon the value of the current), a three-point double-throw switch for quickly changing connections, resistances, and an electrodynamometer. This last piece of apparatus should have sufficient capacity in its current coils to carry the full current without heating. Its hanging or potential coils should be two in number and so arranged as to form a system which is perfectly astatic in respect to the earth's field. The constant of the instrument will then be the same for direct and alternating currents. All good electrodynamometers are constructed in this way. Either the Rowland deflection type or Siemens' type constructed to be astatic, may be used. The method to be described was tested, using a Rowland deflection-type electrodynamometer. This instrument is illustrated in Fig. 1001, and is in part described as follows in The Leeds and Northrup Company 1911 Catalogue, No. 74:

"The electrodynamometer proper consists of two pairs of fixed coils and a swinging coil. One pair of fixed coils on the outside of the case is adapted to carrying currents as great as 50 amperes; one pair on the inside of the case is suitable for currents of 0.1 ampere. The swinging coil is adapted to currents as great as 0.1 ampere. The position of the fixed coils is such that the swinging coil turns in a field of force, which is nearly uniform for the angle thru which the coil moves. The dynamometer has attached to it a scale and telescope which are placed at a fixed distance from the mirror attached to the swinging coil. The scale and telescope rest on an arm which can be swung up out of the way when the instrument is not in use. The scale is not

* This method was first described by the author in a paper presented before the American Institute of Electrical Engineers at the Boston Meeting, June 24–28, 1912.

provided with a lateral adjustment. The coil is brought to zero by turning a micrometer screw which rotates the suspension tube. Thus, when the instrument is adjusted to read zero with no current flowing, the coil is always in the same position in the field. This requirement must be filled in order that the constants of the instrument, after being once determined, shall not alter. The hanging-coil system has two coils connected to form an astatic combination so that the instrument will not be affected in its deflection by the earth's field when used with direct currents."

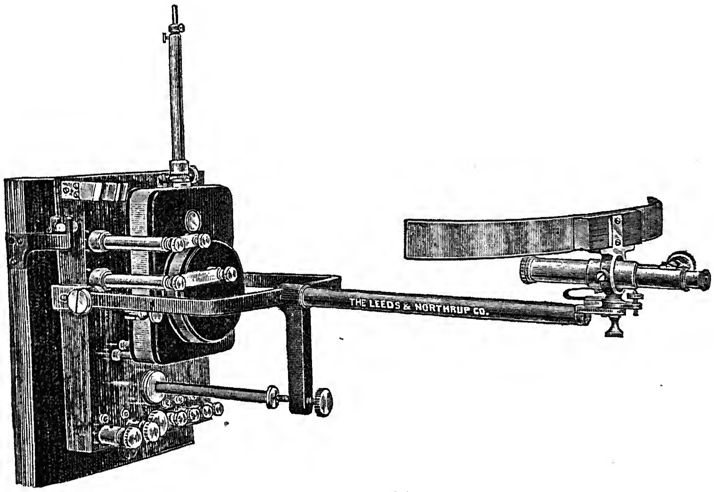

FIG. 1001.

1002. Description of Circuits and Theory of Method. — In I and II, Fig. 1002, GG are the fixed coils and hh the hanging, astatic system of the electrodynamometer. The hanging system has an ohmic resistance α and there is joined in series with this a non-inductive resistance ρ'. Let $\rho' + \alpha = \rho$, the entire resistance of the hanging-coil system. In the instrument, illustrated in Fig. 1001, the resistance α is about 18 ohms. It has a minute inductance, which is approximately 0.00045 henry. When ρ' is moderately large and non-inductive, we may consider, without sensible error, that the alternating current thru the hanging system is in phase with its E.M.F., even when the frequency is high. We shall so consider it in all that follows.

A represents a coil which contains iron. It is assumed that this coil has a certain ohmic resistance R_{dc} as measured by direct current, and a different resistance R as measured by an alternating current of a given value, wave form, and frequency. It is this latter resistance (not the impedance or inductance of A) which the method will enable us to determine. The resistance r is any resistance capable of carrying the full current. It may be a coil inductively wound but it must *not* contain iron or have such a section and resistivity that its resistance on alternating current will be different from its resistance on direct current due to hysteresis, skin effect, or other cause. By a sliding contact p, means must be provided for tapping upon this resistance at any point along its length as the diagram illustrates. ρ is a non-inductive resistance, which is equal to $\alpha + \rho'$, the resistance of the hanging-coil circuit. As will later be shown the connections can instantly be changed from the arrangement shown in I to that shown in II and vice versa.

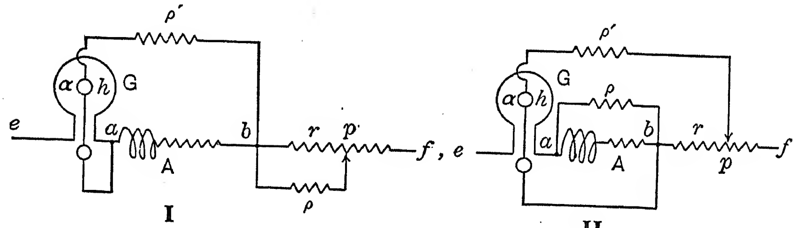

FIG. 1002.

We wish first to find the general expression for the power which the wattmeter measures when the connections are those shown in I, Fig. 1002. Call I the current in the fixed coils of the dynamometer, i the current in the hanging-coil circuit, ϕ the phase angle between the currents i and I. Then the deflection of the dynamometer is

$$D = KIi \cos \phi, \tag{1}$$

where K is the instrumental constant of the dynamometer. This constant in the case of a deflection instrument of the Rowland type will change slightly with the magnitude of the deflection. There is also an inductive action of the current in the fixed coil which tends to induce a current in the hanging-coil circuit when the plane of the movable system is not vertical to the plane of the fixed coils. This inductive action may vary in a complicated

way, so Eq. (1) cannot be taken as strictly true. If, however, the system is deflected by means of the torsion head, when there is no current thru the instrument, so that when current is introduced the system is brought back to the position where its plane is vertical to the plane of the fixed coils, then the inductive action is null and the relation given by Eq. (1) may be considered to hold very exactly. In the use of the dynamometer which follows, the system should be deflected by means of the torsion head, when there is no current flowing, to such an extent that, on introducing current, the instrument reads roughly at the zero of the scale. With this precaution observed the theoretical relations will be found to hold very exactly.

If we call V the impressed E.M.F. between the points a and b, I, then the current thru the hanging-coil circuit will be

$$i = \frac{V}{\alpha + \rho'} = \frac{V}{\rho}. \tag{2}$$

As stated above the current i will be approximately in phase with the E.M.F., V, because the inductance of the hanging coils is very minute.

By Eqs. (1) and (2) we have

$$D = \frac{K}{\rho} IV \cos \phi. \tag{3}$$

But $IV \cos \phi$ is the entire power W_t. This power is the sum of two parts, W the power consumed in A between the points a and b and W' the power consumed in the hanging-coil circuit. The value of this latter is

$$W' = \frac{V^2}{\rho}. \tag{4}$$

Thus we have

$$D = \frac{K}{\rho} W_t, \tag{5}$$

or

$$D = \frac{K}{\rho}\left(W + \frac{V^2}{\rho}\right). \tag{6}$$

From Eq. (6)

$$W = \frac{\rho}{K} D - \frac{V^2}{\rho}, \tag{7}$$

and from Eq. (5)

$$W_t = \frac{\rho}{K} D. \tag{8}$$

$\dfrac{V^2}{\rho}$ is generally a small quantity. ρ is known very precisely and V can be obtained with a voltmeter, hence Eq. (7) enables the true power spent in A to be accurately obtained. It is Eq. (8), however, which we wish to use in measuring the alternate-current resistance of A.

With the connections as shown in I, the torsion head is turned, so that with the current (steady as possible) which is flowing the deflection reads near the zero of the scale. The total power then being registered is given by Eq. (8).

The connections are now quickly changed to those shown in II. The main current will not be altered by this change in connections, for the resistance ρ is simply made to change places with an equal resistance. The total power which is registered, however, will now be

$$W_t' = \frac{\rho D'}{K} \tag{9}$$

when D' is the deflection which the dynamometer now gives. The contact p is moved along the resistance r until the deflection D' is made equal to the deflection D, then W_t' will be equal to W_t.

Since the main current I is the same for the connections I and II we have

$$W_t = I^2R' = I^2r', \tag{10}$$

or

$$R' = r'. \tag{11}$$

Here the quantity R' is not the alternating-current resistance of the coil A but it is the alternating-current resistance of this coil when shunted with the non-inductive resistance ρ. Similarly r' is the alternating-current resistance of r when shunted with the non-inductive resistance ρ.

We can write

$$R' = \frac{\rho R}{\rho + R}K' \text{ and } r' = \frac{\rho r}{\rho + r}k.$$

The alternating-current resistance of two parallel circuits when one or both of the branches contain reactance is not given by the same expression, as applies when the branch circuits are without reactance, hence the ordinary expression for branch circuits without reactance, namely $\dfrac{\rho R}{\rho + R}$, must be multiplied by some factor K', the value of which we now have to determine; also the factor k.

It is shown in "Alternating Currents" by Bedell and Crehore, pages 238 to 241, how the alternating-current resistance, or, as they call it, the equivalent resistance of any number of parallel circuits having self-induction and carrying alternating current, may be expressed. It is there shown that, in general,

$$R' = \frac{A}{A^2 + B^2\omega^2},$$

where

$$A = \frac{R_1}{R_1^2 + x_1^2} + \frac{R_2}{R_2^2 + x_2^2} + \cdots = \sum \frac{R}{R^2 + x^2},$$

and

$$B\omega = \frac{x_1}{R_1^2 + x_1^2} + \frac{x_2}{R_2^2 + x_2^2} + \cdots = \sum \frac{x}{R^2 + x}$$

in which expressions R_1, R_2, etc., are ohmic resistances and x_1, x_2, etc., are reactances of the several branches.

We can now find expressions which will give the values of K' and k. Here we have

$$A = \frac{1}{\rho} + \frac{R}{R^2 + x^2}$$

and

$$B\omega = \frac{x}{R^2 + x^2}.$$

We cannot, because of the necessity of brevity, give here the purely algebraic processes required for obtaining the final expression and so we shall present only the final results, which are as follows:

$$K' = 1 + \frac{\rho x^2}{[(R + \rho)^2 + x^2]\,R},$$

$$k = 1 + \frac{\rho x_1^2}{[(r + \rho)^2 + x_1^2\,]r}.$$

Call the fractional expressions α and α_1 respectively, then $K' = 1 + \alpha$ and $k = 1 + \alpha_1$.

This gives, because of Eq. 11,

$$\frac{R}{\rho + R}(1 + \alpha) = \frac{r}{\rho + r}(1 + \alpha_1).$$

It will be shown that, in general, when a sensitive electrodynamometer is used, α and α_1 are very small quantities which in most cases can be neglected.

We have the following cases:

1st. α and α_1 are negligible. Then

$$R = r. \tag{12}$$

2nd. α and α_1 are not negligible but are very nearly equal. Then, again,

$$R = r.$$

In these two cases the alternating-current resistance sought may be taken as numerically equal to the resistance r.

3d. $\alpha_1 = 0$, but α is not negligible. In this case

$$R = r \frac{1}{1 + \alpha \dfrac{\rho + r}{\rho}}. \tag{13}$$

4th. α and α_1 are not negligible and are unequal, but ρ is very large. Then again we can take $R = r$.

Consideration of a single example under case 3d will suffice to show the magnitude of the error which may be introduced by omitting the correction. The example chosen is from an actual measurement. With the electrodynamometer available, only 0.1 ampere could be passed through the fixed coil and hence, the potential drop over the coil A and over the resistance r being small, the resistance ρ had necessarily to be taken very small to give the requisite sensibility. If the dynamometer coils could have carried several amperes (as is ordinarily the case), ρ would have been much larger and the error would be much less. In the example $\alpha_1 = 0$ and

$$\alpha = \frac{\rho\,(2\pi NL)^2}{[(R + \rho)^2 + (2\pi NL)^2]\,R}$$

$$= \frac{300\,(2 \times 3.14 \times 60 \times 0.036)^2}{[(11 \times 300)^2 + (2 \times 3.14 \times 60 \times 0.036)^2]\,11},$$

or $\alpha = 0.052$, nearly.

Hence,

$$R = r \frac{1}{1 + 0.052 \dfrac{311}{300}} = 0.948\,r.$$

Thus, if we had called $R = r$, the error would have been about 5.2 per cent, R being assumed too large. This conclusion was checked experimentally. Without changing the ohmic resistance of the coil A its inductance, which was capable of variation, was

varied from 0.003 to 0.036 henry, and in the first case, using the uncorrected formula, $R = 10.94$ ohms, and in the second case, using the same formula, $R = 11.62$ ohms, or 6 per cent too large, which is in fairly close agreement with the calculated result of 5.2 per cent.

If the fixed coils of the dynamometer had been made to carry 10 amperes instead of 0.1 ampere, ρ could have been 100 times as large, in which case the correction factor would reduce to about 0.05 of 1 per cent.

The above adjustments having been made, direct current can be made to replace the alternating current and in the same way we find the direct-current resistance of A. It will be

$$R_{dc} = r_1. \tag{14}$$

Hence,

$$\frac{R}{R_{dc}} = \frac{r}{r_1} \tag{15}$$

is the ratio of the alternating-current to the direct-current resistance of the circuit A. This ratio may take a value of 2 or more.

It should be clearly understood just what is meant by the quantity R, which this method measures. It is a quantity which, expressed in ohms and multiplied by the square root of the mean square value of the alternating current thru the circuit, expressed in amperes, will give the square root of the mean square value of that component of the impressed E.M.F. expressed in volts, which is in phase with the current. Or it is the quantity which, when multiplied by the mean square value of the current, will give the power in watts which is being dissipated in the circuit. In drawing the triangle of E.M.F.'s of an inductive circuit one sometimes represents the component of the E.M.F. which is in phase with the current by the product of the current and the direct-current resistance R_{dc}. This procedure may lead to considerable error in circuits in which there are other losses than the I^2R_{dc} losses. In such circuits the alternating-current resistance R should always be used.

1003. Directions for Using, and Test of Method. — For making the above measurement the apparatus is assembled and connected as shown in Fig. 1003.

D is the electrodynamometer with its hanging system h. The heavy fixed coils or the light wire fixed coils are used according to

the magnitude of the current which is selected for the measurement. In the Rowland instrument the light wire fixed coils will carry 0.1 ampere and the heavy wire coils will carry 50 amperes.

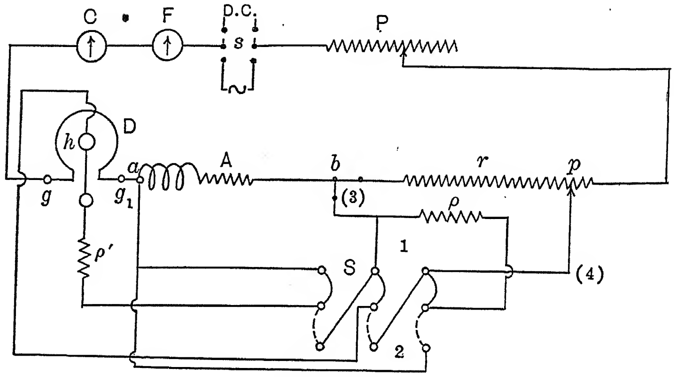

FIG. 1003.

C is the alternating-current ammeter and F the frequency meter. P is a rheostat to control the main current. s is a switch to shift from direct current to alternating current and vice versa. S is the three-point double-throw switch which in position 1 makes the connections shown in I, Fig. 1002 and which in position 2 makes the connections shown in II, Fig. 1002. r is best obtained from a slide-wire rheostat of considerable current capacity. It does not need to be non-inductive, but must contain no iron. If its reactance is just equal to that of the coil being measured $\alpha = \alpha_1$ and $R = r$ exactly.

After the settings for p have been found, the connections are broken at (3) and (4) and the direct-current resistance value of r is measured with a Wheatstone bridge or by any other convenient means.

The resistances ρ and ρ' may be obtained best from plug or dial decade resistance boxes, These may be high, 10,000 ohms or so, depending entirely upon the current used, the magnitude of the resistance being measured and upon the sensibility of the instrument.

The torsion head may be turned so that the no-current-deflection is between 100 and 200 divisions of the scale. By then adjusting P the deflection with current on may be made to come near the zero of the scale.

It will be found, if A consists of an ironless variable standard of inductance, that the variable standard may be set to any inductance value without greatly altering the deflection. The change in the deflection will be less as ρ is made larger.

This method will be found useful in measuring the alternating-current resistance of steel-cored copper or aluminum cables which differ considerably from their direct-current resistance.

The following test was made of this method.

This test was made in a manner to show how large a correction would be required when ρ is chosen only 300 ohms and L is varied between 0.003 and 0.036 henry.

The resistance measured was that of a Hartmann and Braun variable standard of inductance, which should, of course, show the same value on direct and on alternating current and at whatever value its inductance is set.

(a) $L = 0.003$ henry.

With alternating current in circuit,

$$
\begin{aligned}
D_t &= 244 \text{ (no current)},\\
D &= D' = 0 \text{ (current flowing)},\\
R &= r = 10.94 \text{ ohms},\\
I &= 0.08 \text{ ampere},\\
N &= 60.2 \text{ cycles},\\
\rho &= 300 \text{ ohms}.
\end{aligned}
$$

(b) $L' = 0.036$ henry.

With alternating current in circuit,

$$
\begin{aligned}
D_t &= 253 \text{ (no current)},\\
D &= D' = 0 \text{ (current flowing)},\\
R &= r = 11.62 \text{ ohms},\\
I &= 0.08 \text{ ampere},\\
N &= 60.2 \text{ cycles},\\
\rho &= 300 \text{ ohms}.
\end{aligned}
$$

With direct current in circuit,

$$
\begin{aligned}
D_t &= 244 \text{ (no current)},\\
D &= D' = 0 \text{ (current flowing)},\\
R_{dc} &= r = 10.94 \text{ ohms},\\
I_{dc} &= 0.08 \text{ ampere},\\
\rho &= 300 \text{ ohms}.
\end{aligned}
$$

CHAPTER XI.

RESISTANCE MEASUREMENTS WHEN THE RESISTANCE INCLUDES AN ELECTROMOTIVE FORCE.

1100. **Material Included Under this Title.** — We pass now to a consideration of the methods employed for the measurement of resistance, when an electromotive force is included in the portion of the circuit the resistance of which is to be measured.

The methods to be considered are those employed for measuring the insulation resistance of lighting or power circuits while the power is on, the internal resistance of batteries, the resistance of the earth between electrodes where the disturbing effects of earth currents must be considered, and the resistance of electrolytes subject to an E.M.F. of polarization. Resistance measurements of the above character require special consideration at some length.

1101. **Measurement of Insulation of an Electric Wiring System while the Power is On.** — It is not infrequently required to measure the insulation between the gas pipes and each bus-bar of an interior wiring system, as that of a city office building, at a time and under circumstances when it is impracticable to shut off the power.

Two methods are given for making this measurement.*

1102. **Voltmeter Method for Insulation Measurement while the Power is On.** — Let Fig. 1102 represent any wiring system in which X_1 and X_2 represent the insulation resistances between the bus-bars B_1 and B_2, and the earth (the gas or water mains being considered to have the potential of the earth).

The diagrams, I, II, III, of Fig. 207, show circuits equivalent to the circuits represented by Fig. 1102. In these diagrams y represents the unknown resistance of all the lamps, motors, etc., which are connected across the line.

If the bus-bars are supplied with direct current a Weston

* These methods were first described by the author, in the *Electrical World and Engineer*, May 21, 1904, vol. xliii, pages 966–7.

direct-current voltmeter should be used. If the current is alternating then an alternating-current voltmeter of the electrodynamometer type will be required. The resistances X_1 and X_2 are determined by knowing R, the resistance of the voltmeter, and by taking three voltmeter readings d_1, d_2 and D which correspond to three voltages V_1, V_2, and E indicated in Fig. 1102.

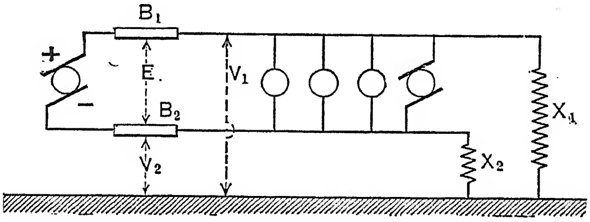

FIG. 1102.

1st. Obtain the deflection D which corresponds to the voltage E between the bus-bars.

2d. Connect the voltmeter between the bus-bar B_1 and the gas or water pipe and obtain the deflection d_1 which corresponds to the voltage V_1.

3d. Connect the voltmeter between the bus-bar B_2 and the gas or water pipe and obtain the deflection d_2 which corresponds to the voltage V_2. The last two readings should follow the first as quickly as possible so that there will be less chance of the line voltage changing in the intervals between the readings.

If the readings in either of the two latter cases are only a fraction of a scale-division, then the insulation resistance is too high to be measured by this method and one must resort to the next method to be described. Having taken the above three readings, we obtain

$$X_1 = \frac{R\,(D - d_1 - d_2)}{d_2} \tag{1}$$

and

$$X_2 = \frac{R\,(D - d_1 - d_2)}{d_1}. \tag{2}$$

The proof of the relations (1) and (2) has been given in par. 207, and a discussion is given of the theoretical accuracy obtainable in par. 104.

The current I which leaks to the ground will be

$$I = \frac{E}{X_1 + X_2}. \tag{3}$$

In a particular case, the insulation resistance of the wiring system of a large office building was determined. A Weston direct-current, 150-volt voltmeter was used and the following readings and resistances were obtained:

$R = 12{,}220$ ohms.

$E = 113$ volts or $D = 113$ scale-divisions.

$V_1 = 1$ volt or $d_1 = 1$ scale-division.

$V_2 = 4$ volts or $d_2 = 4$ scale-divisions.

$$X_1 = \frac{12{,}220\,(113 - 1 - 4)}{4} = 329{,}940 \text{ ohms.}$$

$$X_2 = \frac{12{,}220\,(113 - 1 - 4)}{1} = 1{,}319{,}760 \text{ ohms.}$$

This example shows that where the sum of the resistances X_1 and X_2 is not over a million ohms, the voltmeter method is sufficiently accurate for the purpose of deciding if insulation specifications have been met.

If one side of the line is grounded, that is, if $X_2 = 0$, we have, $V_2 = 0$, and $E = V_1 + V_2 = V_1$ and the method fails to give X_1.

Any instrument, as a galvanometer, in which the deflections are proportional to the current thru it and which has sufficient resistance in series with it so that it will at no time deflect off its scale, may be substituted for a voltmeter.

1103. Galvanometer Method for Insulation-Measurement while Power is On. — This method may be used when greater accuracy is required or when the insulation resistance to earth, of at least one side of the line, is over a megohm.

The wiring system is represented in I, Fig. 1103, and II, Fig. 1103 gives equivalent circuits.

The method is carried out by connecting across the bus-bars a moderately high resistance. A point p is found on this resistance where the potential due to the generator is the same as that of the earth. Then, with the aid of a sensitive galvanometer and an external source of E.M.F., the resistances to earth r_1 and r_2 are measured in the following manner: k is a key and S an Ayrton universal shunt. This latter may be omitted if the source of E.M.F. can be varied in a known manner.

It is evident from II that a balance will be obtained when $\frac{a}{b} = \frac{r_1}{r_2}$, the key k being in its upper position. If k is now depressed, the

resistance R encountered by the current generated by the source e will be

$$R = g_1 + \frac{1}{\frac{1}{b + r_2} + \frac{1}{a + r_1}}, \tag{1}$$

where g_1 is the resistance of the galvanometer; but in comparison with r_1 and r_2, a and b can be neglected, also g_1, then

$$R = \frac{r_1 r_2}{r_1 + r_2}. \tag{2}$$

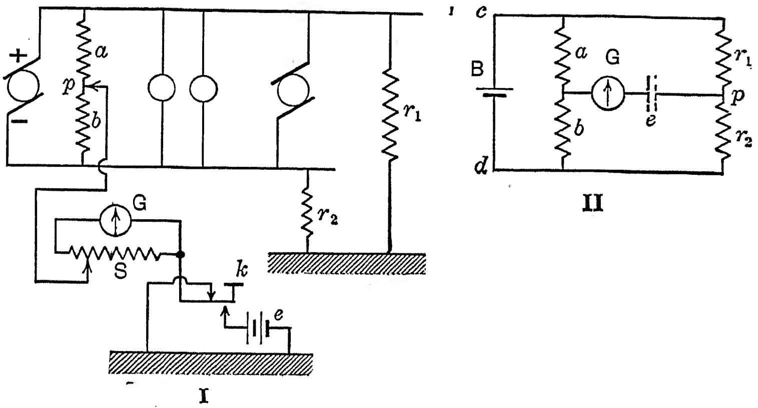

FIG. 1103.

By construction,

$$\frac{r_1}{r_2} = \frac{a}{b} = N, \text{ a known ratio.}$$

From the last two relations we deduce

$$r_2 = \frac{R\,(N+1)}{N}, \tag{3}$$

and

$$r_1 = R\,(N+1). \tag{4}$$

Taking d as the deflection of the galvanometer and K as the galvanometer constant, the current thru the galvanometer is

$$\frac{e}{R} = \frac{d}{K}, \quad \text{or} \quad R = \frac{eK}{d}. \tag{5}$$

K should be defined as the resistance in circuit with the galvanometer (including its own resistance), such that it will give, with one volt, a deflection of one scale-division at the distance at which

the scale is placed from the mirror during the test, usually as one meter.

Then we will have

$$r_2 = \frac{eK(N+1)}{N d},$$

and

$$r_1 = \frac{eK(N+1)}{d}.$$

Taking K equal to 10^8 as an average value for an ordinary sonval galvanometer and $e = 100$ volts, $N = 2$, and $d = 10$ divisions, we have

$$r_2 = \frac{100 \times 10^8 (2+1)}{2 \times 100} = 150 \times 10^6 \text{ ohms, or } 150 \text{ megohm}$$

$$r_1 = \frac{100 \times 10^8 (2+1)}{100} = 300 \times 10^6 \text{ ohms, or } 300 \text{ megohm}$$

This example shows that a galvanometer of very mo sensibility will measure in this way a very high insulation ance. If, on the other hand, the insulation is low, small b power may be used or the deflection of the galvanometer c cut down to 0.1, 0.01, 0.001, 0.0001 by the Ayrton shunt. only difficulty likely to be experienced in applying the method is that, while making the test, the relative values and r_2 keep changing, due to motors or lights being thro or off the line. In this event it is only possible to obtain a s average value for the resistance to earth of each side of the l

1104. **Determination of the Internal Resistance of Batter** The resistance of an electrolytic cell or battery is by no m constant quantity, even approximately. It will change wi temperature, the age of the cell, the current which the giving and with the total ampere-hours of current it has yi It is a quantity which varies greatly with the past hist the cell.

The determination of the internal resistance of a cell on s open circuit requires special methods and the information, over, when obtained has little value because of the variabi the quantity measured. If a cell is closed thru an external ance R and there exists in the circuit an electromotive fo the current I which flows will be

$$I = \frac{E}{R+X},$$

where X is called the internal resistance of the cell. If E' is the fall of potential over the external resistance alone, then

$$\frac{E'}{R} = \frac{E}{R + X} = I, \tag{2}$$

whence,

$$X = \frac{R\,(E - E')}{E'}. \tag{3}$$

We cannot here call X anything more than a quantity which must be added to R to satisfy Eq. (1). It is not an ohmic resistance, as it does not obey Ohm's law, for in general X changes when the current changes.

Nevertheless the current output of a cell, under given circumstances, will depend largely upon this quantity, and it is necessary therefore to be able to determine its value when the cell is subjected to particular conditions.

A determination of the quantity X, to have value and definiteness, really involves the making of a test of the cell in respect to several of its characteristics. These are its open circuit E.M.F. when the cell is fresh and after it has delivered a certain quantity of electricity, its E.M.F. when closed thru a given resistance, its rate of polarization, its rate of recovery from polarization, etc. A description, therefore, of methods of measuring the internal resistance of a battery should begin by showing how a full record of the action of a battery may be obtained. This record is best exhibited in the form of curves. A procedure for obtaining the data for such curves in an accurate and convenient manner will now be explained. It is a well-known method and may be called the condenser method of testing batteries.

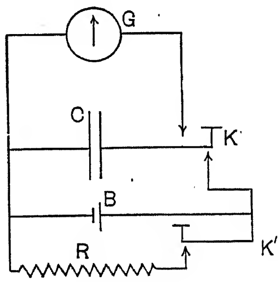

Fig. 1105a.

1105. Battery Tests by Condenser Method. — In the diagram, Fig. 1105a, G is a ballistic galvanometer or other like instrument in which the throw deflections are proportional to the quantity of electricity discharged thru it. B is a cell to be tested, R a known resistance the fall of potential over which is to be measured, K' a key to put this resistance in circuit with the cell B, and K is a charge and discharge key for charging the condenser C and discharging it thru the galvanometer.

The condenser, which should be a mica condenser of about one microfarad, is first charged, K' being open, by means of a standard cell. A Weston cadmium standard cell is recommended. This has an E.M.F. of 1.0183 volts and a zero temperature coefficient between 15° and 35° C. The condenser is then discharged thru the galvanometer and the throw deflection read either with a telescope and scale or lamp and scale.

The standard cell is now replaced by the battery to be tested. With K' open, the condenser is charged and discharged as before. Then K' is closed and after an interval of one minute the condenser is again charged and discharged. K' is maintained closed, preferably for 60 minutes, except that at intervals of two minutes it is opened just long enough to charge and discharge the condenser, at which intervals readings are taken which give the E.M.F. of the cell upon open circuit. The condenser is also charged and discharged, K' being closed, at intervals of two minutes which alternate with the open-circuit readings.

Thus, at time 0 open-circuit reading is taken, K' being open; at end of 1st minute closed-circuit reading is taken, K' closed; at end of 2d minute open-circuit reading is taken, K' momentarily opened; at end of 3d minute closed-circuit reading is taken, K' closed; at end of 4th minute open-circuit reading is taken, K' momentarily opened, and so on until at least 60 minutes have elapsed. At the end of 60 minutes K' is opened permanently and open-circuit readings are taken at intervals of two minutes. In this way data are obtained for plotting the recovery curve.

The deflection corresponding to the E.M.F. of the standard cell having been obtained, the other E.M.F.'s can be calculated from the deflections by simple proportion. Thus, if E_s is the E.M.F. of the standard cell and d_s the corresponding deflection, then any other E.M.F., E_x, which gives a deflection d_x is

$$E_x = \frac{d_x}{d_s} E_s.$$

The internal resistance of the cell at any moment during the 60-minute test is now obtained as follows: Call the E.M.F. of the open-circuit reading at any moment E; that of the closed-circuit reading at the same moment (which is the drop of potential over the resistances R) E_1. Then if X designates the internal resistance sought, the following relations hold. There is a total

E.M.F., E, and a fall of potential over the external resistance E_1. Therefore, there must be a fall of potential $E - E_1$ over the internal resistance X; hence,

$$X : R :: E - E_1 : E$$

or

$$X = \frac{E - E_1}{E_1} R. \tag{1}$$

After the polarization curve given by the different E.M.F.'s called E, and the terminal potential difference curve given by the E.M.F.'s called E_1 are plotted, then the data for solving the equations giving the points for the internal resistance curves can be read directly from the ordinates of these curves. The current flowing at any time is simply

$$I = \frac{E_1}{R}. \tag{2}$$

For a full study of a battery it should be run completely out, tho after the first hour it would be necessary only to take readings at intervals very much longer than two minutes. By joining a number of cells in series (to always have sufficient E.M.F.) and running them completely down thru a voltameter the total number of coulombs which a cell is capable of giving could be computed. Many types of cells should also be given an age-test by giving them a short run and then setting them aside for several months to test if any destructive local action occurs.

In Fig. 1105b are reproduced curves, taken by the author, upon a Barrett silver-chloride cell, like those supplied for portable testing batteries. It will be noted that the internal resistance of this type of cell rapidly falls as the silver chloride of poor conductivity becomes reduced to spongy silver of high conductivity. The polarization is small, the current output increases for the first hour and the recovery is rapid, reaching 1.150 volts. These characteristics, notwithstanding the low E.M.F., have made this type of cell very popular as a small battery for testing purposes.

Dry cells of standard size, as the "Mesco," have become an important commercial factor, and plans for testing and rating them require special consideration. However, the tests which should be made present no problems in measurement which have not been fully discussed and the reader who desires further information upon this subject is referred to a report of a committee of the American Electrochemical Society entitled, "Standard

Methods Recommended for Testing of Dry Cells." This report appeared in the proceedings of the society, vol. XXI, page 275, 1912, and is signed by C. F. Burgess, *Chairman;* J. W. Brown, F. H. Loveridge, C. H. Sharp, *Committee on* Dry Cell *Tests.*

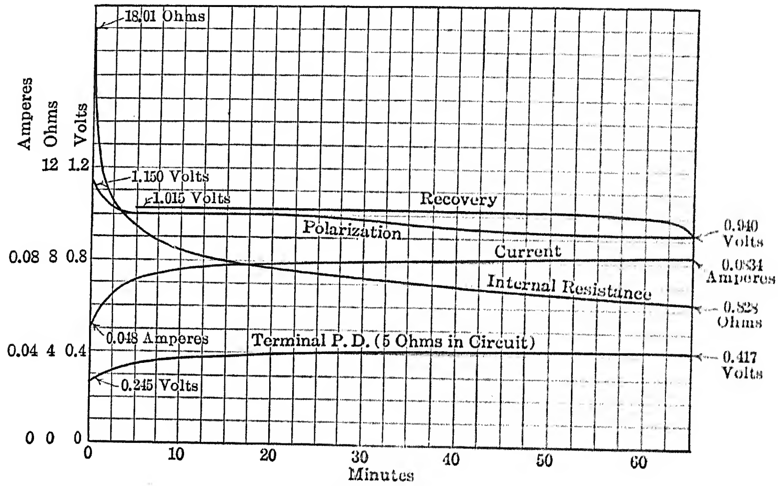

FIG. 1105b.

1106. Mance's Method of Measuring the Internal Resistance of a Battery. — The principle of this method is similar to the one given in par. 404 for measuring the resistance of a galvanometer, and, like it, the method makes use of the "second property" of the Wheatstone bridge.

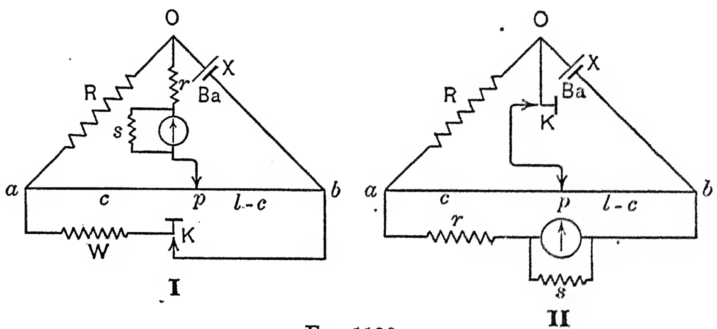

FIG. 1106.

The connections to use are shown in Fig. 1106, I or II. With galvanometers of ordinary sensibility, the current from a battery

placed in the bridge arm Ob would deflect the galvanometer violently off its scale. To avoid this a resistance r is used in series, and a resistance s in shunt with the galvanometer, r and s being so chosen that at no time the galvanometer deflects off its scale. It is also necessary for good results, if the connections I are used, to place a resistance W which is approximately equal to the resistance of the slide wire ab in series with the key K. If this resistance is not used the wire becomes shunted with practically no resistance. The positions of the galvanometer, together with the resistances r and s and the key K may be interchanged as shown in II. In this case the resistance W is not needed. The contact p is moved to a position such that the galvanometer deflection remains unaltered whether the key K is open or closed. When this position is found we have, if l is the length of the slide wire and c the distance of p from a,

$$X = \frac{l - c}{c} R, \tag{1}$$

where R is the fixed resistance in the arm aO and X the resistance of the battery sought.

In applying this method with a slide-wire bridge, it should be noted that, unless the resistance of the slide wire ab is made very high (by winding in a helix as described in par. 401), the battery is yielding considerable current which, in some types of cells, would probably affect the internal resistance, making it different than it would be if the cell were yielding a less current.

In Mance's method, just given, as well as in Kelvin's method for measuring the resistance of a galvanometer, greater precision may be obtained by using two equal extension coils, as shown by n_1 and n_2, Fig. 401a (§ 401). In this case calling n the value of each extension coil, in terms of equivalent length of bridge wire, we should use the formula for Mance's method,

$$X = \frac{n + l - c}{n + c} R. \tag{2}$$

Also we should use the same formula (§ 404) for Kelvin's method, in which we replace X, the resistance of the battery, by g, the resistance of the galvanometer.

1107. Voltmeter and Ammeter Methods of Measur Internal Resistance of a Battery.

Method I. With K' open (Fig. 1107a) close K and rea open circuit E.M.F. of the battery Ba. With K' closed r the drop of potential over R. Then the current is

$$I = \frac{E_1}{R} = \frac{E}{R + X},$$

whence

$$X = \frac{E - E_1}{E_1} R.$$

This method assumes, first, that the voltmeter takes so lit rent that, with K' open and K closed, the cell may be con to be upon open circuit, and second, that the polarization cell is so trifling that when K' is closed the E.M.F. of remains unchanged. Neither assumption is justified in t of many types of cells that polarize readily and is probabl wholly justified with any type of cell. However, for a estimate of the condition of dry batteries, etc., it is a satis test. The voltmeter should have a full scale reading of on 2 to 5 volts for accuracy and the resistance R should be equal to the internal resistance X of the cell. The total i resistance of a battery of cells joined in series may be meas the same way, a voltmeter with a scale reading higher bein required.

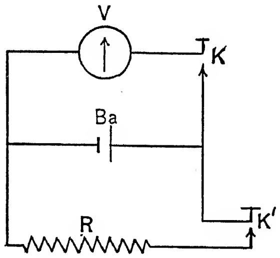

Fig. 1107a.

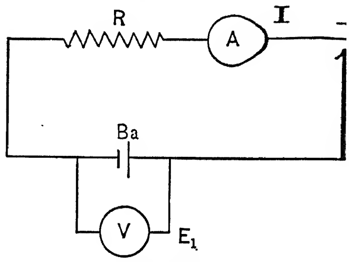

Fig. 1107b.

Method II. The measurement may be made using a vol and an ammeter.

The cell Ba (Fig. 1107b), a resistance R, the ammeter A key K are joined in series. A low reading voltmeter V

nected to the cell terminals. With K open, read E, the open circuit E.M.F. Close K and read E_1 and the current I. Then,

$$I = \frac{E}{X + R} = \frac{E - E_1}{X};$$

hence,
$$X = \frac{E - E_1}{I}. \tag{1}$$

This result, in which two E.M.F.'s and a current are measured, assumes also that the voltage E is not altered by polarization of the cell.

1108. A Word on Polarities. — In Fig. 1108 let a line which carries a direct current i have introduced in it, in series, a direct-current ammeter A and a cell B. This is assumed to have an unalterable E.M.F., e, and a zero internal resistance. Also insert an ohmic resistance X. Connect a direct-current voltmeter V at the points 1 and 2 to measure the fall of potential between the points 1 and 2 or 2 and 1. Let a be the positive terminal and b the negative terminal of the voltmeter. Let the positive terminal of the cell be joined to the resistance X.

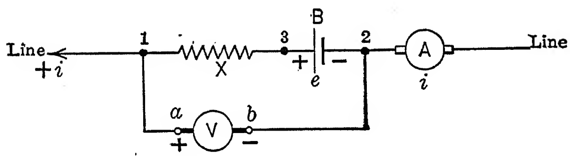

FIG. 1108.

The magnitude and direction of the current i in the line will depend both upon the E.M.F., e, of the cell B and upon other E.M.F.'s which are in the rest of the circuit. Let the line current i when flowing in the direction from 2 to 1 be called positive, and negative when flowing in the opposite direction. When the potential (with respect to the earth) at 1 is greater than the potential at 2, call the reading e_1 of the voltmeter positive, and when the potential at 2 is greater than the potential at 1, call the reading e_1 of the voltmeter negative. First, assume that the current i flows from 2 to 1. Then the fall of potential from 3 to 2 is such as to tend to send a current thru the voltmeter from a to b and the fall of potential from 3 to 1 is such as to tend

to send a current thru the voltmeter from b to a. Hence, the voltmeter will read

$$e_1 = e - iX, \tag{1}$$

from which we deduce

$$X = \frac{e - e_1}{i}. \tag{2}$$

Second, assume that the current i flows from 1 to 2. Then the fall of potential from 3 to 2 is such as to tend to send a current thru the voltmeter from a to b, as before, but now the potential rises from 3 to 1 and the potential fall thru X will be such as to tend to send a current thru the voltmeter from a to b. Hence, the voltmeter will read

$$e_1 = e + iX. \tag{3}$$

We shall know, however, that the current in the second case is opposite to the current in the first case because the terminals of the ammeter will have to be reversed to obtain a reading. Therefore, if we follow the convention of calling the current positive when flowing from 2 to 1, and negative when flowing from 1 to 2 (or against the polarity of the cell), then we should write, Eq. (3),

$$e_1 = e - iX, \text{ as in the first case.}$$

In Eq. (1) if $i = 0$, $e_1 = e$ as it should, also e_1 will remain positive as long as iX is less than e. If iX becomes greater than e then e_1 will be negative, which fact will be known from the necessity of changing the voltmeter terminals in order to obtain a reading.

The above principles must be kept in mind when applying the following volt and ammeter methods of measurement: We shall adopt the convention that the current in the line is to be regarded positive if it has the direction it would have if the E.M.F. in the circuit being tested were the only E.M.F. acting and the voltmeter reading will be regarded as positive if it reads with its positive terminal joined to the positive terminal of the circuit which contains an E.M.F. and is under test.

1109. Voltmeter and Ammeter Method; Principle of Polarities Illustrated. — In Fig. 1109a, e is a source of E.M.F. which has a resistance represented by X. V is a voltmeter. E is an auxiliary cell, as a storage-battery cell. A is an ammeter. With the key

K closed and the polarities of the two sources of E.M.F. as shown, we have

$$e_1 = e - iX, \tag{1}$$

where e is the E.M.F. that the voltmeter reads when K is open, and e_1 the E.M.F. that it reads when K is closed. Also i is the current which the ammeter reads, regard being given to the sign of i. From Eq. (1) we thus obtain

$$X = \frac{e - e_1}{i}. \tag{2}$$

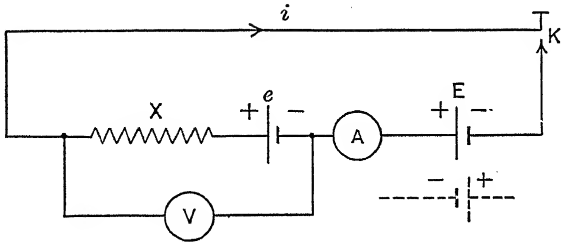

FIG. 1109a.

The polarity of E is now reversed, as indicated in the figure by the dotted lines, and the value of X is then found to be

$$X_1 = \frac{e - e_2}{i_1}. \tag{3}$$

The first and second values of the resistance will probably not agree on account of polarization of the cell being tested. The mean value, however, of X and X_1 should be taken as representing the most probable value of the resistance.

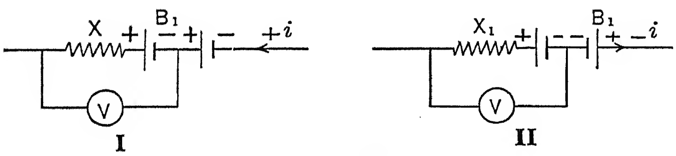

FIG. 1109b.

This method was tried with the following observations and results. Connections and polarities were made first as in I, and second as in II, Fig. 1109b.

For polarities as in I,

$$i = +\,0.1287, \quad e = +\,2.16, \quad e_1 = -\,2.64.$$

Hence, $$X = \frac{2.16 - (-2.64)}{0.1287} = 37.24 \text{ ohms.}$$

For polarities as in II,

$$i_1 = -0.0635, \quad e = +2.16, \quad e_2 = +4.50.$$

Hence, $$X_1 = \frac{2.16 - 4.50}{-0.0635} = 36.85 \text{ ohms.}$$

The mean of X and X_1 is 37.05 ohms.

In this experiment X was a metallic resistance and B_1 a small storage cell. The resistance X was measured upon a bridge, after the test, and found to equal 37.25 ohms. Hence, the error in the measurement by method II was a little over one half of 1 per cent, a fair result, considering that the instruments used were a commercial voltmeter and ammeter.

1110. Total Resistance of a Network between Two Points when the Branches of the Network Contain Unknown E.M.F.'s. — This method was devised by the author. In Fig. 1110a let

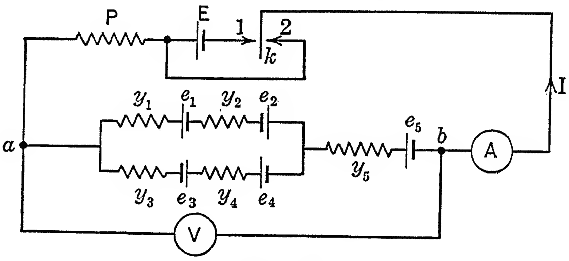

FIG. 1110a.

$y_1, y_2, y_3, \ldots y_n$ be any combination of resistances joined together in a network in any manner whatever. Let $e_1, e_2, e_3, \ldots e_n$ be E.M.F.'s associated with the branches of the network having any values and polarities. The problem presented is to determine the resistance between the points a and b. The quantity R to be measured should be the same as the quantity which would be obtained if e_1, e_2, e_3, etc., were all zero and R is defined as

$$R = \frac{E}{I}. \tag{1}$$

Let P be any resistance and A an ammeter which will measure I. Let k be a switch or key which will make connection with either the point 1 or the point 2.

First put k to point 1 and read the current I_1 on the ammeter A and the voltage V_1 on the voltmeter V. Then put k to 2 and read the current I_2 on the ammeter and the voltage V_2 on the voltmeter. In the first case, if we call E_1 the resultant E.M.F. at the points a and b of all the E.M.F.'s, e_1, e_2, e_3, etc., then, as shown in connection with Fig. 1108,

$$V_1 = E_1 - I_1R, \tag{2}$$

and in the second case

$$V_2 = E_1 - I_2R. \tag{3}$$

From Eqs. (2) and (3)

$$R = \frac{V_1 - V_2}{I_2 - I_1}. \tag{4}$$

In applying this method careful attention must be given to the convention of signs as explained in par. 1108.

The only assumption made here, which bears upon the precision of this method, is that the resultant E.M.F. at the points a and b is not altered by polarization of the sources of E.M.F.'s, e_1, e_2, e_3, etc., when the main current is changed from I_1 to I_2.

In practice this method gives good results under certain circumstances that often arise. It is adapted to the measurement of the resistance between two conductors, as between a gas and water pipe main buried in the earth, when the resistance path in the earth is subject to many local and unknown E.M.F.'s which correspond to the E.M.F.'s, e_1, e_2, e_3, etc., of Fig. 1110a.

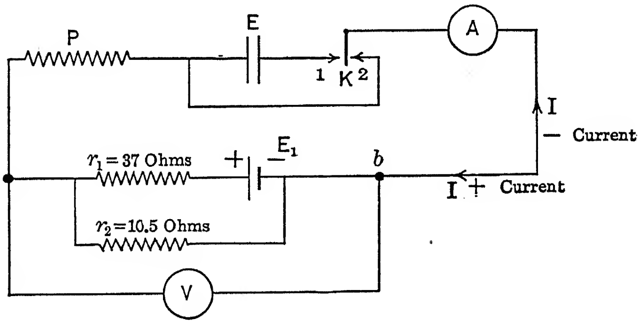

FIG. 1110b.

The following trial of the method was made by the author: Circuits were made up as indicated in Fig. 1110b. E_1 was a small storage-battery cell and E was also a source of E.M.F.

from a storage battery. P was a resistance to vary the current in the line. r_1 and r_2 were metallic resistances. It was required to determine, by this method, the resistance between the points a and b. The following table exhibits the readings and the results obtained:

Here, V_1 and I_1 are the voltage and current with K on 1, or E in circuit, and V_2 and I_2 are the voltage and current with K on 2, or E out of circuit.

V_1	V_2	I_1	I_2	Ohms a to b	Notes
−7.48	0.000	0.9160	0.0000	8.17	E_1 made zero.
−7.50	0.100	0.9734	0.0553	8.28	
−7.50	0.100	0.9751	0.0504	8.22	$P = 0$
					$P = 5.4$ ohms.
3.87	0.365	−0.3660	0.0623	8.18	$E = 0.75\times$ original E.
3.86	0.360	−0.3650	0.0623	8.19	$E_1 = 2\times$ original E_1.
4.23	0.150	−0.4597	0.0395	8.16	$P = 3$ ohms.
4.24	0.150	−0.4620	0.0395	8.15	E_1 reduced.
−3.93	0.150	0.5375	0.0394	8.19	Polarity of E reversed.
−3.93	0.150	0.5376	0.0388	8.18	
....	.....	......	......	8.19	Mean.
....	.....	......	......	8.178	True value.

1111. Alternating-current Methods of Measuring the Resistance of a Battery. — While several other direct-current methods of measuring the internal resistance of a battery are described in the older treatises, they will not be mentioned here, as they are all very much inferior to alternating-current methods and more troublesome. Of the alternating-current methods of value, two are bridge methods and one an electrodynamometer method, in which the value of the resistance is given as equal to a known metallic resistance.

1112. Bridge Method. Telephone Detector. — This method is similar to the method of Kohlrausch, described in connection with Fig. 1120c, for measuring the resistance of an electrolyte (see also par. 1124). The arrangement provided is intended for the measurement of the internal resistance of a battery when this is yielding a known current. As the resistance of any battery, and especially one which polarizes readily, varies with the current which it gives, it is practically useless to obtain its internal resistance if the current to which this resistance corresponds is not known.

The arrangement shown in the diagram will enable the internal resistance to be obtained and the current which the battery gives to be measured or calculated.

The source of the measuring current, which is rapidly alternating, is a small induction coil I which is operated by one or two cells of battery B. The bridge is an ordinary slide-wire bridge which will be found very suitable for the measurement. The detector to indicate when the bridge is balanced is a telephone. It is convenient if this is provided with a head band.

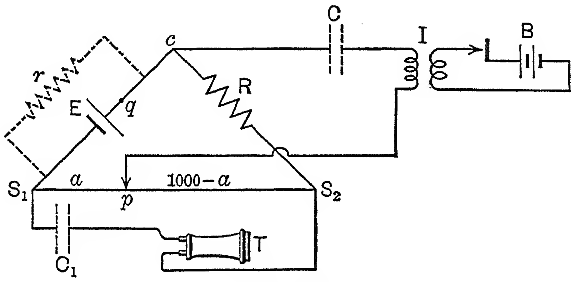

FIG. 1112.

Connections are made as indicated in Fig. 1112. There is shown in the diagram in dotted line a condenser C in the main circuit and a condenser C_1 in the telephone circuit. The object in using these (which may be cheap paper condensers of one microfarad capacity) is to confine the direct current produced by the cell being measured, to the circuit $c\ S_2\ S_1$ around the bridge. If this is done it becomes very easy to calculate the current which the cell is giving, if its E.M.F. has been determined by means of a voltmeter. The condensers will not interfere with the passage of alternating current sufficient for the measurement. If the condensers are omitted the current from the cell may still be determined by inserting a low-reading ammeter or millimeter at q.

The bridge is balanced for a minimum sound or no sound in the telephone by moving the contact p upon the slide wire S_1, S_2. As the scale of a slide-wire or meter bridge is usually divided into 1000 divisions, we have for the internal resistance of the cell, the expression

$$X = \frac{a}{1000 - a} R, \tag{1}$$

where a is the reading from the end of the scale nearest the cell

and R is the fixed resistance in the bridge. This latter should be quite non-inductive and preferably of about the same magnitude as the resistance of the cell. For values of $\frac{a}{1000-a}$, see Appendix I, 1. The current from the cell (when the condensers are used) is now simply

$$i = \frac{E}{X + R + l}, \tag{2}$$

where l is the resistance of the bridge wire.

It may be required to determine the value of X when the cell is in circuit with a lower resistance than the arms of the bridge. The cell may be made to yield a larger current by joining to its terminals a known resistance r (shown in the diagram in dotted line). If the bridge is now balanced we have

$$\frac{rX}{r + X} = \frac{a}{1000 - a}R, \tag{3}$$

or

$$X = \frac{Rra}{(1000 - a)\,r - Ra}. \tag{4}$$

The current may be measured with an ammeter or milliammeter inserted at q, or, if E is measured with a voltmeter and the condensers are used, it may be calculated from the formula

$$i = \frac{E\,(R + r + l)}{X\,(R + r + l) + r\,(R + l)}. \tag{5}$$

For the information given by this measurement to have the highest value, the temperature of the cell, the current which it gives, and its corresponding resistance should all be recorded. The author made a careful test of this method which developed points of interest.

A meter bridge with a slide wire of 14 ohms resistance was selected for the test. The alternating current was supplied by a very small induction coil about 7 cms long and 2.5 cms in diameter. It was first run by one and later by two small cells of storage battery. Its vibrator gave a high-pitched note but not loud. The condensers used were a 1 microfarad and a 0.5 microfarad mica condenser, the latter being in the current circuit and the former in the telephone circuit. The telephone was supplied with an ivory plug attached to a cord which could be used with advantage to stop up one ear to keep out the sound of the coil.

The method was first tried, using for the R and the X resistances two 10-ohm non-inductive manganin coils. It was found under these circumstances that the sound in the telephone was sufficiently loud, and that a point could be found upon the slide wire which gave complete silence. The loudness of the sound was scarcely affected by cutting out of circuit the two condensers. The setting could be made to within 0.5 of a millimeter. The bridge balanced at $a = 501$, showing that, as the coils were equal, the wire was practically of equal resistance either side of its middle point.

A new Columbia dry cell was now tested, first without a shunt and later with a shunt of 1 ohm. The resistance R was made 0.123 ohm. It was found now that it was impossible to obtain silence in the telephone and that it was difficult to set the sliding contact closer than 2 or 3, and sometimes 7 or 8 millimeters. The continuance of the sound in the telephone was attributed to the electrostatic capacity of the cell, and this was shown to be the case by putting an equal number of cells, joined in series, in the two arms of the bridge when a balance giving complete silence could be obtained, as in the case of metallic resistances. It should be remembered that to accurately balance a bridge with alternating current it is necessary that the "time constant" of its adjacent arms shall be the same. The smaller the cell and the higher its resistance the more accurately and easily can it be measured by this method. A set of six dry cells, some very old, were joined in series and with polarities mutually opposed, and it was found easy to balance the bridge accurately because by the arrangement in series the resistance was increased more than the capacity. Thus, when one has several cells it is easier to measure the resistance of a number joined in series than to measure the resistance of one. Some of the results obtained are recorded below:

Columbia dry cell (standard size)	Initial E.M.F. $E = 1.47$ volts
$R=0.123$ ohm (1) Settings, a 373 372 372 372.3 Mean 0.0729 ohm, calculated resistance of cell.	$R=0.223$ ohm (2) Settings, a 239 238 238 237 235 237.2 Mean 0.0693 ohm, calculated resistance of cell.

Mean resistance obtained by (1) and (2) is $X = 0.0711$. Departure from mean is 2.5 per cent. The current flowing was 0.1 ampere. Same cell as above shunted with 1 ohm, or $r = 1$.

Settings, a. $R = 0.123$ ohm.
377
370
380
375.6 mean. This gives $\frac{rX}{r + X} = 0.0740$ ohm,

or $X = 0.0799$ ohm. The current was 1.4 amperes.

An old Mesco dry cell was also measured. Its resistance was found to be about 5 ohms, but this resistance rose during the measurement.

From these measurements it is to be concluded,

1st. That with cells of moderate size a close setting for a balance with complete silence in the telephone is impossible.

2d. That the resistance of a dry cell is an extremely variable quantity.

3d. That the method is well adapted to metallic resistances, small cells of high resistance or to a number of cells in series, but is not accurate to more than from 3 to 5 per cent for low resistance, single cells.

1113. **Bridge Method; Electrodynamometer Detector.** — This method may be applied in precisely the same way as the method for measuring the resistance of an electrolyte described in connection with Fig. 1120d, except that a condenser of considerable capacity should be in circuit with the fixed coil of the electrodynamometer. No current from the battery can then flow thru both the fixed and movable coil of the electrodynamometer and so influence its deflection. If this instrument is of the suspended coil type (as designed by the late Prof. Henry A. Rowland and described in par. 1001) it will have ample sensibility when used in this way. Thus on a circuit of 60 cycles and 100 volts the current thru the fixed coil of the dynamometer will be very approximately

$$i = 2\pi NVC = 6.28 \times 60 \times 100 \times C = 3768\,C \text{ amperes},$$

where C is the capacity in farads in the circuit. If C is 2.5×10^{-6} the current will be 0.0942 ampere, which is sufficient. The direct

current from the cell is determined most simply by measuring it with an ammeter or milliammeter in circuit with it.

1114. Electrodynamometer Substitution Method (Author's Method). — This method has a much wider range of usefulness than for the particular measurement here described. Its application to the measurement of the effective resistance of a circuit containing iron when carrying alternating current has been already described in Chapter X, and therefore its application to the determination of the internal resistance of a battery may be indicated very briefly.

The electrodynamometer should be of the Rowland type. The cell under test and accessory apparatus are connected as in I, II and III, Fig. 1114.

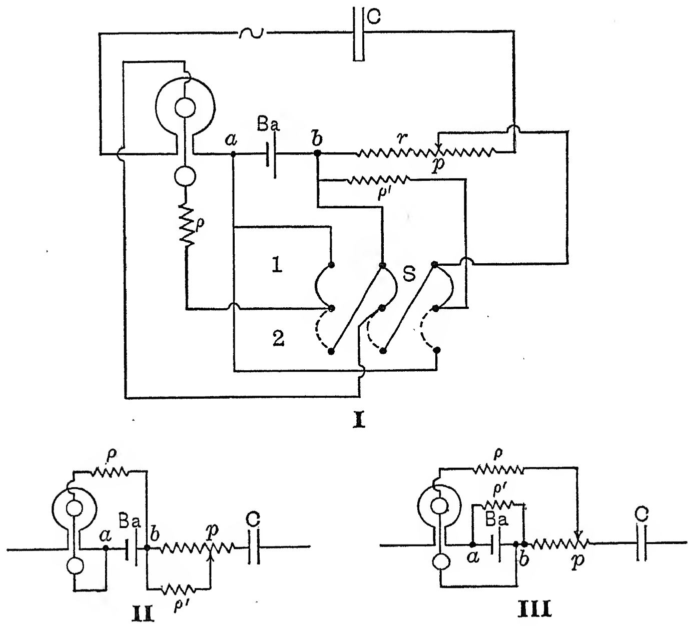

FIG. 1114.

Referring to diagram I it will be noted that the three-point double-throw switch *S* when in position 1, shown in full line, makes the connections indicated more simply in diagram II, and when in position 2, shown in dotted line, makes the connections indicated more simply in diagram III.

The introduction of the condenser C in the main circuit is to prevent any direct current from the battery passing thru the fixed coil of the dynamometer. The alternating-current mains of 110 volts may be used as the source of E.M.F.

Now it is evident by inspecting diagrams II and III that the resistance of the cell Ba is equal to r, provided the point p has been adjusted upon the resistance r until the deflection of the electrodynamometer is the same when, with the switch S, the connections are changed from II to III and vice versa. This equivalance of the resistance of the cell Ba and the resistance r will hold accurately provided the resistance ρ' is made equal to the total resistance ρ of the hanging coil circuit. The further assumption must also be made that the electrostatic capacity of the battery is small. A considerable capacity reactance in the battery would necessitate a small correction of the same general character as that discussed in par. 1002. It is thought, however, that the magnitude of this correction would in general be so small that it could be entirely disregarded. When the measurement can be made upon a number of cells of the same size and kind at the same time the effect of reactance can be reduced to any desired extent by joining a number of cells in series, for the resistance measured will increase directly with the number of cells joined in series while the capacity will decrease.

A trial of this method was made by the author. It gave excellent results and a brief description of the test follows:

Four Daniell cells were made up in glass jars with porous cups. These were joined in series and not opposing. The internal resistance was measured, using the method and connections shown in Fig. 1114. The electrodynamometer was of the Rowland type. Both its fixed coils and hanging coil system had a carrying capacity of 0.1 ampere. The source of current was 120 volts A.C. and the frequency, at the time of the test, was 59.9 cycles per second. The condenser C was a mica condenser of 1.75 microfarads. The current flowing in the main circuit was 0.077 ampere. The dynamometer deflection was 218 divisions. To make this deflection the same with the connections first as in III, and then as in II, it was necessary to make $r = 7.64$ ohms; hence, the resistance of the four cells in series was 7.64 ohms, making the average resistance of each cell 1.91 ohms. This method gave good results without any difficulty arising and the sensibility was

found to be ample. At the time of the test the resistance thru which the battery could flow was 300 ohms, this being the value given to ρ. The method gave in another trial with these same cells so connected that their E.M.F.'s opposed, under which circumstances the cells yielded practically no current, the value 1.81 ohms as the mean resistance for each cell.

1115. Galvanometer Deflection Methods for Obtaining the Resistance of a Battery. — Though the following methods are well known they are not much used, for the primary battery has assumed a subordinate position as a source of electric current. However, for the sake of completeness in the treatment of this subject we shall describe them briefly.

Many types of modern cells, especially storage-battery cells, have an extremely low internal resistance, and in any of the methods for measuring this resistance it is very advantageous, when one has a number of similar cells, to join as many of them as possible in series opposing their E.M.F.'s. In this way the resistances of the cells are added in series, the electrostatic capacity is reduced and the resultant E.M.F. is small, which permits of smaller resistances in the circuits being used. Even when there is an even number of cells, the resultant E.M.F. is usually sufficient to furnish enough current for the measurement if an ordinary D'Arsonval galvanometer is the measuring instrument. It must be remembered that the internal resistance measured includes the connecting wires to the cells and the contact resistance under the binding posts. These resistances must be taken into account and allowed for whenever great accuracy is desired.

1116. Diminished Deflection Method. — The battery, of which the resistance X is to be determined (Fig. 1116), is joined in series with a resistance and a galvanometer. Ordinarily the galvanometer must be shunted with a low-resistance shunt, but where a low-sensibility galvanometer is used, as a tangent galvanometer, and the battery has a comparatively high resistance, the shunt may be omitted. Calling g the resistance of the galvanometer, and s the resistance of its shunt,

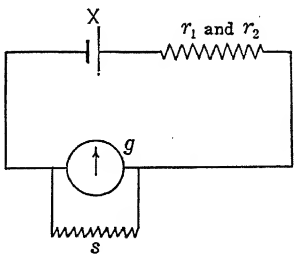

FIG. 1116.

E the E.M.F. of the battery, and r_1 the resistance used, the value of the current which flows is

$$i_1 = \frac{E}{X + r_1 + \dfrac{gs}{g+s}} = Kd_1. \tag{1}$$

Here d_1 is the deflection of the galvanometer and K is a constant. The value of the resistance is now changed from r_1 to r_2 and the current which then flows is

$$i_2 = \frac{E}{X + r_2 + \dfrac{gs}{g+s}} \; Kd_2. \tag{2}$$

If the galvanometer is a tangent galvanometer, then we must write, instead of Kd_1 and Kd_2, $i_1 = K \tan \theta_1$ and $i_2 = K \tan \theta_2$, where θ_1 and θ_2 are angular deflections.

Eqs. (1) and (2) make the assumption (never strictly true) that the E.M.F. of the cell remains unchanged when the current is changed from i_1 to i_2. From the two relations above, we easily derive

$$X = \frac{r_1 d_1 - r_2 d_2}{d_2 - d_1} - \frac{gs}{g+s}. \tag{3}$$

If r_2 is chosen so that $d_2 = \dfrac{d_1}{2}$, Eq. (3) becomes

$$X = r_2 - \left(2\, r_1 + \frac{gs}{g+s}\right). \tag{4}$$

It should be recalled that r_1 is the resistance that gives the deflection d_1, and r_2 is the larger resistance which halves it.

It sometimes happens that a calibrated galvanometer is used to read the E.M.F. of a thermocouple. Now the resistance of a thermocouple will change with its depth of immersion in the hot place, and with the length of the lead wires used. The above method could be conveniently employed to determine the resistance of the thermocouple circuit from binding post to binding post of the galvanometer. In this case the galvanometer would have no shunt and its resistance g would be known. Also the resistance r_1 would be zero and thus, if a resistance r_2 is inserted quickly in the thermocouple circuit before the temperature of the hot junction has time to change we would find, by Eq. (3),

$$X_c = \frac{d_2}{d_1 - d_2} r_2 - g. \tag{5}$$

Or if r_2 is so chosen as to halve the deflection d_1,

$$X_c = r_2 - g. \tag{6}$$

The above measurement is made under the best conditions when

$$r_1 + \frac{gs}{g+s} \text{ is less than } X.$$

1117. Kelvin's Method. — The object of this modification of the reduced deflection method is to maintain the deflection of the galvanometer unchanged and then it makes no difference what the law of the deflection of the galvanometer may be. With the circuits arranged as in Fig. 1117 we have, for the current thru the galvanometer,

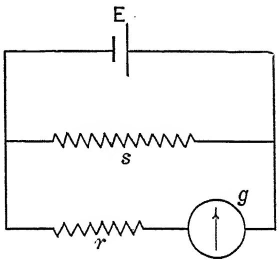

FIG. 1117.

$$i = \frac{Es}{X(s+g+r) + s(g+r)}. \tag{1}$$

The value of the shunt is now changed to s_1 and r is changed to r_1, so the same current as before goes thru the galvanometer. Then,

$$i = \frac{Es_1}{X(s_1+g+r_1) + s_1(g+r_1)}. \tag{2}$$

From Eqs. (1) and (2)

$$X = \frac{ss_1(r_1 - r)}{s_1(r+g) - s(r_1+g)}. \tag{3}$$

If in the second case the shunt s_1 is made infinity then Eq. (3) becomes

$$X = \frac{s(r_1 - r)}{g+r}. \tag{4}$$

The method, like the former, assumes that the E.M.F. of the battery remains unaltered when the current which it delivers is changed.

For this method to be applied practically, the galvanometer must be very insensitive or shunted, or two cells of nearly equal E.M.F. must be joined in series with their polarities opposed. If the galvanometer is shunted, then in place of g we must use the shunted resistance of the galvanometer.

1118. Siemens' Method. — The arrangement of circuits for applying this method is shown diagrammatically in Fig. 1118. Here the circuit *a c b* is a slide wire of uniform resistance upon which a contact *c* may be moved. *Ba* is the battery of which the

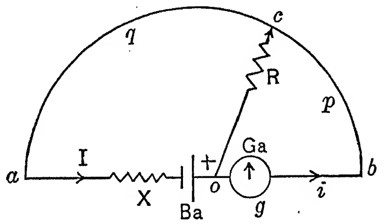

Fig. 1118.

internal resistance is to be determined and *Ga* is a galvanometer which has its sensibility reduced by any means which does not include a series resistance. It may be shunted, in which case the value assigned to its resistance must be that of its shunted resistance.

Let X = the internal resistance of *Ba*, to be found,
g = the resistance of the galvanometer (or, if shunted, its shunted resistance),
p = resistance from point c to point b,
q = resistance from point c to point a,
R = resistance from point c to point o,
E = E.M.F. of battery,
I = current from battery, and
i = current thru galvanometer circuit.

For brevity, write $Q = q + X$ and $P = p + g$.
Then $Q + P = K$, a constant.

By inspection of the diagram it will be seen that if c is moved to b the current thru the galvanometer will be greater than if c is at some intermediate point between a and b. Also if c is moved to a the current thru the galvanometer will again be greater than if c is at some intermediate point between a and b. Hence, generally, there is some point c between a and b where the current thru the galvanometer is a minimum. It is this point, which, when

found by trial, will give the value of the resistance sought. With the contact at any point c on the slide wire, we have

$$I = \frac{E\,(P + R)}{PR + Q\,(P + R)}, \tag{1}$$

and

$$i = \frac{R}{P + R} I, \tag{2}$$

or

$$i = \frac{ER}{PR + Q\,(P + R)}, \tag{3}$$

or, as

$$Q = K - P,$$

$$i = \frac{ER}{PK - P^2 + KR}. \tag{4}$$

It is required to move c until i is a minimum or until $\frac{1}{i} = i_1$ is a maximum.

Thus we have

$$\frac{\delta i_1}{\delta P} = \frac{1}{ER}(K - 2P),$$

and this expression is a maximum when

$$K = 2P, \tag{5}$$

or when

$$Q + P = 2P, \quad \text{or} \quad Q = P$$

That is, when

$$q + X = p + g, \quad \text{or} \quad X = g + p - q. \tag{6}$$

Thus the value of X is found by moving the contact c upon the slide wire until the deflection of the galvanometer is reduced to a minimum. As all the resistances are known we can calculate the current which the battery is yielding, provided we know its E.M.F.

Thus, Eq. (1) becomes

$$I = \frac{E\,(p + g + R)}{(p + g)\,R + (q + X)\,(p + g + R)},$$

or

$$I = \frac{E\,(p + g + R)}{(p + g)\,(2R + p + g)}. \tag{7}$$

Analysis shows (see Kempe, "Handbook of Electrical Testing," p. 161) that the measurement is made under the best conditions when $p + q$ is not less than the greater of the two quantities $R + X$ and $R + g$. Also R should be less than the greater

of the two quantities X and g, and the galvanometer resistance should preferably not exceed X.

1119. Resistance of Electrolytes. — The resistance of electrolytes, as sulphuric acid, salt solutions, etc., could be measured with a Wheatstone bridge in the usual way if it were not for the fact that, as soon as a measuring current passes thru the electrolyte the electrodes polarize and an E.M.F. is developed which opposes the E.M.F. which sends current thru the electrolyte. To understand this clearly consider the diagram, Fig. 1119.

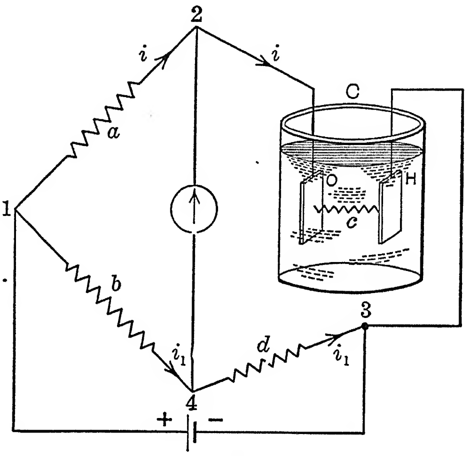

FIG. 1119.

Let C be an electrolytic cell in one arm of a Wheatstone bridge. Let a, b, c, d be resistances, and i and i_1 be currents. If the resistances a, b and d are always so chosen that the bridge is balanced, we shall have the potential drop V_a from 1 to 2 equal the potential drop V_b from 1 to 4. Also the potential drop V_c from 2 to 3 will equal the potential drop V_d from 4 to 3. Or $ai = bi_1$, and $V_c = di_1$, whence

$$\frac{ai}{V_c} = \frac{bi_1}{di_1} \quad \text{or} \quad V_c = \frac{d}{b}ai. \tag{1}$$

Now the potential V_c will be equal to the current i times the resistance c of the cell, less the opposing E.M.F., E, of polarization of the cell,

or

$$V_c = ic - E. \tag{2}$$

It may be assumed that for a very small current i which has flowed for a *short* time t the E.M.F. of polarization is proportional to the quantity of electricity that has passed thru the cell, or, what is the same thing,

$$E = Kit. \tag{3}$$

Hence,

$$V_c = ic - Kit. \tag{4}$$

Putting this value of V_c in Eq. (1) we obtain

$$c = \frac{da}{b} + Kt. \tag{5}$$

This last relation shows that the resistance c of an electrolyte, which would be measured by a balanced Wheatstone bridge, will seem to increase with the time that the current i is kept flowing thru the electrolyte, and that it will always be higher than the true resistance of the electrolyte. For this reason the use of the Wheatstone bridge with direct current is not suited to the measurement of the resistance of an electrolyte. If, however, an alternating current be substituted for a direct current and a detector, responsive to alternating current, be substituted for the galvanometer, the principle of the Wheatstone bridge may be used with convenience and accuracy. This is because the E.M.F. of polarization, produced by the current in one direction and which would lead to a balancing of the bridge giving too high a value of the resistance, will, upon the reversal of the current, either be neutralized or, if not neutralized, will lead to a balancing of the bridge giving too low a value of the resistance. Thus the setting actually obtained for a balance is the same, whether polarization is neutralized or not, as would be required were there no polarization.

Thus, to merely measure the resistance of an electrolyte, without attempting to determine its specific resistance, it is only necessary to place the electrolyte in a vessel provided with two electrodes of thin gold or platinum and connect this vessel into one arm of a Wheatstone bridge. The other arms are resistances which are *highly non-inductive.* The bridge is balanced for alternating current. A telephone is a suitable detector and a small induction coil with a secondary winding furnishes from its secondary a very suitable source of alternating current. The alternating current obtained from a small induction coil is filled with harmonics and gives a clearer and sharper sound in the telephone, conse-

quently a more accurate balance, than an alternating current from ordinary sources which is approximately sinusoidal. It is usually required, however, to obtain the resistivity as well as the resistance of an electrolyte, and to determine this at a very exactly known temperature. The apparatus and method of working devised by Kohlrausch are, with slight modifications, best adapted for this determination and may be described as follows:

1120. The Method of Kohlrausch for Measuring the Resistivity of an Electrolyte.

I. Direct determination of resistivity, telephone employed as detector.

The vessel to contain the electrolyte must be so shaped and the electrodes so located that the length and cross-section of the electrolyte measured can be ascertained with precision. For this purpose nothing is better than a cylindrical glass tube of, as nearly as possible, uniform bore. This may, for example, be a glass tube 20 cms long and 1 cm internal diameter. It may be open at both ends. In the bottom end is fitted an electrode of gold or platinum which completely fills and is parallel to a right cross-section of the tube, but may with advantage have some very small holes thru it to allow liquid to rise in the tube when this is partly submerged in a vessel of liquid. The other electrode is of the same material and should be arranged to be movable along the length of the tube by means of a rod of metal (preferably gold plated). The rod or the tube should have a scale cut upon it so that the distance between the two electrodes may be read from the scale. Diagrammatically the arrangement would be as suggested in Fig. 1120a. If this tube is insulated in an electrolyte so that the upper electrode, but not the upper end of the tube, is submerged, the current will only have the one path, within the tube, from one electrode to the other.

The accurate calibration of this tube for cross-section and distance between the two electrodes, for any setting of the upper electrode, must be made with care. Thus a plug may be turned to fit accurately into each end of the tube if this is round, and these plugs may have their diameters determined with a micrometer caliper. This will give the cross-section where the plugs are fitted. The distance between the electrodes may be determined with a cathetometer, or the volume of the tube, for a given length, may be determined by filling it with mercury and weighing this. From

data so obtained the mean cross-section is readily calculated. It is unusual to find a tube which is not more or less conical. Assuming that it is conical and that S_1 is the cross-section at one electrode, S_2 the cross-section at the other electrode, and that l is the distance between the two electrodes, then it can easily be shown

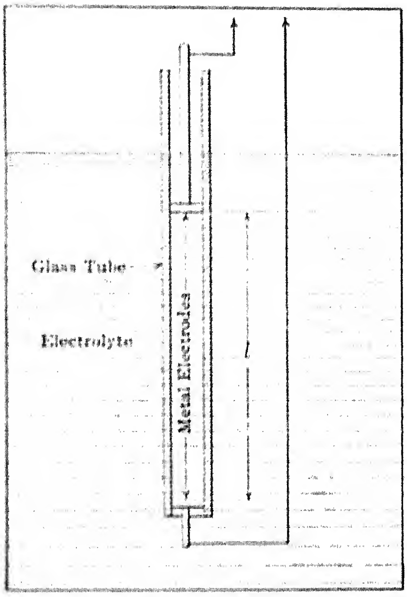

FIG. 1120a.

that the expression for the resistance from one electrode to the other, when the tube is filled with a liquid of specific resistance ρ, is

$$R = \frac{l}{\sqrt{S_1 S_2}}\rho, \tag{1}$$

$\frac{l}{\sqrt{S_1 S_2}}$ is called the resistance capacity of the tube. Call this quantity r_r. Then,

$$\rho = \frac{R}{r_r}. \tag{2}$$

To complete the arrangement the outer vessel, containing the electrolyte, should be provided with a stirrer for stirring the electrolyte to insure a uniform temperature throughout. The

temperature may then be read with any type of accurate thermometer placed close to, but outside of, the measuring tube.

To make a measurement the two electrode-terminals are connected in one arm of a Wheatstone bridge, which is preferably of the slide-wire type. A special form of slide-wire bridge with a long slide wire wound spirally upon a marble cylinder has been upon the market for several years. A type of bridge of this kind, in which the marble cylinder is stationary and the contact made movable, was designed by the author and is shown in Fig. 1120b.

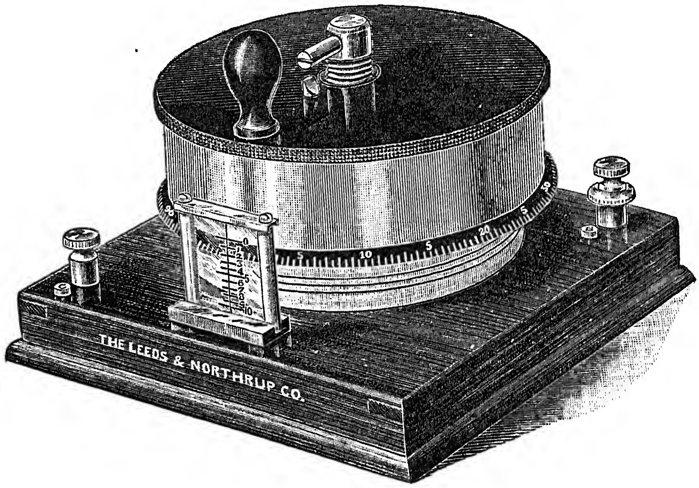

Fig. 1120b.

The cylinder upon which the bridge wire is wound is 15 cms in diameter. There are ten turns of wire, giving a total length of wire of 470 cms. The contact point consists of a minute piece of hardened steel mounted in a short rod of manganin (the same material as the slide wire, so as to avoid thermal E.M.F.'s where the apparatus is used for other purposes with direct currents). This steel piece slides upon the wire without abrading it. It is not visible in the illustration, being inside the protecting hood. The hood revolves upon a threaded spindle, the pitch of the thread being equal to the pitch of the groove in the marble block on which the manganin wire is wound. The resistance of this wire is approximately 5 ohms. The position of the contact is read by means of the vertical glass scale shown in the illustration. Com-

plete turns are read upon the horizontal lines of the glass scale and fractions of a turn are read from the scale upon the lower rim of the hood. The latter scale is divided into 100 scale divisions, each of about 0.6 cm. These are divided into halves, so that it is easily possible to estimate to thousandths of one complete revolution. The wire is made very uniform in resistance.

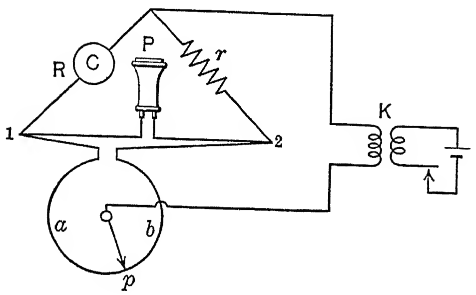

FIG. 1120c.

The connections to use are diagrammatically shown in Fig. 1120c. It is generally better to join the secondary of the induction coil K to the sliding contact p and the point between the cell C and the rheostat r, and join the telephone P to the other two points 1, 2 of the bridge, than vice versa. A balance can generally be easily obtained with almost complete silence in the telephone. The resistance R of the electrolyte is then $R = \frac{a}{b} r$. But as the scale of the slide wire is divided into 1000 divisions, $a + b = 1000$ or $b = 1000 - a$, whence

$$R = \frac{a}{1000 - a} r. \tag{3}$$

As the fraction $\frac{a}{1000 - a}$ occurs frequently in bridge measurements a table is given for all values of a from 1 to 1000 (see Appendix I, 1). It is desirable, for precision, to choose r as nearly as possible equal to the resistance of the electrolyte being measured, and then, for a balance, p will come near the middle of the slide wire. This will give greater accuracy to the measurement than if it came near either end. The resistance r should be, as far as possible, non-inductive and free from electrostatic capacity. The specific resistance at a particular temperature is obtained by

dividing the value of R, found for a given temperature, by the resistance capacity r_c of the tube.

If one is not in possession of the special Kohlrausch bridge illustrated in Fig. 1120b, very good results may be obtained by using an ordinary straight wire slide-wire bridge. For accuracy in reading to not better than 0.1 to 0.2 of 1 per cent the circular slide wire shown in Fig. 401b, may be used with advantage. As the scale connected with this slide wire is laid off to read the resistance directly in per cent of the standard resistance all calculation is avoided by its use.

II. Electrodynamometer employed as detector:

In place of a small induction coil for the current source and a telephone for a detector one may use alternating current and a sensitive electrodynamometer as the detector. The proper way to connect this instrument in circuit is shown in Fig. 1120d.

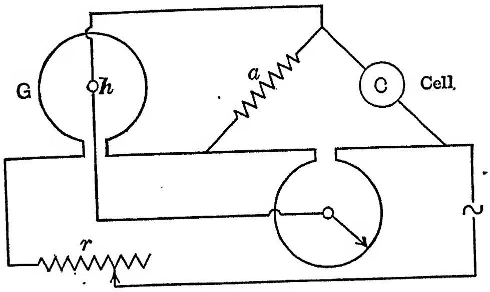

FIG. 1120d.

The reason for placing the fixed coil G of the electrodynamometer in the main circuit is to increase the sensibility of the apparatus, which would be very small if the fixed and swinging coils were joined in series and the electrodynamometer then connected in the bridge in the usual way. In this method the alternating current may be taken directly from the mains and its value reduced by a suitable resistance r. The method is otherwise carried out in the same manner as when a telephone and small induction coil are used.

The type of deflection electrodynamometer recommended is the one described in par. 1001.

1121. Determination of Relative Resistivities of Electrolytes. — For this purpose the methods of making the measurements are not different from those just given. A different

form of cell for holding the electrolyte is, however, to be preferred.

Suppose the containing cell to have any shape and that it is filled with an electrolyte of known specific resistance ρ_t at temperature $t°$ C. If S is the effective cross-section of the cell and l its length then $R_t = \frac{l}{S}\rho_t$ is the resistance of the electrolyte at temperature $t°$ C. Let the resistance be accurately determined at the temperature $t°$ C.

If the cell is now filled with an electrolyte of unknown specific resistance γ and the resistance of this electrolyte is measured at temperature $t_1°$ C., we have, as before,

$$R_{t_1}' = \frac{l}{S}\gamma_{t_1}.$$

Whence, taking the ratio of the two resistances so obtained, we have

$$\gamma_{t_1} = \frac{R_{t_1}'}{R_t}\rho_t. \qquad (1)$$

Eq. (1) gives the specific resistance of the electrolyte being measured at the temperature $t_1°$ C., in terms of the two resistance measurements and the specific resistance ρ_t of the standard electrolyte at the temperature $t°$ C. This last value can be taken from a table of specific resistances of electrolytes at different temperatures. But if the specific resistance of the electrolyte being measured is to be compared at the same temperature with that of the standard electrolyte it is necessary to adopt either of two procedures. One, is to arrange that the temperature at which R is determined shall be the same as the temperature at which R' is determined; or R' may be measured at two temperatures, one a little above the temperature $t°$ C. and one a little below this temperature. Then the resistance that the electrolyte would have at the temperature at which the standard electrolyte was measured can be determined by a simple calculation. Assuming that R' has been determined in either of these ways we have,

$$\gamma_t = \frac{R_t'}{R_t}\rho_t. \qquad (2)$$

The form of cell which is found very convenient for determinations of the above class is shown in Fig. 1121.

Contact with the platinum or gold electrodes is made permanently with mercury in bent glass tubes as shown in the figure, and temporary lead wires dip into the mercury. During the measurements the cell should be suspended in a vessel containing water which is well stirred and of which the temperature can be accurately taken.

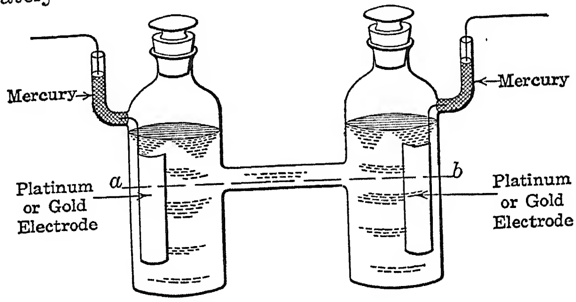

FIG. 1121.

The resistivities of a saturated solution of sodium chloride at various temperatures are given in Appendix IV, 4.

The methods described of measuring resistivities of electrolytes furnish a means of ascertaining the concentration of a solution. Tables have been constructed which give the relation which has been found to exist at some standard temperature between the resistivity and the concentration of many solutions. Hence, if the nature but not the concentration of any solution is known, for which tables exist, the latter can be simply and accurately found by measuring the resistivity of the solution.

The tables usually give not the resistivity but the conductivity which is the reciprocal of the resistivity. The standard temperature for which the conductivities are given in most tables is 18° C., and hence measurements should be made as near this temperature as practicable.

For tables of the electrical conductivity of solutions, the reader is referred to "Physical and Chemical Constants," by G. W. C. Kaye and T. H. Laby, pages 86–87. Also to "Physikalisch-Chemische Tabellen," by Landolt, Börnstein and Meyerhoffer, page 735 and following, third edition.

It may be remarked here that the resistivity of a saturated solution of sodium chloride (NaCl) at 20° C. is 4.4248 ohms. This is the resistance between opposite faces of a centimeter-

cube of the solution. The resistivity of 100 per cent conductivity copper at the same temperature is 1.7215×10^{-6} ohm. Hence the salt solution has a resistivity which is 2.570×10^{6} times that of copper. In general, electrolytes have, roughly, a million times the resistivity of metals.

1122. **Hering's Liquid Potentiometer Method for Determining Electrolytic Resistances.** — Dr. Carl Hering has shown* that the principle of the potentiometer may be employed for determining the resistance, and, if the necessary dimensions of the containing vessel are known, the resistivity of an electrolyte. The method is said to avoid the errors which in direct-current methods are ordinarily introduced by polarization of electrodes. This method permits the resistance to be measured between two selected points of a quantity of electrolyte contained in a tank. It is applied as follows:

The apparatus employed consists of a tank of rectangular form built of insulating material. A suitable scale is fastened to the upper and longer edge of the tank for measuring the distance between the potential electrodes at the moment a balance is obtained. Two current electrodes are fitted into each end of the tank which reach across it, and above the surface of the liquid. The liquid may fill the tank to any convenient height.

It is important to choose potential electrodes which are as inert as possible in the electrolyte. Two gold coins or thin strips of gold are recommended for these electrodes. The instruments required are an ammeter, a galvanometer or millivoltmeter, and means for determining the E.M.F. of the small cell which is used as the standard of E.M.F. Porous diaphragms should be fitted in the ends of the tank to prevent the products of decomposition at the current electrodes from entering the main body of the electrolyte. The polarities of the two batteries are arranged as in the ordinary use of the potentiometer. The "setting" is made by varying the distance between the two gold electrodes until the galvanometer shows no deflection. At the final setting one of the potential electrodes should be oscillated thru a small amplitude in the direction of the fall of potential so the small galvanometer deflections are equal on both sides of the zero. This setting made, the current is read upon the ammeter, also the distance

* Transactions of the American Institute of Electrical Engineers, February 28, 1902, page 827.

between the electrodes. If E is the E.M.F. of the standard cell and I the current read on the ammeter then the resistance between the electrodes is $R = \frac{E}{I}$. If l is the distance between the electrodes and S the cross-section of the tank the specific resistance of the electrolyte is

$$\rho = \frac{SR}{l} = \frac{SE}{lI}. \tag{1}$$

The author has not tried this method and so cannot speak from personal experience as to its accuracy and value. Further details may be obtained by reference to Dr. Hering's original paper.

1123. **The Substitution Method.** — The principle of this method has already been described, par. 206, Fig. 206b. The apparatus shown in Fig. 1120a is the same in principle as that shown in Fig. 206b and may be used for the measurement. The substitution method of measuring electrolytic resistances is inferior to alternating-current methods but may be used with advantage, perhaps, for quick and rough determinations of the resistivities of electrolytes in battery jars, electroplating tanks, etc. The apparatus required is generally at hand and the tube or cell may be submerged in any body of electrolyte and the measurement be very quickly made *in situ*.

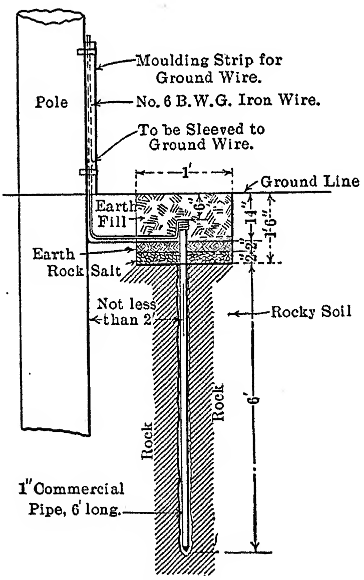

Fig. 1124a.

If the electrolyte is a silver, copper or nickel solution it is well to use electrodes of these metals. For other solutions gold or platinum electrodes are more suitable. It is recommended to make the electrodes of fine wire-mesh, as bubbles from polarization will more readily escape from the electrodes.

1124. **Resistance of "Grounds"** (Bell Telephone Method). — There is used in telephone practice what is termed a "Cable protector ground." These are ground connections made at tele-

phone poles to afford protection for aerial, as well as underground cable plants, against lightning. The ground connection is carried to an "open space cutout," and is made in the manner shown in Fig. 1124a. It is necessary that the resistance of such grounds should not exceed a proper limit and tests and reports are fre-

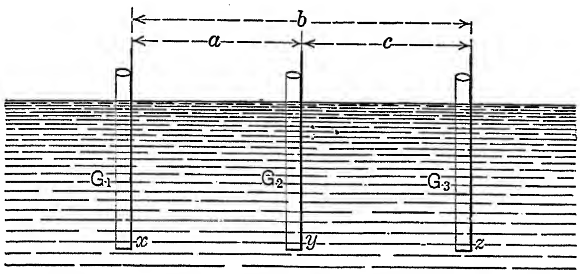

FIG. 1124b.

quently made upon such grounds. The method adopted by the telephone company for measuring the ground resistance is known as the "three ground method" the principle of which may be explained as follows:

Referring to Fig. 1124b, G_1, G_2, G_3 are three ground connections, having resistances to ground x, y, z, respectively. The ground G_1

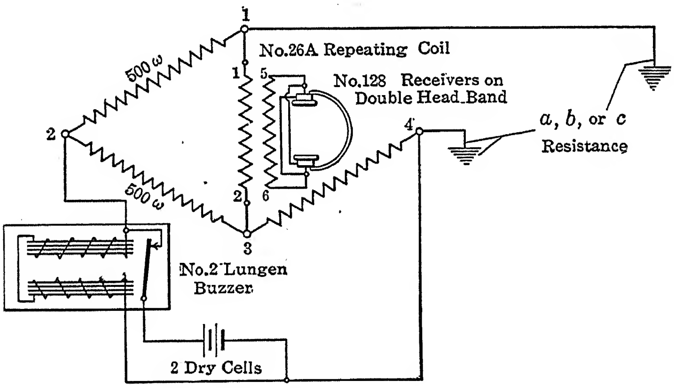

FIG. 1124c.

is the permanent ground. It is required to determine the resistance x of this. The grounds G_2 and G_3 are auxiliary or temporary grounds which are constructed in order to effect the measurement. First measure (by the principle of the method shown in Fig.

1120c) the resistance between G_1 and G_2, then between G_1 and G_3, and lastly between G_2 and G_3. Calling a, b, and c these resistances respectively, we have

$$x + y = a,$$

$$x + z = b,$$

and

$$y + z = c.$$

From these three equations we derive

$$x = \frac{a + b - c}{2}. \tag{1}$$

The exact manner, as applied by the telephone company, of making the measurements is clearly explained by the diagram shown in Fig. 1124c. The ground resistance may vary between such limits as 1 and 1000 ohms.

CHAPTER XII.

ELEMENTARY PRINCIPLES OF FAULT LOCATION.

1200. **Fault Location.** — All electric lines, whether used for the transmission of intelligence or power, are subject to what is technically designated "faults." These faults consist, in general, of a complete breaking down or a serious deterioration of the insulation of the line, or of a break in the conductor. If the defect develops at a definite point it becomes important to be able to locate its place, as determined in distance from some station along the line. If the location of a fault can be quickly and accurately effected, the time and expense of making a repair is greatly reduced.

The methods which have been developed for locating faults from a station on the line chiefly embody some form of resistance measurement and are carried out with resistance-measuring apparatus. A description of these methods properly belongs, therefore, to a work of this character. The full development and application of the methods when applied to submarine cables in service is complex and extensive, and should be confined to works devoted especially to this phase of the subject. The fundamental principles of fault location upon land lines, however, are easily understood and may be properly described here. In many cases their application is quite simple. In other cases, however, the conditions under which the relatively simple principles have to be applied are complicated by networks of conductors, multiplicity of faults, earth currents, variations in the size of wires in the same circuit, and other causes which become at times very puzzling. The majority of faults may be located, however, by one familiar with the fundamental principles and moderately practiced in their application. We proceed to classify and tersely describe these fundamental principles. For detailed descriptions of the special apparatus which instrument makers have developed for fault location the reader must be referred to the trade publications which advertise and often very fully describe this class of apparatus.

1201. Faults Occurring on Land Lines.

(a) *Grounds.* — This is a common fault which is a partial or complete breaking down of the insulation whereby the conductor becomes connected to the ground or to the sheath of the cable. If the ground connection is localized at a single point the fault may be definitely located, but not infrequently a ground connection occurs at two or more points. In this event a precise location of each point where the conductor is grounded is, in general, not possible.

(b) *Crosses.* — These are faults in which two or more conductors in the same sheath, or on the same pole lines, become connected together or crossed. As in the case of grounds this fault may occur at one place or several places. In the former case the localizing of the fault is easy and locations are usually made by the same methods as are used for locating grounds.

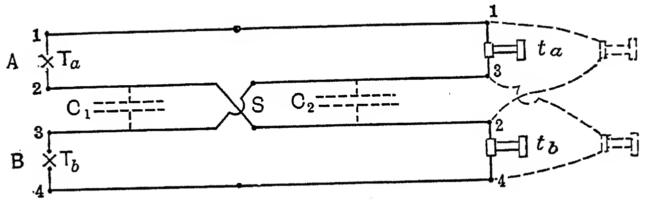

FIG. 1201.

(c) *Opens.* — An open is produced when the conductor-circuit becomes broken or open. If the circuit is not at the same time grounded or crossed, the point at which the circuit is open may usually be located. The methods differ, however, from those used for grounds or crosses.

(d) *Inductive crosses.* — These faults consist of a transposition in telephone cables of single sides of adjacent pairs of conductors. They result from incorrect cable-splicing. This fault is illustrated by Fig. 1201.

Here one wire of the pair A is transposed with one wire of the pair B at the splice S. As there is an electrostatic capacity between pairs of wires as represented by the condensers C_1 and C_2 shown in dotted line, conversation might be carried on between T_a and t_a, likewise between T_b and t_b, but there will also be bad cross talk between T_b and t_a, and T_a and t_b. Workmen often attempt to correct this fault by connecting the telephones as indi-

cated in dotted line, but the cross talk remains. A special method will be given for locating the position of an inductive cross.

1202. Problems in Fault Location. — The chief problems and tests, treated under the subject of fault location, may be summarized as follows:

(a) Identification of faulty wires.

(b) Determination of the resistance of conductor loops.

(c) The location, in distance from a station, of grounds, crosses, opens or inductive crosses, on telephone or telegraph lines.

(d) Fault locations when loops are made up of wires of different sizes and lengths.

(e)Insulation resistance tests of installed and uninstalled cables. Here either the insulation may be defective in particular places or the defective insulation may be distributed along the conductor.

(f) The location of grounds or crosses in heavy power cables, requiring special apparatus.

(g) The location of grounds or crosses in very heavy, short, underground cables. A special method is here required.

(h) Location of grounds or crosses in high-tension cables which are subject to inductive disturbance from parallel alternating-current lines.

(i) Location of faults in submarine cables, during manufacture, test, and after being installed. These problems are special and are not considered here.

1203. Relation of Principles to Practice in Fault Location. — Fault location depends upon certain fundamental principles which must be clearly understood for intelligent work. They should be studied before any consideration is given to the details of specific apparatus and methods. There are, however, many practical points which must be considered in the successful application of the fundamental principles. These practical points and familiarity with fault-locating apparatus are best acquired, however, by practice and experience in the field. The principles of the subject, therefore, should chiefly concern us here and we proceed to their elucidation.

1204. Location of a Ground upon a Single Line with Only an Earth Return. — Single lines with only earth return are found in overhead telephone and telegraph lines in unsettled country and in single lines laid under water. The two methods to be given are considered as having more of a theoretical interest than

practical value, altho they work out very well with artificial lines made up in a laboratory.

(a) *By testing from each end of line.*

Signal from A to B, Fig. 1204a, to open K_1 and k_1, assuming that ground is not so bad but that it is possible to drive a signal

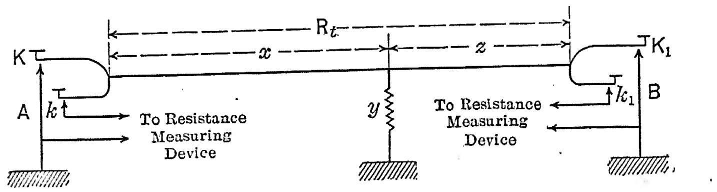

FIG. 1204a.

through. Measure with resistance-measuring device, k being closed and K open, the resistance

$$x + y = R.$$

Leave K open and open k. From end B, with k_1 closed and K_1 open, measure resistance

$$z + y = R_1 = R_t - x + y$$

where R_t is the total resistance of the line.

Then the resistance to the fault from end A is

$$x = \frac{R - R_1 + R_t}{2}. \tag{1}$$

The distance to the fault may then be calculated when the feet per ohm of the wire is known.

The difficulties which would be encountered in this test would arise (a) from earth-currents; (b) from variations in the resistance y while the test is in progress, and (c) from the resistance of y at times being so high that the resistances x or z would be small in comparison.

The method recommended for measuring the resistances R and R_1 is the "voltmeter and ammeter method" given in par. 209. If the measuring current sent over the line is large it will give the double advantage of making the effect of earth currents relatively small and of breaking down the ground resistance y to a low value. The method should prove most useful and relatively easy of application in a wild country free from disturbing earth currents and when the single line is of great length.

The method may be used to locate a grounded point in a coil of insulated conductor in a tank, or when wound upon a metal frame. In this case, however, both ends of the conductor would be at the same place and the location could be made more easily and accurately by the Murray-loop test given in par. 1205.

(b) *By testing from one end of line only.* (Author's method.)

It is of theoretical interest to show that a ground may be located from one end only of a long, single conductor line when the only resistance known is that of the line itself.

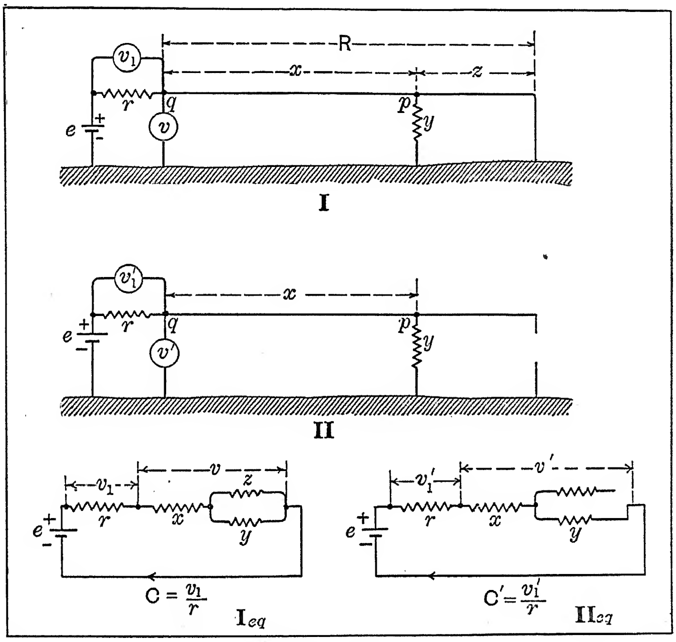

FIG. 1204b.

The apparatus required is two voltmeters, a known resistance (preferably one which can be given different values), and a source of steady E.M.F.

The fundamental assumptions made are: (a) that the total resistance R of the line is known; (b) that the ground is not so bad but that a signal may be driven through the line to give directions for earthing and unearthing it at its far end at proper times, and (c) that the ground connection is located at one point only and that its resistance remains steady during the test.

Represent the line and connections for the test, Fig. 1204b, at I with line grounded at both ends, at II with far end of line open, and let I_{eq} and II_{eq} represent equivalent circuits.

First, a signal is sent to ground the line, as represented at I, and the voltmeter readings v_1 and v are made with the two voltmeters.

Second, a signal is sent to disconnect the line at the far end, as represented at II, and readings v_1' and v' are made with the two voltmeters. We now have $R = x + z$, the total resistance of the line. The auxiliary resistance r is known and v_1, v_1', v, and v' are measured quantities. We then have (case I or I_{eq})

$$z = R - x, \tag{1}$$

$$v = \frac{v_1}{r}\left(x + \frac{zy}{z+y}\right), \tag{2}$$

and (case II or II_{eq}).

$$v' = \frac{v_1'}{r}(x + y). \tag{3}$$

In these three equations there are only the three unknown quantities x, y, and z; hence it is possible to find the value of each of them. The value of x, the resistance of the line from q to p, may be obtained as follows:

For brevity, write $\frac{v_1}{r} = C$ and $\frac{v_1'}{r} = C'$.

These are the currents in the line in cases I and II respectively. Then, from Eq. (2),

$$y = \frac{z(Cx - v)}{v - Cx - Cz}, \tag{4}$$

and from Eq. (3)

$$y = \frac{v' - C'x}{C'}. \tag{5}$$

Replacing z in Eq. (4) by its value $R - x$ we obtain

$$\frac{(R - x)(Cx - v)}{v - Cx - C(R - x)} = \frac{v' - C'x}{C'}. \tag{6}$$

This is a quadratic equation in x. Its solution is

$$x = \frac{v}{C} \pm \sqrt{\frac{v^2}{C^2} + \frac{v'CR - C'Rv - vv'}{C'C}}. \tag{7}$$

(See Appendix II, 7, Eq. 16.)

Since from Eq. (2) $x = \frac{v}{C} - \left|\frac{zy}{z+y}\right.$, we see that in Eq. (7) the negative sign should be used before the radical.

If in Eq. (7) we replace C and C' by their values $\frac{v_1}{r}$ and $\frac{v_1'}{r}$ respectively, and use the negative sign before the radical, we obtain

$$x = r\left[\frac{v}{v_1} - \left\{\frac{v^2}{v_1^2} + \frac{\frac{R}{r}(v'v_1 - vv_1') - v'v}{v_1v_1'}\right\}^{\frac{1}{2}}\right]. \tag{8}$$

Eq. (8) gives the resistance of the line to the point where it is grounded in terms of voltmeter readings and known resistances.

Two important simplifications may be made:

(a) The resistance r can be chosen equal to the total resistance R of the line.

(b) The current flowing can be maintained the same (by means of a small rheostat in circuit with the battery on its grounded side) when the connections are changed from I to II. In this case $v_1 = v_1'$.

Assuming that conditions (a) and (b) are fulfilled, Eq. (8) reduces to

$$x = \frac{R}{v_1}\left[v - \sqrt{(v' - v)(v_1 - v)}\right]. \tag{9}$$

It should be possible to always meet, in practice, these two requirements for simplicity, and occasions might arise where the method would yield practical results of importance.

The method was tested in the laboratory as follows: 470 feet of No. 17 soft iron wire were strung around the sides of a room. A fault was made. Its position was varied and also the resistance y of the fault to earth, and locations made with each arrangement.

The total resistance of the wire was 13.8 ohms and the resistance r was given this value. Locations were generally made with an error of less than 5 feet in 335 feet.

A sample set of readings and the calculation of the value of x are given below:

Voltage drop over r, v_1	Voltage drop over $x + z$ and y in parallel, v	Voltage drop over $x + y$, v'	Calculated value of x in ohms and feet	True value of x in ohms and feet
2.000	1.886	3.740	9.84 ohms	9.87 ohms
.....	.....	.....	335.1 feet	336.2 feet

The calculation made by Eq. (9) is

$$x = \frac{13.8}{2}\left[1.886 - \sqrt{(3.740 - 1.886)\ (2.000 - 1.886)}\right] = 9.84 \text{ ohms.}$$

1205. Loop Methods for Locating Grounds or Crosses. — In most cases where a fault is to be located, other wires, not faulty, run between the same points as the faulty wire, and one or more of these good wires may be used in making a test. Good wires are usually to be found in the same lead sheath with the faulty

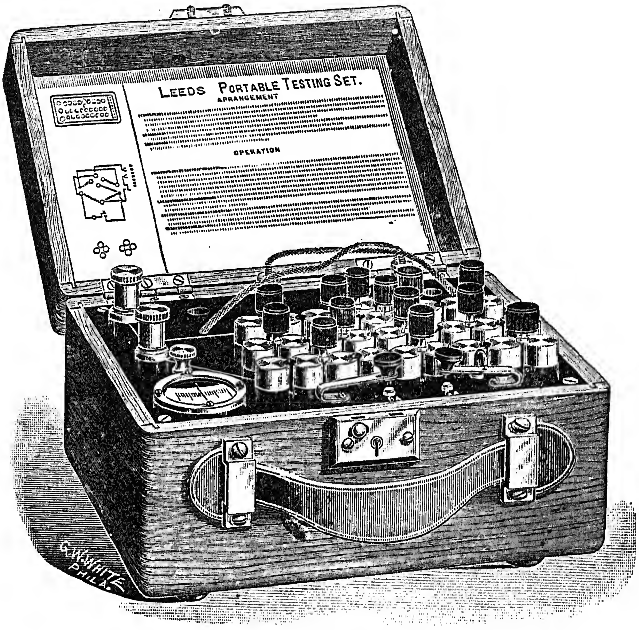

Fig. 1205a.

wire. An end of a good wire may be connected with an end of the faulty wire. When a good wire and a faulty wire are so connected, the combination forms a loop. Hence the name, where this combination is used, of "loop test."

The "Murray loop" and the "Varley loop" are the two loop tests best known and most frequently employed.

The Murray Loop. — In this method two parts of the conductor-loop make up two arms of a Wheatstone bridge. The other two arms are obtained from the testing apparatus. The testing apparatus usually employed is a portable Wheatstone bridge with keys,

a galvanometer, and several cells of small dry battery mounted in the same case with the coils. The rheostat arm is made variable, in some constructions with plugs and in others with dials. Apparatus of this character is popularly known as " A portable testing set." The oldest, simplest, and least expensive type of portable testing sets, thousands of which are in use, is shown in Fig. 1205a. The more expensive, but far more convenient, sets are made with decade rheostats, some operated with plugs and others, better still, with dials.

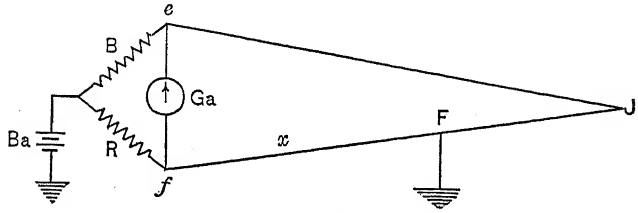

FIG. 1205b.

The arrangement employed in the Murray-loop test, when the testing apparatus is a Wheatstone bridge with a variable rheostat, is shown as a theoretical diagram in Fig. 1205b.

Here fJ is the faulty wire; in this case the fault is indicated as a ground at the point F.

eJ is the return wire, which is free from a fault. The " loop " is eJf. The two arms of the bridge, made by the conductors of the loop, are eJF and Ff, or x.

The two arms, made by the testing apparatus, are B and R. In this application of the method the arm B is given a fixed value, and the arm R is varied to obtain a balance.

The galvanometer Ga is joined between the points e and f, and the battery has one terminal, either positive or negative, joined between the arms B and R and the other terminal to the earth. The relative positions of the galvanometer and battery should always be chosen as above; because, if the galvanometer is not joined to the earth, it will not be affected by earth currents. There would be a very disturbing factor if the galvanometer were put to earth.

(a) Call r the resistance of the loop eJf and assume this as previously known or obtained by a measurement at the time of the test.

Then, if the resistance B is maintained fixed and R is varied until the bridge is balanced, we have

$$\frac{B}{R} = \frac{r - x}{x},$$

whence,

$$x = \frac{R}{B + R} r. \tag{1}$$

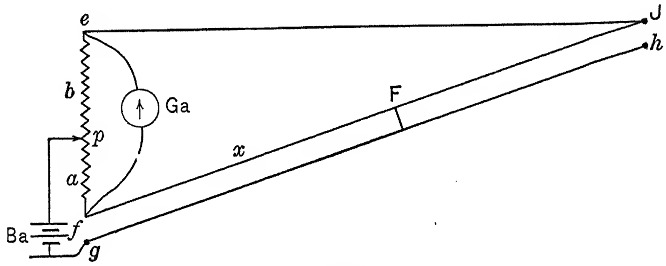

FIG. 1205c.

(b) If the resistance of the wire is uniform throughout its length the resistance will be proportional to the length. Then, if L equals the total length of the loop, in any chosen units of length, and d is the distance to the fault, expressed in the same units of length, we have

$$\frac{B}{R} = \frac{L - d}{d},$$

whence,

$$d = \frac{R}{B + R} L. \tag{2}$$

When the loop is made up of wires of known different lengths of different sizes, these can be reduced by calculation to equivalent lengths of one of the wires of the loop (see § 1210).

In using formula (1) in order to calculate the distance to the fault, the resistance of the wire per 1000 feet must be obtained from a wire table (for copper wire, see Appendix III, 1). A slight error in gauging the diameter or in estimating the temperature of the wire may cause a considerable error. If the length L of the loop is known, so that formula (2) may be used, the distance to the fault may be obtained with much greater accuracy, as all errors in wire-size, temperature, and specific resistance eliminate.

An arrangement employed for the Murray-loop test, when some type of slide-wire bridge is used for the testing apparatus, is shown in Fig. 1205c.

Here the faulty wire fJ is shown crossed with another wire gh at F. In this case one terminal of the battery is connected to the crossed wire instead of being put to the earth. Otherwise there is no difference in procedure for locating a cross or a ground.

The slide wire of the bridge is ef and the balance is obtained by moving the contact p along the slide wire, thus varying the ratio $\frac{b}{a}$.

(a) If r is the total resistance of the loop, we have

$$\frac{b}{a} = \frac{r - x}{x},$$

or

$$x = \frac{a}{a + b} r. \tag{3}$$

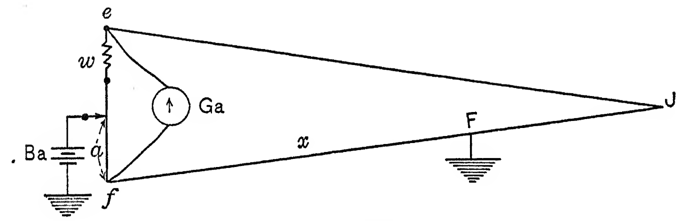

FIG. 1205d.

(b) Or, if L is the total length of the loop and d is the distance to the fault,

$$d = \frac{a}{a + b} L. \tag{4}$$

It is usual to make $a + b$ equal to 1000 scale divisions. In this case

$$d = \frac{a}{1000} L. \tag{5}$$

Thus suppose $L = 5286$ feet and the scale reading is $a = 236$, then the distance to the fault is

$$d = 0.236 \times 5286 = 1247.5 \text{ feet}.$$

(c) It is customary in order to gain the advantage of a slide wire of double length to employ the following modification of the above method:

In Fig. 1205d, w is an extension of the slide wire. This extension is a wire equal in length and size to the slide wire or it is a spool resistance which is made exactly equal to the resistance

of the slide wire itself. Then the relation which holds for a balance of the bridge is

$$\frac{w + w - a}{a} = \frac{r - x}{x}.$$

From this relation

$$x = \frac{a}{w}\frac{r}{2}. \tag{6}$$

The quantities a and w are resistances, but if a is proportional to lengths read upon the scale of the slide wire and if this scale has 1000 scale divisions, the distance to the fault is

$$d = \frac{a}{1000}\frac{L}{2}. \tag{7}$$

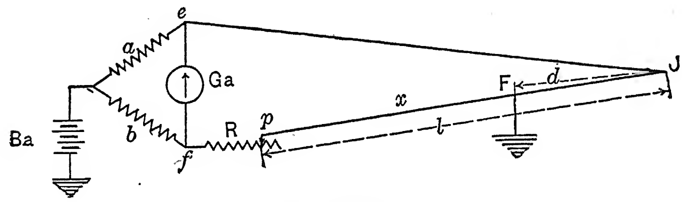

FIG. 1205e.

The rule for locating a fault by this arrangement of the Murray loop then becomes: Connect the faulty wire to the free end of the slide wire and the good wire to the end of the extension resistance. Obtain a balance and read the number of scale divisions from the end of the slide wire joined to the faulty wire. This scale reading divided by 1000 and multiplied by one half the length of the loop is the distance to the fault.

This modification, if used without corrections, also assumes that the resistance of the wires in the loop is uniform and proportional to length of wire.

The Varley Loop. — The Varley loop differs from the Murray loop in having a portion of the resistance of the loop in the testing apparatus.

The Varley-loop test is generally made with a portable testing set. It is most conveniently applied when the ratio arms in the set can be given ratios, 1, 10, 100, 1000, etc.

The connections are shown schematically in Fig. 1205e.

Here a, b are the ratio arms of a Wheatstone bridge (or portable

testing set) which are set at a fixed ratio which we shall call $\frac{a}{b} = D$. R is the rheostat of the set which can be varied, as at p, by plugs or dials. The complete loop is now $eJpf$, and the fault, a ground or cross, is at F.

For a balance

$$\frac{a}{b} = D = \frac{r - x}{R + x},$$

where r is the resistance of the conductor loop ejp. From this relation

(a) $$x = \frac{r - DR}{D + 1}. \tag{8}$$

For unity ratio, or $D = 1$,

$$x = \frac{r - R}{2},$$

or

$$\frac{R}{2} = \frac{r}{2} - x = K_1 d, \tag{9}$$

where d is now the distance to the fault *reckoned from J*, the point of union of the good and bad wires, and K_1 is a constant of proportionality. If the wire in the loop is uniform and the faulty and good wires of equal cross-section, then,

$$\frac{r}{2} = K_1 l, \tag{10}$$

where l is the length of one wire.

From Eqs. (9) and (10)

(b) $$d = \frac{R}{r} l. \tag{11}$$

Thus the rule to follow in making a test becomes: Set the ratio arms of the bridge at unity ratio. Vary the rheostat until a balance is obtained. Then the distance to the fault, reckoned from the *far* end of the loop, is the ratio of the resistance of the rheostat to the resistance of the conductor loop multiplied by the length of one wire.

(c) The gauge of the wire but not its length may be known. In this case we have the length of the conductor loop, or $2\,l$, proportional to the resistance of the conductor loop, or $2\,l = Kr$, or $l = \frac{r}{2} K$.

Here K is a constant which expresses feet per ohm or meters per ohm. Knowing the gauge and kind of wire, the value of this constant may be obtained from a wire table. Putting in Eq. (11) the above value of l, the distance to the fault reckoned from J becomes

$$d = R\frac{K}{2}. \tag{12}$$

If K is given as feet per ohm, the distance in feet to the fault from the far end is the product of the rheostat resistance in ohms and one half the number of feet per ohm of the particular size of wire in the loop.

The relation given in Eq. (12) is one which is very commonly used by telephone line testers.

(d) Another relation sometimes used is obtained thus: for a balance,

$$\frac{a}{b} = \frac{r - x}{R + x},$$

or,

$$x = \frac{br - aR}{a + b}. \tag{13}$$

Eq. (13) expresses the resistance to the fault, reckoned from the end of the line where the test is made.

Another method of using the Varley-loop test is applied as follows:

(e) From Eq. (8) we found when $D = 1$ that $x = \dfrac{r - R}{2}$. Here again, if the gauge and kind of wire is known, we have the distance to the fault, reckoned this time from the *instrument* end,

$$s = Kx = \frac{r - R}{2}K.$$

If K is given as feet per ohm the distance in feet to the fault from the instrument end is

$$s = \frac{r - R}{2} \times \text{feet per ohm}. \tag{14}$$

Illustrative Examples.

(1) Illustration of (a), Eq. (8). The measured resistance r of the loop was 32 ohms. The ratio $\dfrac{a}{b} = D$ used was 0.1, and a balance was obtained with a rheostat setting of 184 ohms. Then

by Eq. (8) the resistance to the fault from the instrument end was

$$x = \frac{32 - 0.1 \times 184}{0.1 + 1} = 12.36 + \text{ohms}.$$

(2) Illustration of (b), Eq. (11). The total resistance r of a cable loop was 267 ohms. The length of one wire, or the cable, was 40,650 feet. The resistance in the rheostat which gave a balance was $R = 27$ ohms. Hence by Eq. (11) the distance to the fault was

$$d = \frac{27}{267} \times 40{,}650 = 4111 \text{ feet}.$$

(3) Illustration of (c), Eq. (12). In a circuit the part of the faulty wire from where the wires were joined was composed of No. 14 copper wire. A balance was obtained with 25 ohms in the rheostat. By a wire table it is found that No. 14 wire has 396.6 feet per ohm. Then the distance from J to the fault is

$$d = 25\,\frac{396.6}{2} = 4957.5 \text{ feet}.$$

(4) Illustration of (e), Eq. (14). The resistance of a loop measured 74 ohms, and a balance was obtained with the rheostat set at 23 ohms. The size of the wire in the loop was No. 14, which has 396.6 feet per ohm. Then by Eq. (14) the distance to the fault from the instrument end was

$$s = \frac{74 - 23}{2} \times 396.6 = 10113.3 \text{ feet}.$$

1206. Notes on the Varley Test.* — "This test is extremely useful, particularly on multiplied telephone cables. By multiplied one is to understand that the same pair of wires is tapped into a number of different terminals, as shown in Fig. 1206. This pair is multiplied at four points, A, B, C, and D.

When the arms of the Wheatstone bridge are made even in the Varley test, the formula $R = r - 2x$ means that the resistance in the rheostat when balance is obtained is equivalent to that of both sides of the pair from the end where the helper makes his connection to the fault.

It is an easy matter for the tester to memorize the constants

* These notes were written by J. W. Wright, of the Bell Telephone Company of Pennsylvania for The Leeds and Northrup Company, and with this company's permission are here reproduced.

for the number of feet per ohm of the three or four most common sizes of wire used for telephone cables.

Knowing the gauge of the wire it is merely necessary to multiply one half the number of feet per ohm of that size wire by the rheostat reading in order to get the distance to the fault in feet.

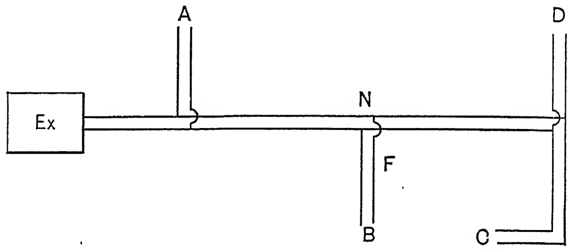

Fig. 1206.

The application of this to multiplied cable may be readily shown. Assume a fault at F. The tester is at Ex and the helper connects the two wires together, say at C. The Varley test then gives the ohms back from C to the apparent location of the fault (in this case at N where the pair to B is tapped on the main cable). Having the resistance from C to N, a rough calculation involving feet per ohm multiplied by Varley reading gives the distance from C to the apparent fault.

From the cable diagram one can read this distance, which should be about that between C and N. The connection at C is then removed and replaced on the same pair at B and the above process repeated. In order not to be misled by the variation in resistance due to changes in temperature, it is well for the tester to measure some known length once each week or two and divide distances by resistance to obtain the proper constant.

That this is quite important may be understood from the fact that for underground cable the constants will vary 10 per cent between summer and winter temperatures.

In cases where there are cables of two different gauges spliced together it is easy to figure out the location without making any gauge correction.

For instance, consider the case where a pair of wires of one gauge is attached to another pair of equal length but different gauge, the total loop resistance being, say, 40 ohms and the Varley test showing a balance at 10 ohms with equal bridge arms.

This would mean that the fault was 10 ohms from the far end. Knowing the two gauges one can estimate mentally if this amount of resistance will carry the location beyond the junction point of the two sizes. If not, then multiply by the constant for the gauge wire on the far end, which will give directly the distance from that end to the fault.

In any case where the balancing resistance carries the location into the section nearest the locator, then instead of multiplying the constant by the rheostat reading, subtract this reading from the total loop resistance and multiply the difference by the constant for the gauge wire nearer to the tester. This method is lengthy of explanation but when once one gets the idea, these processes are mainly mental and really take very little time.

Tests may be made this way on all but the very shortest cables and not then, for the reason that ordinary bridge sets are not subdivided below one ohm in the rheostat.

Very often, however, one is able to interpolate proportionally to the deflection of the galvanometer when a close approximation is necessary.

However, when it comes to a question of inches on a short length of wire, either the Murray test or the Varley test with unequal bridge arms should be used for accuracy.

The formula $R = r - 2x$ is the universal Varley formula for equal bridge arms. Knowing the gauge of the bad wire it is possible to obtain the location by solving for x. When a good wire of large size is used and the fault is near the far end of the smaller wire it sometimes happens that a balance cannot be obtained with the bad wire in series with the rheostat. In these cases it is necessary to reverse the wires on their respective binding posts. Then balance as usual and use the formula with R negative or $-R = r - 2x$,

or $$x = \frac{R + r}{2}.\text{"}$$

1207. Modified Loop Methods to Meet Special Conditions. — *Faulty wire of known length and two good wires of unknown length and resistance.* (H. W. Fisher's method.)

This method is widely applicable. The faulty wire may be an aërial of known length or resistance, or a wire in a cable of known length. The two good wires (either aërials or wires in a cable)

must meet the one necessary condition of terminating at the same points as the faulty wire.

The meaning of the symbols is apparent from an inspection of the diagrams I and II, Fig. 1207a.

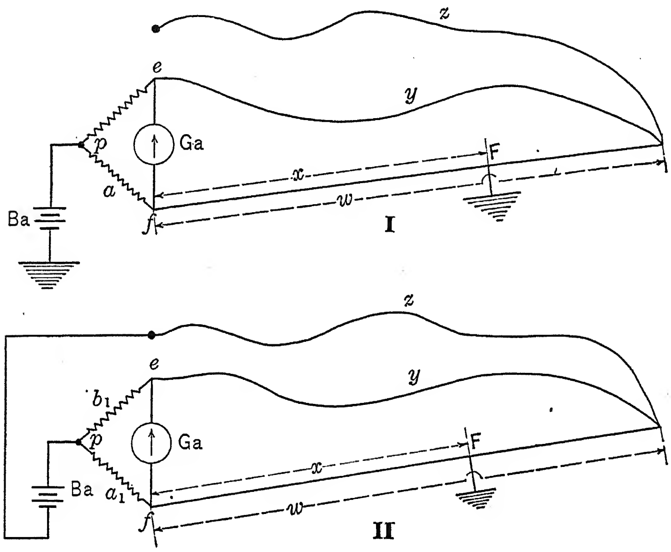

FIG. 1207a.

(a) For connections made as in I a balance of the bridge is obtained when

$$\frac{b}{a} = \frac{y + w - x}{x}, \tag{1}$$

and for connections made as in II a balance is obtained when

$$\frac{b_1}{a_1} = \frac{y}{w}. \tag{2}$$

Eliminating y from Eqs. (1) and (2) the resistance from point f to the fault F is

$$x = \frac{a\,(a_1 + b_1)}{a_1\,(a + b)}\,w. \tag{3}$$

Ordinarily when a balance is obtained in case II the arm b of the bridge would not be changed, in which case

$$x = \frac{a\,(a_1 + b)}{a_1\,(a + b)}\,w. \tag{4}$$

(b) This method is more simply applied in the slide-wire type of bridge. In this case the point of attachment p of the battery is moved over a uniform resistance to secure a balance. Then the resistance $a + b = a_1 + b_1$ at all times, and Eq. (3) becomes

$$x = \frac{a}{a_1} w. \tag{5}$$

If the length l of the faulty wire is known and d is the distance from f to the fault F Eqs. (3), (4), and (5) may be written

$$d = \frac{a\,(a_1 + b_1)}{a_1\,(a + b)}\,l. \tag{6}$$

$$d = \frac{a\,(a_1 + b)}{a_1\,(a + b)}\,l. \tag{7}$$

$$d = \frac{a}{a_1} l. \tag{8}$$

(c) This method may be modified as follows:

1st. Join the two good wires at their distant end and ground them at the point of connection.

2d. Measure the resistance of the loop so formed. Call $z + y = r$.

3d. By means of a Murray or Varley test ascertain the individual resistance of each wire. Thus $z + y = r$ and $\frac{y}{z} = \frac{a}{b}$, whence

$$z = \frac{b}{a + b} r \text{ and } y = \frac{a}{a + b} r.$$

4th. Connect at the far end one of the good wires with the faulty wire and measure the total resistance of the loop. Then obtain the resistance of the faulty wire by subtracting the resistance of the good wire from the resistance of the loop.

5th. The resistance of the faulty wire now being known, the distance to the fault is determined by either the Murray or Varley-loop method.

(d) The Fisher method described above is very conveniently applied when the *length of the cable* which contains the faulty wire is known, but not necessarily the length of the faulty wire itself. The faulty wire may twist in the cable and therefore be longer than the cable sheath. If this twist is *uniform* thruout the length of the cable, the method correctly locates the fault as a certain distance measured along the cable sheath from the tester's end.

Furthermore, the slide wire of the bridge may have an extension resistance q of any value upon one end.

This modification may be briefly described as follows:

In I and II, Fig. 1207b, q is the extension resistance. b, a, b_1, a_1, y, w, and x are resistances. K and k are constants. l is the length

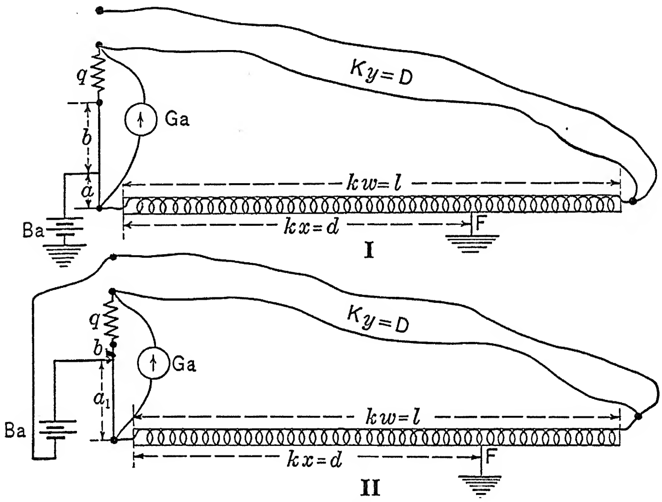

FIG. 1207b.

of the cable sheath and d is the distance to the fault F. With the connections as in I,

$$\frac{q+b}{a} = \frac{y+w-x}{x}. \tag{9}$$

With the connections as in II,

$$\frac{q+b_1}{a_1} = \frac{y}{w}. \tag{10}$$

Whence, from Eqs. (9) and (10),

$$x = \frac{q+a_1+b_1}{q+a+b}\,\frac{a}{a_1}w.$$

But $\quad q + a_1 + b_1 = q + a + b;$

hence, $$x = \frac{a}{a_1}w.$$

Also, $$x = \frac{d}{k} \text{ and } w = \frac{l}{k};$$

hence, $$d = \frac{a}{a_1}l. \tag{11}$$

Thus the distance to the fault equals the ratio of the two readings multiplied by the length of the cable sheath. By using an extension resistance of the right magnitude, a point can always be found upon the bridge wire in case II which will give a balance whatever may be the resistance of the wire y. When $y = w$ and $q =$ the resistance of the bridge wire, the greatest length possible of the bridge wire will be utilized. By using an extension resistance q greater precision may be secured.

The general method of par. 1207 is very usefully applied for locating faults in large cables which are wound upon reels and lie in the factory waiting to be tested. One end only of the cable is brought into the testing room. Two wires, no special regard being given to their resistance, lead from the testing room and are connected to the other end of the cable. The distance to the fault is then obtained from the simple relation given in Eqs. (8) and (11).

1208. Where the Faulty Wire is of Known Length and there is Only One Good Wire of Unknown Length and Resistance. — There are two methods of finding the resistance or the distance to

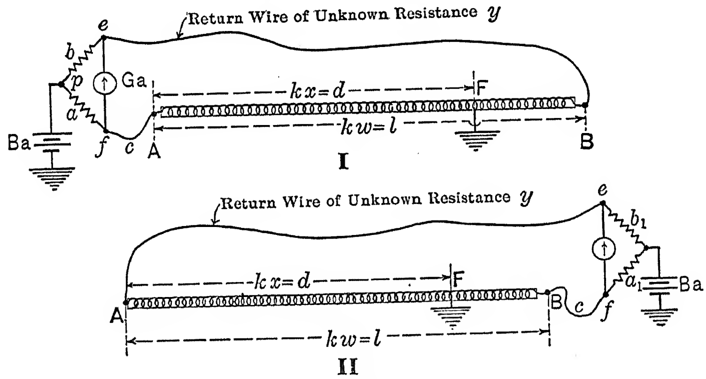

FIG. 1208a.

the fault in this case. The first method described is well known. It requires that a test be made from each end of the faulty wire. The second method described is the author's method and by its means the fault may be located by testing from one end only.

(a) *Testing from both ends of faulty wire.*

1st. Join the faulty wire and good wire together at end B, as in I, Fig. 1208a, and make connections with the testing apparatus as shown in the diagram. For precise work it is necessary to consider

the influence on the result of the lead wire c which joins the point f to an end of the faulty wire. Obtain a balance by varying the bridge arms. Let the resistance values obtained be a and b.

2d. Join the faulty and good wire together at end A, as in II, Fig. 1208a, and complete the connections to the testing apparatus at end B, using the same, or an equivalent, lead wire c. Obtain a second balance of the bridge. Let a_1 and b_1 be the resistance values which give a balance this time.

Let $l = kw =$ the length of the *faulty cable*, where $w =$ the resistance of the *faulty wire* in the cable and k is a constant.

Let $d = kx =$ the distance to the fault from end A, where $x =$ the resistance of the faulty wire from end A to the fault.

Let $y =$ the unknown resistance of the good wire.

Let $c =$ the resistance of the lead wire used.

The resistance of the wire to the fault is now obtained as follows:

In case I,

$$\frac{b}{a} = \frac{y + w - x}{x + c}. \tag{1}$$

In case II,

$$\frac{b_1}{a_1} = \frac{y + x}{w - x + c}. \tag{2}$$

Eliminating y from Eqs. (1) and (2), we finally obtain

$$x = \frac{a\,(a_1 + b_1)}{a\,(a_1 + b_1) + a_1\,(a + b)}\,w + \frac{ab_1 - ba_1}{a\,(a_1 + b_1) + a_1\,(a + b)}\,c. \tag{3}$$

With the slide-wire type of bridge, we would have $a + b = a_1 + b_1$, in which case

$$x = \frac{a}{a + a_1}\,w + \frac{ab_1 - ba_1}{(a + b)(a + a_1)}\,c. \tag{4}$$

Eqs. (3) and (4) show that for exact work the resistance c cannot be neglected unless it is made small. We shall assume that this can always be done so that it is only necessary to use the first term of the right-hand member of Eq. (3) and of Eq. (4).

If c is made negligible, and l the length of the cable which contains the faulty wire is known, then, as resistances are proportional to lengths, Eqs. (3) and (4) become

$$d = \frac{a\,(a_1 + b_1)}{a\,(a_1 + b_1) + a_1\,(a + b)}\,l, \tag{5}$$

and

$$d = \frac{a}{a + a_1}\,l. \tag{6}$$

(b) *Testing from one end of faulty wire.* (Author's Method, I.) In this method a known auxiliary resistance P is required. When a slide-wire bridge is used the test is made as follows: With the connections as in I, Fig. 1208b,

$$\frac{s-a}{a} = \frac{P}{y+w}, \tag{7}$$

where s is the length of the bridge wire and a the reading from the end f, both in scale divisions.

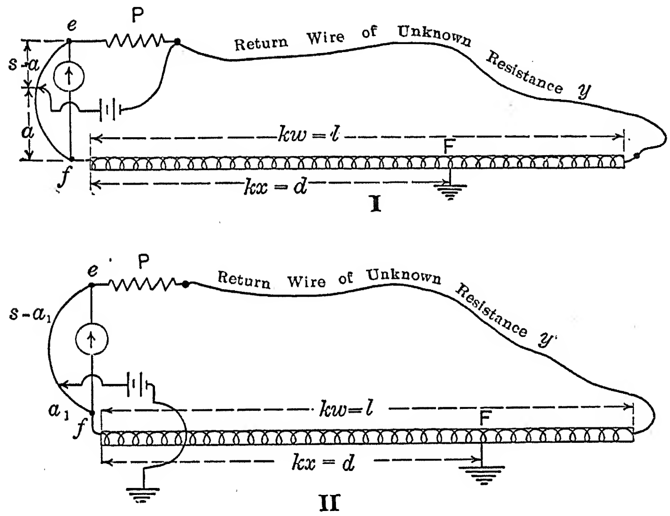

FIG. 1208b.

With the connections made as in II,

$$\frac{s-a_1}{a_1} = \frac{P+y+w-x}{x}. \tag{8}$$

From Eqs. (7) and (8) we have, by eliminating y,

$$x = \frac{a_1}{s-a}P. \tag{9}$$

Eq. (9) gives the *resistance* to the fault F from the end where the test is made.

If the bridge wire has a length of 1000 scale divisions, $s = 1000$ and

$$x = \frac{a_1}{1000-a}P. \tag{10}$$

It will be noted that in this method it is not even necessary to know the resistance of the faulty wire to obtain the resistance to the fault. If, however, the resistance w of the faulty wire is known then the distance to the fault will be

$$d = \frac{a_1P}{1000 - a}\,\frac{l}{w}, \tag{11}$$

where l is the length of the cable which contains the faulty wire. The auxiliary resistance P should be given the same order of magnitude as the resistance of the wire y or w.

Example taken from an actual test to illustrate Eqs. (10) *and* (11).

The length of the faulty cable was $l = 1252$ meters.

The resistance of the faulty wire was $w = 8.34$ ohms.

$P = 13.2$ ohms.

$a = 399$ scale divisions.

$a_1 = 111.7$ scale divisions.

$s = 1000$ scale divisions.

Therefore, by Eq. (10),

$$x = \frac{111.7}{1000 - 399} \times 13.2 = 2.453 \text{ ohms.}$$

By Eq. (11) the distance to the fault was

$$d = \frac{111.7}{1000 - 399}13.2 \times \frac{1252}{8.34} = 2.453 \times \frac{1252}{8.34} = 368.2 \text{ meters.}$$

The actual distance to the fault was afterward determined and found to be 374 meters. This makes the error 5.8 meters or a little over 0.5 of 1 per cent of the length of the cable.

(c) *Testing from one end of faulty wire.* (Author's Method, II.) This method is only a modification of I, above. It will be noted that the method may be applied by employing the same apparatus, and connections similar to those used in making a Varley-loop test.

In this modification the ratio arms a and b are maintained fixed and a balance is secured by varying an auxiliary resistance R. This resistance may be the rheostat of the portable testing set. Call z the unknown resistance of the good wire plus the faulty wire from its far end J to the fault F.

Call x the resistance of the faulty wire from the near end f to

the fault F. Then with the connections as in I, Fig. 1208c, we have for a balance, obtained by varying R,

$$\frac{a}{b} = \frac{z}{x + R}. \tag{12}$$

With the connections as in II, we have for a balance, obtained with a new value of the resistance R_1,

$$\frac{a}{b} = \frac{z + x}{R_1}. \tag{13}$$

For brevity call the ratio $\frac{a}{b} = N$.

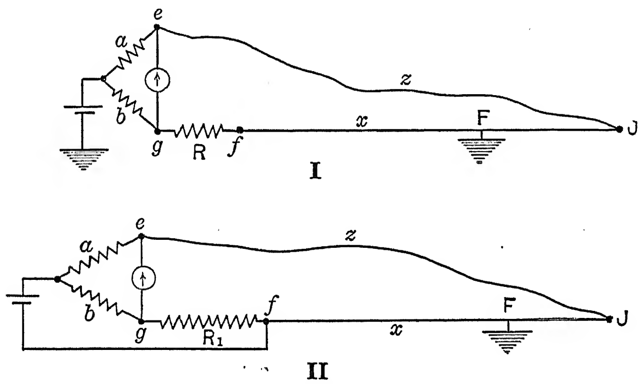

FIG. 1208c.

Then from Eqs. (12) and (13) we obtain, by eliminating z,

$$x = \frac{N(R_1 - R)}{N + 1}. \tag{14}$$

If the ratio $\frac{a}{b} = N$ is made unity, then

$$x = \frac{R_1 - R}{2}. \tag{15}$$

Eq. (14) or (15) gives the *resistance* from the testing end to the fault, when the resistance of the faulty and single good wire are both unknown. This method illustrates the very general applicability of the Varley-loop test when modified to meet particular conditions. Apparatus suitable for a Varley-loop test will serve for practically all cases of grounds and crosses on telephone or telegraph lines. By the use of a little ingenuity the standard

Varley-loop method may be modified to take care of such special conditions as are illustrated in the method above.

1209. One Good Wire of Unknown Length and Two Faulty Wires Equal in Length and Resistance.

(a) *Mr. Henry W. Fisher's Method.**

It sometimes happens that all of the wires in a cable become defective and it may be difficult to secure two good wires and apply the test given in par. 1207.

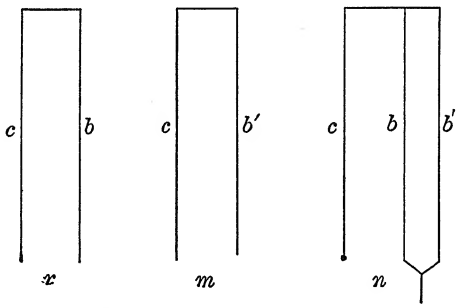

FIG. 1209.

If one good wire can be obtained for a loop test, the separate resistances of the good and bad wires can be determined by a method devised and used by Mr. H. W. Fisher, at a time when all the conductors in a cable were bad and there was only one aërial wire available for the test.

Let b and b' represent the bad wires and c the good wire.

Measure the combined resistance of c and b and call it r.

Measure the combined resistance of c and b' and call it m.

Measure the combined resistance of c in series with b and b' in multiple and call it n.

Fig. 1209 shows the connections and underneath each is the letter designating the resistance.

$$\text{The resistance of } b = r - n + \sqrt{n(n - r - m) + rm}. \quad (1)$$

$$\text{The resistance of } b' = m - n + \sqrt{n(n - r - m) + rm}. \quad (2)$$

Where the resistances r and m do not differ by more than 2 per

* The description of this method, as first used by Mr. H. W. Fisher, is taken from published literature of The Leeds and Northrup Company, with its permission.

cent or 3 per cent, the following approximate and much simpler equations may be employed:

$$b = \frac{3\,r + m}{2} - 2\,n, \tag{3}$$

$$b' = \frac{3\,m + r}{2} - 2\,n. \tag{4}$$

This method has given good results where used. It is only strictly applicable when the bad wires b and b' are faulty at the same point.

Leading wires can be used without in any way affecting the result, the total measured resistance of each loop being taken as represented by the letters r, m and n. The same leading wires, however, must be used throughout the tests.

Having thus determined the resistances of the faulty wires, a Loop Test can be applied and the fault located.

(b) *When there are one or more faulty wires of unknown length and resistance and only one good wire of unknown length and resistance.*

The author's methods (b) and (c), par. 1208, can be applied generally in the above cases.

Referring to I and II, Fig. 1208b, we may call the total resistance w of the faulty wire unknown. Eq. (9) or (10) of par. 1208 gives the *resistance* to the fault in terms of two scale readings and the auxiliary resistance P. The size and temperature of the faulty wire can be determined and then the distance to the fault can be calculated with the aid of a wire table. The wire, however, may twist in the cable and be longer in reality than the cable itself. The tendency, therefore, would be to place the fault at too great a distance from the testing end.

The unknown resistance w of the faulty wire may be determined if a duplicate test is made at the other end of the line. In this case the resistance x_1 from the other end of the line will be given in the same manner as the resistance x from the first end of the line. Then $x + x_1$ will be w, the total resistance of the line. If l is the length of the *cable*, $\frac{x}{x + x_1}\,l$ will be the distance measured along the cable to the fault from one end, and $\frac{x_1}{x + x_1}\,l$ will be the distance to the fault measured along the cable from the other end.

A test of this character would become useful when one aërial is

available and all the wires of unknown length in a cable of known length have become grounded at some *one* point.

1210. Methods of Applying Corrections in Loop Tests. —

(a) *When the good and faulty wires differ from each other in both size and length, but where the size and length of each is known.*

If the two wires of the loop differ in the above respects, the fault will be incorrectly located by a Murray or Varley-loop test unless a correction is applied.

To make this correction, multiply the length of the good wire by its rated resistance per unit of length and divide the product by the rated resistance per unit of length of the faulty wire, add to this result the length of the faulty wire and call the final result the total length of the loop.

In applying the Murray and Varley-loop tests in the ordinary way this equivalent length of loop should be used, when the wires differ, for L in the regular formulæ.

The above statement becomes when expressed in symbols,

$$l_b + l_g \frac{c_g}{c_b} = \frac{l_b c_b + l_g c_g}{c_b} = L_e, \tag{1}$$

where l_g and l_b are the lengths respectively of the good and faulty wires, and c_g and c_b are the resistances per unit length of the good and faulty wires. L_e is the equivalent loop resistance to use in place of L.

Example.

Refer to (b) par. 1205, Fig. 1205c.

Let the length $eJ = l_g$, and let $r_g = c_g l_g$ be the resistance of the good wire.

Let the length $fJ = l_b$, and let $r_b = c_b l_b$ be the resistance of the faulty wire.

Then $x = c_b d$ is the resistance to the fault when d is the distance to the fault.

We now have

$$\frac{b}{a} = \frac{r_g + r_b - x}{x} = \frac{c_g l_g + c_b l_b - c_b d}{c_b\, d}, \tag{2}$$

whence

$$d = \frac{a}{a+b} \frac{c_g l_g + c_b l_b}{c_b} = \frac{a}{a+b} L_e. \tag{3}$$

(b) *Faulty wire of two different sizes.*

The methods of par. 1207, Eq. (8), and of par. 1208, Eq. (6), will

give accurate results when the faulty wire is of two different sizes, provided the length and size of each section is known and a proper correction is applied. This correction is made as follows:

Let Fig. 1210a represent the faulty wire of two sizes,

l = the length of the entire wire,
d = the length of one section,
d_1 = the length of the other section,
s and s_1 = the resistance per unit length of the first and second sections respectively and
w = the total resistance of the faulty wire.

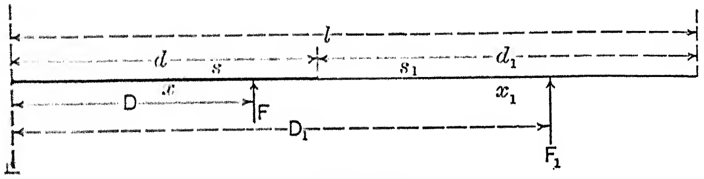

Fig. 1210a.

We have two cases to consider. First, where the fault F is in the first section and, second, where the fault F_1 is in the second section.

Let D = the distance to the fault F,
x = the resistance to the fault F,
D_1 = the distance to the fault F_1 and
x_1 = the resistance to the fault F_1.

Then in the first case, as $x = Bw$ (where $B = \dfrac{a}{a_1}$ or $\dfrac{a}{a + a_1}$ according as the method used is that of par. 1207 or par. 1208), we have

$$x = sD = B\,(sd + s_1 d_1),$$

or

$$D = B\frac{sd + s_1 d_1}{s}. \tag{4}$$

In the second case where F_1 is located beyond the first section of the wire, we have

$$x_1 = sd + (D_1 - d)\,s_1 = B\,(sd + s_1 d_1),$$

or

$$D_1 = B\frac{sd + s_1 d_1}{s_1} + \frac{s_1 - s}{s_1}d. \tag{5}$$

It will be noted that the values of D and D_1 are not altered when s and s_1 are multiplied by the same constant; hence we can call s and s_1 ohms per foot, ohms per meter, or ohms per 1000 feet.

In Eqs. (4) and (5) if $s = s_1$, that is, if the two sections are of the same size wire, we have

$$D = B\,(d + d_1) = Bl = \frac{a}{a_1}\,l, \quad \text{or} \quad \frac{a}{a + a_1}\,l.$$

If the test shows that the resistance to the fault is equal to or less than the resistance sd of the first section Eq. (4) should be used, but if it is greater than this then Eq. (5) should be used.

(c) *Where the loop consists of conductors of different sizes.**

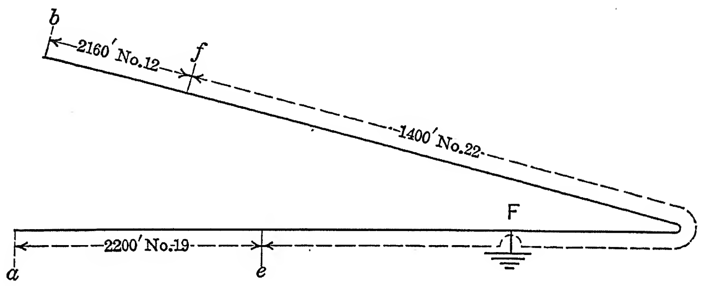

FIG. 1210b.

The loop may consist of several lengths of conductor of different sectional areas, as frequently occurs when cable circuits are joined to toll lines. The distance to the fault in this case may be calculated by expressing the lengths, having different cross-sections, in terms of what would be equivalent lengths of any one of the conductors in the loop. To do this the cross-sections and the lengths of each of the sections in the loop must be known. The procedure is to multiply the length of each conductor by its resistance per unit length and divide the product by the resistance per unit length of the conductor of the size to which the others are to be reduced. The following example will further explain the process:

In the loop shown in Fig. 1210b the conductor in the cable section a to e consists of 2200 feet of No. 19 B. & S. copper wire, the conductor in the section e to f consists of 1400 feet of No. 22 B. & S. wire and the section f to b, which may be considered part of a toll line, consists of 2160 feet of No. 12 B. & S. wire. We shall reduce

* The description of this correction is taken without material modification from the published literature of The Leeds and Northrup Company, with its permission.

the No. 19 and the No. 12 to equivalent lengths of the No. 22 wire. Using 1000 feet as the unit of length, we have

$$\frac{2200 \times 8.038}{16.12} = 1097 \text{ feet of No. 22 wire, which}$$

is equal in resistance to 2200 feet of No. 19 wire.

Also

$$\frac{2160 \times 1.586}{16.12} = 212.5 \text{ feet of No. 22 wire, which}$$

is equal in resistance to 2160 feet of No. 12 wire. This makes the total length of the loop equivalent to $1097 + 212.5 + 1400 = 2709.5$ feet of No. 22 wire. If the test shows the fault F to be 1346 equivalent feet from a, then 1097 feet is in the section a to e. Consequently the fault must be $1346 - 1097 = 249$ feet from e, or $2200 + 249 = 2449$ feet from a.

(d) *Lead Wires.*

The testing apparatus cannot always be brought close to the ends of the cable and it then becomes necessary to use lead wires of considerable length and resistance. In this case the simplest procedure is to use lead wires of equal length and of the same size as the wires in the conductor. The distance to the fault will then be given from where the instrument is located.

If lead wires cannot be obtained of the same size as the conductors in the cable, then multiply the length of each by its resistance per unit length and divide the product by the resistance per unit length of the wire in the cable. The values thus found represent the equivalent length of the wire in the cable which has the same resistance as each lead wire.

If occasion arises where the separate resistance of each lead wire is unknown, it can be determined by fastening the two lead wires together at their far ends and measuring the resistance of the loop. The point of junction is then put to earth and the separate resistances determined by a Murray-loop test.

1211. **Location of Grounds on High-tension Cables.** — High-tension power lines carried on poles may become grounded thru a high-resistance ground at some one point. This may result from the breaking of an insulator. It is often possible to locate a ground of this character by means of a Murray or Varley-loop test. The power must, of course, be taken off the conductors while the test is being made. Under these circumstances, however, there

is usually another line on the same poles which is kept alive while the test is being made on the faulty line. When this is the case the test is made difficult from two causes: Grounds which are serious for the high voltages employed in the transmission may, nevertheless, have a very high resistance, which makes it difficult to pass enough direct current into the line to give sufficient sensibility for the test. Then, further, the live wires which run on the same pole line with the line under test induce in the latter alternating currents of considerable magnitude. These induced currents may cause a serious disturbance of the galvanometer system, causing it to vibrate so violently that the scale cannot be read.

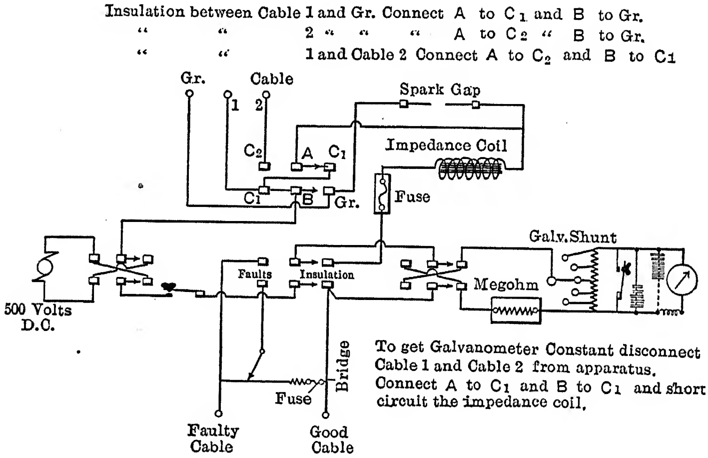

Fig. 1211.

These two difficulties, however, can generally be overcome. The testing galvanometer should be of the high sensibility type with a suspended coil system, and the testing current should be obtained from a small generator which can supply direct current at 500 volts or over. The second difficulty can be avoided as follows: There is joined in series with the high sensibility galvanometer a large ironless inductance (the larger the better, but there is no gain by attempting to increase the inductance by using iron). A cheap paper condenser, of 10 or more microfarads, is then connected as a shunt around the inductance and the galvanometer in series. This condenser absorbs much of the alternating current and effectually prevents any serious disturbance of the galvanometer system.

A complete cable-testing and fault-locating outfit for high-tension lines was designed in part by the author and supplied by The Leeds and Northrup Company to one of the power companies at Niagara Falls. The layout of circuits employed in this case with a little study is self explanatory. It is given in Fig. 1211.

1212. Location of Faults upon Low-tension Power Cables. — In locating grounds upon cables, as trolley line feeders, electric light and power cables, etc., satisfactory results cannot be obtained by the Murray or Varley-loop method unless special apparatus of heavy construction is used. The resistance of the contacts where the wires are joined to the bridge and the small current carrying capacity of a small diameter slide wire make accuracy and sensibility impossible with light apparatus of ordinary construction.

To overcome these difficulties a special bridge for locating faults in power circuits has been placed upon the market. In this

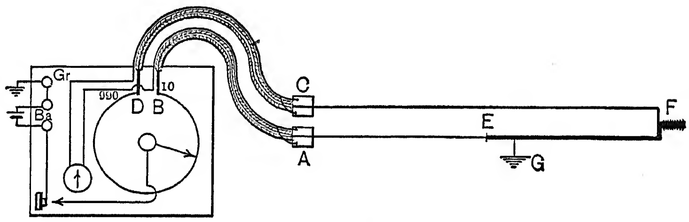

Fig. 1212.

bridge, which operates upon the Murray-loop principle, a heavy manganin slide wire is used, bent into circular form. It is placed underneath the top of the instrument and a circular scale and index are mounted upon the top. The bridge is provided with heavy flexible leads about 7 feet long. The terminals of these leads end in clamps of heavy construction into which the ends of the cable can be securely fastened. The contact resistance is thus made very low. Two other light flexible leads go from the pointer galvanometer, which is self-contained in the bridge case, to the clamps which terminate the two lead conductors. The scale, which otherwise is divided into 1000 divisions, has 10 divisions removed from each end. These correspond to a length of bridge wire which has the same resistance as the leads which clamp to the conductors. The normal current to use with this bridge is 5 amperes. In Fig. 1212 the bridge is shown joined

to a cable of two different sizes of wire. Locations can be made in this case as explained above. When the entire loop is a conductor of one size the location is calculated by the ordinary Murray-loop formula without modification.

1213. **Method of Locating Grounds upon Heavy, Short, Underground Cables.** — Sometimes very heavy feeder lines, laid in a trench, become grounded. In this case all loop methods will fail to locate the ground. Mr. Felix Wunsch has described * a method by which grounds, under the above circumstances, may be quite accurately located.

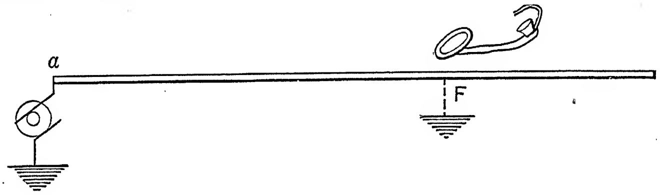

FIG. 1213.

The essential principle of the method is indicated in Fig. 1213. An alternating current is sent into the line at one end a and leaves the line at the fault F, returning to the generator thru the earth. An exploring coil, about two feet in diameter, which consists of about 500 turns of No. 18 wire, is used. The terminals of this are joined to a telephone with a head band. The coil is carried above ground along the path of the cable until a point is reached where the sound in the telephone, caused by the induction of the current in the cable upon the exploring coil, either ceases or becomes very greatly diminished. This method is reported by Mr. Wunsch to have given very good results. If the fault is of high resistance it can be broken down by applying a high voltage. The alternating current needed is not much above one half ampere, and the exploring coil can be from 10 to 15 feet above the faulty conductor.

A similar method is used by the telephone companies in locating faults before cutting open the sheath of the cable. The source of current is a small induction coil and cell of battery. The method is extremely accurate and is readily applied.

1214. **The Location of Opens.** — An aërial wire, or a conductor in a cable, may be severed at some point so the circuit is

* *Electrical World*, February, 1909, vol. xxi, page 118.

completely interrupted. This is called an open and it is important to determine, from either end of the line, the distance to the break; or to locate the "open." The possibility of doing this depends upon the fact that every linear conductor has an electrostatic capacity. It is a condenser. The conductor itself is one "coating" or plate of the condenser; the dielectric is the insulation surrounding the condenser, be this air or insulating covering upon the wire, or both; the other coating or plate of the condenser is any conductor which may lie parallel with the other thruout its length. In the case of an aërial this would be the earth (capacity to ground) or another conductor on the same poles (capacity to a conductor). In the case of telephone or telegraph wires in a cable with a lead sheath, the other plate of the condenser would be the lead sheath of the cable (capacity to ground) or any one of the other wires in the cable (capacity to a conductor).

A telephone wire in a cable is usually twisted with a return wire which constitutes its "mate." If both wires of a pair are open at both ends, then the two constitute a condenser; the two conductors, in this case, being the condenser plates, and the double thickness of insulation which separates the conductors being the dielectric.

Now, the capacity of a combination of this kind is quite approximately proportional to the length of the conductor and, therefore, if the capacity of a conductor (which is the broken conductor) of unknown length is compared with the capacity of a conductor of known length, then the distance to the open point is determined.

The comparison of two capacities may be made very simply with circuits arranged in the manner of a Wheatstone bridge.

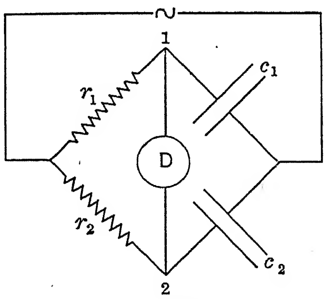

FIG. 1214a.

In Fig. 1214a, r_1 and r_2 are two ohmic resistances. These should be as free as possible from electrostatic capacity or self-induction and preferably should be fairly high resistances, of the order of 1000 ohms. c_1 and c_2 are the two capacities to be compared. D is some form of detector, usually a telephone, which is responsive to an alternating, interrupted, or rapidly varying current of any kind. This bridge arrangement is supplied with a

current of this character. By varying the ratio of the two resistances r_1 and r_2 a value of the ratio may be found such that at all times the potential at point 1 is the same as the potential at point 2. This equality of potentials will be indicated by the detector D; in the case of a telephone, by silence in the telephone.

The condition for a balance is

$$\frac{r_1}{r_2} = \frac{c_2}{c_1}. \tag{1}$$

Note that the capacities are in reciprocal relation to the resistances.

In the application of this principle to fault location a telephone is invariably used as the detector. The source of variable current is a battery and a buzzer, or a battery and a small hand commutator for quickly reversing the current. In an emergency the current may be rapidly interrupted by drawing a metal piece over the surface of a coarse file.

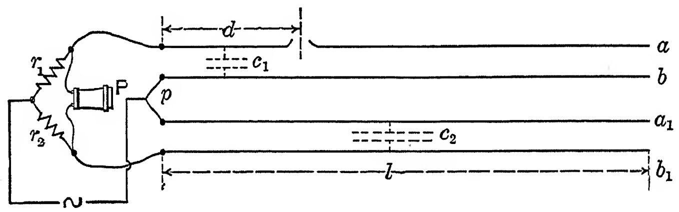

Fig. 1214b.

(a) *Good wires available.*

To locate an open, say, in a conductor in a telephone cable which contains the mate of the broken wire and another good pair, the connections are made as in Fig. 1214b.

Here the capacities of the pair a, b (wire a broken) and a_1, b_1 are indicated by the hypothetical condensers drawn in dotted line. r_1 and r_2 are varied together, or either of them alone, until the telephone P is silent, or nearly so.

Then,

$$\frac{r_1}{r_2} = \frac{c_2}{c_1}.$$

As d, the distance to the fault, is proportional to c_1, and as l, the length of the good pair, is proportional to c_2,

$$d = \frac{r_2}{r_1} l. \tag{2}$$

If the wires a and b_1 are aërials, as a telegraph or electric light wire, then p, the point of attachment to b and a_1, would be joined to the earth or to another good wire on the same poles, which runs the full length of, and is separated the same distance from, both the good and the faulty conductor.

For this location to be successful the conductors must not be grounded or crossed and their far ends must be completely open.

(b) *When no good wire is available.*

In this case it is necessary to make a test, first at one end of the line and then at the other end. Also an auxiliary condenser must be used. The capacity of this need not be known and its value may be chosen between wide limits, but the same condenser must be used thruout the test. A suitable value would be one half microfarad.

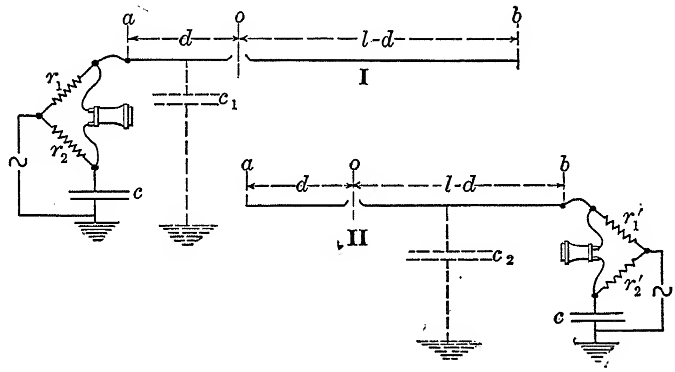

FIG. 1214c.

Referring to Fig. 1214c the connections are made first as in I. Here c_1, shown in dotted line, represents the capacity to ground of the open section a to o of the conductor ab. c is the capacity of the auxiliary condenser. For a balance,

$$\frac{r_1}{r_2} = \frac{c}{c_1} = \frac{c}{k\,d},$$

or

$$c = k\,d\,\frac{r_1}{r_2}, \tag{3}$$

where d is the distance from the end a to the open, and k is a constant of proportionality. The connections are made next at the other end of the line as in II. Here c_2, shown in dotted lines, represents the capacity to ground of the open section b to o.

For a balance, in this case,

$$\frac{r_1'}{r_2'} = \frac{c}{c_2} = \frac{c}{k(l-d)},$$

or

$$c = k(l-d)\frac{r_1'}{r_2'}, \tag{4}$$

where l is the length of the open wire ab.

From Eqs. (3) and (4),

$$d = \frac{r_2 r_1'}{r_1 r_2' + r_2 r_1'} l. \tag{5}$$

(c) Another test, used by the Bell Telephone Company, which is said to be extremely useful and exceedingly simple to apply, is the following: A telephone cable which is carried into a building and is *not* covered with a lead sheath may have a break in a wire underneath the insulation. The exact position of this break, within an inch or two, is located by the use of a telephone, a buzzer, and a battery. The buzzer has one terminal put to earth and the other to the wires in the cable at a free end. The tester attaches one terminal of a telephone to the earth and the other terminal to his body. He then places his hand upon the cable containing the broken wire. If he is on the side of the break to which the buzzer is attached he will hear a sound in the telephone. He moves his hand along the cable and when he has passed the break the sound ceases. In this way the position of the break is narrowed down and finally located within an inch or two of its exact position. The cause of the sound in the telephone is the condenser current which flows thru the telephone. The wire of the cable forms one plate of the condenser and the tester's hand, which grasps the insulated conductor, forms the other plate of the condenser. This test is very much used.

1215. Location of Inductive Crosses. — An inductive cross (defined in par. 1201) may be located by a procedure similar to that employed in locating an open. It is necessary in applying the test to have in the same cable sheath a pair of good conductors. The method requires a comparison of capacities, when the conductors used for the test are connected, first in one manner and then in another. A setting of the ratio arms to give a balance is made for each connection, and from these two settings the necessary data are obtained for calculating the distance to the fault or inductive cross. The method is carried out as follows:

Connections are made first as in I, Fig. 1215. The hypothetical condensers, represented in dotted line, are drawn to indicate the

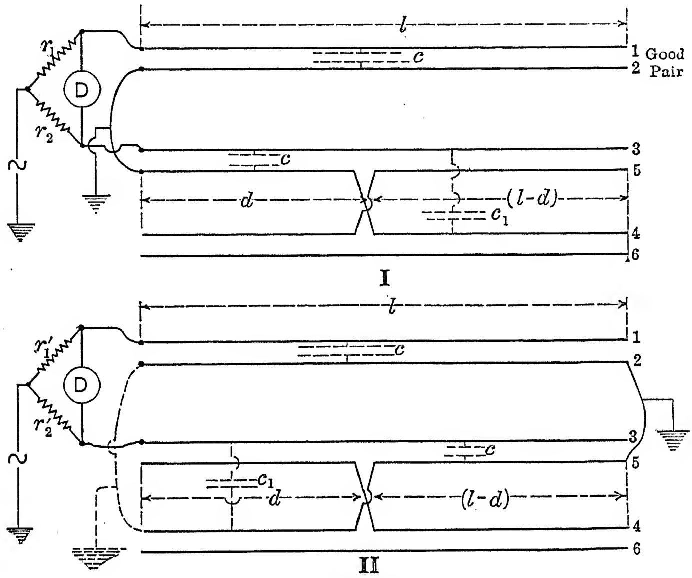

FIG. 1215.

capacity per unit length of conductor between the pair of good wires 1, 2; a good wire and its mate 3, 4; and this same good wire and the wire 4, mated beyond the fault with wire 6 of another pair.

A balance is obtained with these connections, by varying the resistances r_1 and r_2. Connections are then made as in II, Fig. 1215, the dotted lines indicating an equally good alternative arrangement. The hypothetical condensers drawn in dotted line show the capacities per unit length of conductor as they would become with these connections. A balance is again obtained, the ratio arms taking the values r_1' and r_2'. The distance to the fault, by an approximate formula, is now calculated as follows:

Let l = the length of the cable,
d = the distance to the fault,
c = the capacity per unit length of a conductor and its mate and
c_1 = the capacity per unit length of a conductor and a conductor of another pair.

In case I,

$$\frac{r_2}{r_1} = \frac{cl}{cd + c_1(l-d)}. \tag{1}$$

In case II,

$$\frac{r_2'}{r_1'} = \frac{cl}{c_1 d + c(l-d)}. \tag{2}$$

For brevity let

$$\frac{r_2}{r_1} = a, \quad \text{and} \quad \frac{r_2'}{r_1'} = b.$$

Then, from Eq. (1),

$$c_1 = \frac{c(l-ad)}{a(l-d)}, \tag{3}$$

and from Eq. (2)

$$c_1 = \frac{c[l - b(l-d)]}{bd}. \tag{4}$$

Hence,

$$\frac{l - ad}{a(l-d)} = \frac{l - b(l-d)}{bd}. \tag{5}$$

From Eq. (5) we find

$$d = \frac{a(b-1)}{b(2a-1) - a} l. \tag{6}$$

If in Eq (6) we replace a by its value $\frac{r_2}{r_1}$ and b by its value $\frac{r_2'}{r_1'}$, we obtain for the distance to the fault,

$$d = \frac{r_2(r_2' - r_1')}{r_2'(2r_2 - r_1) - r_2 r_1'} l. \tag{7}$$

Eq. (7) is not rigidly true, because all of the capacity relations between different conductors were not taken into account. It is, however, sufficiently exact for practical purposes.

1216. Comments on Practice and Accuracy in Fault Location. — Tho the principles of fault location and the formulæ used are relatively quite simple, difficulties are apt to arise in their application in the field. The chief cause of these difficulties is that conditions, which are assumed to be constant in deducing the formulæ, prove variable in practice. Thus in settled districts no two widely separated points upon the surface of the earth are at exactly the same potential. For this reason, if a conductor makes contact with the earth at two points, stray currents will flow in the line. Then also the proximity of other lines carrying currents which alternate or large direct currents which vary will often induce stray currents in the testing circuit. These may cause

erratic movements of the galvanometer which seriously interfere with the measurement.

The resistance of a fault, especially a ground, may vary greatly while the test is in progress. A ground may be due to a moist condition of the insulation which the testing current dries out, and the ground will disappear while the test is in progress. This is known as a disappearing ground.

Then another serious cause of trouble, which may become very puzzling and exasperating, is a bad contact resistance at some unknown point in the loop circuit. If this contact resistance is constant false results will be obtained, but if, as often happens, the bad contact is variable, it becomes impossible to obtain a balance while the cause of the difficulty is misjudged.

Again two faults may be present on a conductor. The location then will only give some intermediate point between the faults. If the resistance of one of the two faults is steady while that of the other varies, the point of balance on the bridge will shift in a puzzling way. The cause of this would be difficult to distinguish from the effects of a poor contact. The best procedure, when there are two faults, is to cut the wire between the faults and locate each one separately. The existence of two faults may be disproved by testing from each end of the line. If both locations place the fault at the same point there is only one.

Any method which gives only the resistance to the fault is inferior to one which gives the distance to the fault as a fraction of the total length of the cable. In calculating distances from resistances it should be remembered that copper wire varies in resistance about 0.4 of 1 per cent per degree C., and the temperature of a long conductor may vary considerably from one point to another. Then also a small variation from standard gauge in the conductor may mislead one in calculating the distance from the resistance.

Experience shows that copper wire in telephone cables laid under ground will run about 10 per cent higher in resistance in summer than in winter in the State of Pennsylvania.

The most important error, however, is likely to arise from the fact that the conductors are usually longer than the cable sheath, since pairs of conductors are twisted together in telephone cables. Even aërials will be longer, due to the sag of the wire, than the distance measured along the pole line.

It will be noted that in the methods which have been given, the

first two excepted, the resistance of a fault, a cross or a ground, does not enter into any of the measurements. Also that the galvanometer is so placed that neither the potential differences existing in the earth, nor any electromotive force at the fault itself, can send a current thru the galvanometer. Many methods which might be given for locating crosses or grounds have not been mentioned because they involve measuring the resistance of the fault itself or expose the galvanometer to possible earth currents or electromotive forces. Such methods, some of which are well known in connection with fault locations upon marine cables, work well with artificial lines in the laboratory, but they give uncertain and unsatisfactory results when used upon land lines in the field. For this reason we have omitted giving them, but the interested student will find the standard methods of this character fully explained in Kempe's "Hand Book of Electrical Testing," and in other works upon marine cable testing.

If a helper has been instructed to make a connection at the far end, it is possible for the tester to prove or disprove that he has done so. One method of ascertaining whether a connection has been made at the far end, to a wire which can be connected to the testing set, is carried out as follows: Prepare for a test of the electrostatic capacity of the conductor in question by the deflection method. Take a deflection before and after the supposed connection has been made. If upon closing the circuit, the latter deflection is the larger the connection has been made.

Another and preferable method is to join the two wires, which are to be connected by the helper at the far end, to the X posts of the testing set. The switches of the set are arranged for making a loop resistance measurement. A resistance is unplugged in the rheostat which is greater than the resistance of the loop can possibly be. Before the two wires are joined at the far end the pointer of the galvanometer will deflect to one end of the scale corresponding to infinite resistance for X. As soon as the helper makes the connection the pointer will deflect to the opposite end of the scale corresponding to a resistance less than that unplugged in the rheostat.

In giving the precision with which a fault is located it is customary to give not the relative, or per cent, value but the absolute precision expressed in feet or meters. In measurements of this character the important matter is the actual distance in feet or

meters that the location is out, regardless of the length of line. It is well, however, to state this latter as giving additional information regarding the circumstances under which the location was made.

1217. A Word on Fault-locating Apparatus. — Fault locations upon land lines can be made, if necessary, with comparatively

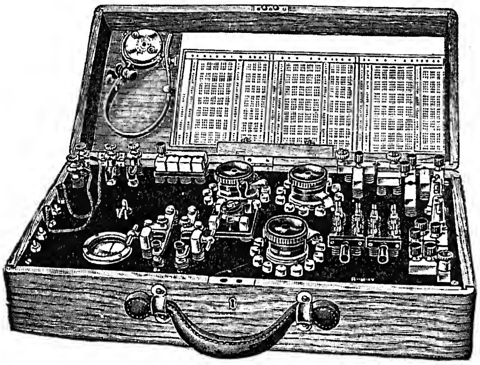

FIG. 1217a.

simple apparatus which a skillful tester can devise and assemble from material usually to be found about an electric station. However, in connection with the work of a large telephone equipment, the locating of faults is an important and frequent operation. It is economy, therefore, to use fault-locating apparatus devised especially for portability, speed and precision.

It is impossible to give the space here required to describe, even in outline, the many different forms of fault-locating and cable-testing apparatus which instrument makers, here and abroad, have placed upon the market. We shall merely mention two which have had an extensive use in this country. One is the well-known portable cable-testing set designed by Henry W. Fisher. With this set, which is robust and complete in every respect, the following tests are readily effected:

Location of crosses and grounds.
Location of breaks or opens in cables.
Conductor resistance measurements.
Liquid resistance measurements.
Insulation resistance measurements.
Capacity measurements.

The outside appearance of this set is shown in Fig. 1217a. The other apparatus referred to, with the design of which the author was largely connected, is the "Lineman's Fault Finder." This is shown in Fig. 1217b.

Fig. 1217b.

The essential feature of the apparatus is a uniform resistance, which lies in a circle and is about 100 ohms. By a special construction, it is arranged so that contact can be made at any point along it, and it is therefore equivalent to a very high resistance slide wire. It has a moving contact and a uniform scale of 1000 divisions. In series with this, there are two resistances, which may be short-circuited by switches. One has exactly the same resistance as the wire. There is a resistance of 100 ohms, and it is the fixed resistance of the bridge arrangement for resistance measurements. Resistances of 1000 ohms and 9000 ohms are connected to the battery post to protect the battery and the apparatus from excessive current. The 9000 ohms may be short-circuited by a switch. Other features are a self-contained battery

and a galvanometer of the type described in par. 1501, and three switches which permit the connections to be quickly and unmistakably made for the following uses:

Measurement of conductor resistances.

Murray and Varley-loop tests, and, when a telephone and buzzer are used as accessories, the location of opens.

Both of the above sets are manufactured by The Leeds and Northrup Company, of Philadelphia, Pa.

CHAPTER XIII.

MEASUREMENT OF TEMPERATURE BY THE MEASUREMENT OF RESISTANCE.*

1300. Remarks on Temperature and Thermometry. — The measurement of a physical quantity implies, generally, the numerical comparison of the quantity with a certain selected quantity of the same kind taken as a unit. Temperature, however, cannot be treated as a quantity in the same sense. It is rather to be considered as a state in which matter is found, and all temperature measurements are made by comparing the changes in some form of matter produced by heat. As shown by Lord Kelvin as early as 1848, temperature may be expressed on a scale which is independent of any particular form of matter, but this thermodynamic scale cannot be used in actual temperature measurements, which, in practice, consist in comparing the change in some particular form of matter produced by changes in temperature.

Certain gases change in volume under constant pressure or change in pressure under constant volume in a nearly regular manner with equal increments of temperature, as estimated on the thermodynamic scale. Gas thermometers have, therefore, naturally been chosen as standards with which to compare the changes in various forms of matter, which changes may then serve as a convenient means of temperature measurement. The present upward range of the gas thermometer scale is 1550° C., with a probable error of plus or minus 2° C.,† and the melting-point of pure platinum is known within plus or minus 5° C. and is assigned the value 1755° C. The melting-points of many other metals are known with varying degrees of accuracy,† and these melting-points of the metals constitute fixed temperatures which may be used for the calibration of various temperature-measuring devices.

The science of thermometry, especially its extension into high-temperature pyrometry, is far too extensive to be even touched

* Portions of this chapter are taken from an article by the author in the *Proc. of the A. I. E. E.*, 1906.

† Dr. A. L. Day, *Trans. of the Faraday Soc.*, Nov., 1911, pages 142 and 144.

upon here, and its consideration does not belong to a work of this kind, but the resistance thermometer, which is one of the best devices for the measurement of temperature, may with propriety be briefly described as well as the methods employed for determining temperature by its use.

1301. Electrical-resistance Thermometry.* — Electrical resistance thermometry is possible because very many electrical conductors change in resistance with change of temperature in a perfectly definite manner.

The percentage change in resistance of the pure metals with temperature is larger than that in the volume of gases, and over twenty times as great as the volume change in mercury. Thus, the coefficient of expansion of nitrogen gas is 0.00367+, and of mercury 0.00018+, while the coefficient of increase of resistance of pure nickel is about 0.0041 per degree C. between 0° and 100° C.

A change in electrical resistance can be measured with greater ease and far greater precision than a change in volume of a liquid or a gas. A change, in either a high or a low electrical resistance, can be measured when it is one part in a hundred thousand. Thus, the sensitiveness of the electrical-resistance method of measuring temperature is very great. In the use of the bolometer, where the electrical-resistance method of measuring temperature is carried to its greatest sensitiveness, temperature changes as small as one ten-millionth of a degree C. are said to be detectable.

For the electrical-resistance method of measuring temperature to be of utility the resistance which is measured must always return to the same value when brought back to the same temperature. Fortunately, experience has shown that when the proper resistance materials are chosen, and due precautions in their treatment have been used, the reliability of the method in this respect is very satisfactory. A properly constructed resistance thermometer, if not exposed to too high a temperature, will maintain its calibration better and longer than the best mercury thermometer, which is usually subject to small alterations and irregularities due to elastic after-effects in the glass.

* A valuable treatment of this subject may be found in the *Bulletin of the Bureau of Standards*, Vol. 6, Nov., 1909, page 149. Article by C. W. Waidner and G. K. Burgess. A bibliography of the subject is given there, pages 223–230. See also, "Measurement of High Temperatures," Burgess and Le Chatelier, 1912 edition.

As the pure metals are greatly elevated in temperature, the rate of increase in resistance with temperature generally changes. Thus, over extensive temperature ranges there are no metals of which the resistance is even approximately a linear function of temperature. Small impurities in the pure metals affect also the amount as well as the law of their change.

These facts make it unlikely that an electrical-resistance temperature scale will be found bearing such definite relations to the absolute-temperature scale that it will serve conveniently for a standard scale of reference in the same manner as does the scale of the gas thermometer. When, however, the means are available, it is relatively easy to determine experimentally the relation between the electrical resistance of any particular specimen of wire and the temperature for a working range of the gas thermometer of 900° or 1000° C. An electrical-resistance thermometer can then be made of this specimen of wire, and it will serve as a standard with which other resistance thermometers may be very simply and easily compared.

The law of variation of electrical resistance with temperature in the case of platinum has been investigated by Callendar and Griffiths, and several others. It has been shown that in the case of platinum the following relation exists between the temperature t, as measured on the air thermometer, and the resistance of platinum:

Let p_t be a so-called "platinum temperature" as defined by the relation

$$p_t = \frac{R_t - R_0}{R_{100} - R_0} 100, \tag{1}$$

where R_0 is the resistance of a given specimen of platinum at 0°, R_{100} at 100°, and R_t at t°, all measured on the centigrade scale. It has been shown that placing

$$t - p_t = \delta\left[-\frac{t}{100} + \left(\frac{t}{100}\right)^2\right], \tag{2}$$

expresses the difference between the "platinum temperature" and the temperature as measured on the air thermometer. This "difference formula," as it is called, holds to within 0.1° C. up to 500° C. and within 0.5° C. up to 1000° C. In this formula δ is a coefficient which varies with the particular specimen of platinum used. For very pure platinum it is about 1.5, and larger for impure specimens. To determine δ the resistance of the thermometer

is measured at the three known temperatures, 0° C., 100° C., and 444.6° C., the boiling-point of sulphur. The authors referred to give convenient methods of using the difference formula to convert the temperatures as given by the platinum-resistance temperature scale to degrees centigrade as given on the scale of the air thermometer.

In the relation (1) above the quantity $R_{100} - R_0$ is called F_i, *the fundamental interval*. It is a constant quantity for any particular thermometer.

The quantity $C = \dfrac{F_i}{100\,R_0}$ is called *the fundamental coefficient*.

As
$$p_t = 100\,\frac{R_t - R_0}{F_i},$$
we have
$$Cp_t = \frac{R_t - R_0}{R_0}. \tag{3}$$

Thus, the purer the platinum the greater will be the coefficient C. As examples of the values of the above constants we give the following data taken from tests* made by the National Bureau of Standards upon two platinum-resistance thermometers, called A

Thermometer A	Thermometer B
R_0=21.3476 F_i= 4.4067 C= 0.00206426 δ= 1.571 Diameter of wire=0.01 cm.	R_0=3.48779 F_i=1.34298 C=0.00385052 δ=1.504 Diameter of wire=0.015 cm.

and B. The current thru the thermometers in the above test was 0.004 ampere and 0.010 ampere respectively.

If we plot resistance as ordinates and gas thermometer degrees as abscissæ, the curve obtained for platinum is always slightly concave toward the axis of X and is parabolic in form.†

When pure nickel is used for resistance thermometers, the resistance variation obeys another law. Prof. C. F. Marvin has

* Bulletin of the Bureau of Standards, Vol. 6, page 156, 1909.

† For a more extended discussion of formula (2) and for a description of methods for reducing platinum temperatures to the gas scale, consult "Measurement of High Temperatures," Burgess and Le Chatelier, 1912 edition, Chapter V.

shown * that the nickel-resistance curve is very closely represented by the equation

$$\text{Log}_e R = a + mt, \tag{4}$$

where R is resistance of thermometer in ohms, t the temperature in degrees C., and m and a are constants. In some particular cases this equation became

$$\text{Log}_e R = 1.0854 + 0.001699\, t,$$

and again,

$$\text{Log}_e R = 1.9004 + 0.001818\, t,$$

and again,

$$\text{Log}_e R = 0.9614 + 0.001450\, t.$$

These equations were tested in the range $-25°$ C. to $75°$ C. with an error never greater than $0.1°$ C. and again in the range $0°$ C. to $375°$ C. with an error not exceeding $0.9°$ C.

The simple meaning of the above relation [Eq. (4)] is that pure nickel wire increases in resistance by *the same per cent* of its resistance at the beginning of an increment of temperature for every *equal increment* in temperature anywhere in the range $-25°$ C. to $350°$ C.

The law is sufficiently accurate to be relied upon for work not requiring a precision greater than $1°$ C. over the range mentioned above, and for short ranges of $50°$ C., or less, reliance may be placed upon the law to $0.1°$ C., or better.

As far as known, no other metal obeys the law of nickel. Because of this law for nickel the temperature-resistance curve may be located by observing the resistance at only *two* temperatures, say $0°$ C. and $100°$ C. When resistance is plotted as ordinates and temperature as abscissæ the curve will always be convex toward the axis of X. It follows that, if a certain length of nickel wire is joined in series with a certain length of platinum wire, a combination resistance thermometer may be made which, over short ranges, will have a change with temperature which is practically linear.

To engineers and those who make industrial uses of resistance thermometers the theoretical side of the subject is of minor interest. There is a practical procedure which may be adopted that makes it unnecessary for manufacturers or users to give consideration to these methods of standardization of resistance

* *Physical Review*, April, 1910, pages 522–528.

thermometers. The instrument maker may carefully construct a resistance thermometer to serve as a standard and send this from time to time to the National Bureau of Standards at Washington. The Bureau will measure the resistance of this thermometer over a wide range, at several known temperatures given by their standard resistance thermometers, and furnish a certificate giving the relations which are found between temperature and resistance of the thermometer submitted for calibration. The instrument maker may then use this thermometer as a standard with which other thermometers are easily calibrated. This is done by direct comparison in an oil bath for medium temperatures, and in a specially constructed electric furnace for high temperatures. Cold brine, or liquid air, or other means may be used for making the comparison at low temperatures.

The feature of paramount importance in the use of electrical resistance thermometers is the constancy with which they maintain their calibration. This subject has received considerable attention, especially in the case of thermometers made of platinum wire, and the results observed have proved the entire reliability of this material for temperatures not exceeding 1000° C. It is highly probable that other materials will behave in an entirely regular manner if not subjected to too high temperatures.

Careful investigations of the constancy of other materials than platinum that are suitable for resistance thermometers are needed. But the investigations so far made show that where permanent alterations in resistance occur these may usually be traced to causes which proper precautions may avoid. Thus, the material selected for the thermometer may be by nature of an unstable character. Iron, for example, is an unsuitable metal to use. The material may contain impurities which by vaporization, crystallization, or otherwise, cause the resistance to alter gradually. The wire of which the thermometer is made may have been subjected to mechanical strains which gradually work out with repeated heatings, thus altering the resistance. If the material is one which does not oxidize, it may still be greatly affected at high temperatures by absorbing gaseous impurities. Thus, a nickel-wire or a platinum-wire thermometer heated to 400° C. in a brass tube is ruined by absorbing the metallic vapors given off. For the same reason all metal solderings near the resistance wire are liable at high temperatures to give off vapors which affect the permanent

resistance, besides making liable the formation of local resi at the joints.

Proper construction and choice of materials can remove : above causes of permanent alterations. It may be that case of platinum, to some extent at least, and more so in materials, slow permanent alterations in resistance occu cause of which is not known. Only extended investigatio give the limits of these possible alterations. Enough wo been done, however, to show that for even very refined wo reliability of platinum and some other materials is suffic temperatures too high are not exceeded.

In resistance thermometry practical details of constructi all important. The chief of these will now be considered.

1302. **Construction of Resistance Thermometers.** — Th material of which to construct a resistance thermometer de upon the temperature range to be measured, as well as up physical qualities of the available materials.

Constancy of composition and other practical consider seem to limit the choice to a few of the pure metals, usually form of wire. The metal which has received the most st platinum. It can be used over a very wide temperature and can be obtained under the name of Heræus platinum in a of great purity. This material answers every requirement sistance thermometry, except that it is very costly. A subs for platinum should, therefore, be sought and used where will serve as well. This substitute should be inexpensiv obtainable in a pure state. It is desirable that it should l high specific resistance, combined with a large temperature cient. It should be unoxidizable under usable conditions withstand a high temperature without deterioration or perm alteration in resistance.

An examination of the pure metals shows that these conc are best met by nickel. The author has had many thermo constructed of this wire for temperatures ranging from — to 300° C., and has found it reliable in this range. It has a coefficient than the purest platinum, that of nickel being 0.0041 per degree between 0° C. and 100° C., pure platinum 0.0039, and commercial platinum but about 0.002. The s resistance of pure nickel and pure platinum is in the ratio of 933 to 1000.

It may here be remarked that a determination of the temperature coefficient of the metallic elements offers usually a very delicate test of their purity, and specimens of nickel and platinum which show a low temperature coefficient can positively be considered as impure and inferior for use in resistance thermometers.

Another test of interest, especially on wires intended for use in thermocouples, is to attach the two ends of a short length to the terminals of a very sensitive galvanometer, and to pass a flame along the wire. If the galvanometer gives positive and negative deflections of considerable magnitude, the wire may be known to be unhomogeneous, and liable to have parasitic currents set up in it when exposed to high temperatures. A pure nickel and a pure platinum wire should show little of this effect.

The particular purpose for which a resistance thermometer is to be used largely determines its special features of construction. Broadly classified, resistance thermometers are particularly useful in the following cases:

1. Measurement of all temperatures below − 40° C., the freezing point of mercury.
2. Measurement of all temperatures up to 1000° C., when the temperature is to be taken at a place where it cannot be directly observed.
3. Measurement of temperatures below 1000° C., and above the range of the mercury thermometer.
4. Measurement of all temperatures below 1000° C., which must be photographically or otherwise recorded.
5. Determinations of small temperature differences or variations for which the mercury thermometer is not sufficiently sensitive.

It is evident from the above classification that there can be no general form or type of construction of a resistance thermometer. Each special requirement must be met by the instrument maker, who should be guided in his designs by experience and a study of the conditions. The form of thermometer having been chosen, the particular method of reading the resistance variations and of expressing them in degrees should have particular care, for in nearly every case which arises different requirements must be met.

Resistance thermometers for use below 140° C. are of relatively simple construction, for in this case silk-insulated nickel wire may be used. Certain precautions, nevertheless, need attention. The

mass of the wire used and that of the body on which it is wound should be small, or the temperature of the resistance wire will lag behind any changing temperatures which are being measured, and lead to erroneous indications. The wire must be so chosen in respect to size and resistance that the heating of the wire by the measuring current shall be negligible.

The constancy of any wound resistance depends largely upon the treatment to which it is subjected after being wound. The winding of the wire introduces strains, which gradually work out, causing variations in the permanent resistance. This certain result is avoided by an artificial "aging," which consists in maintaining the wire for several hours or days, before the thermometer is calibrated, at a temperature higher than that at which it will be used.

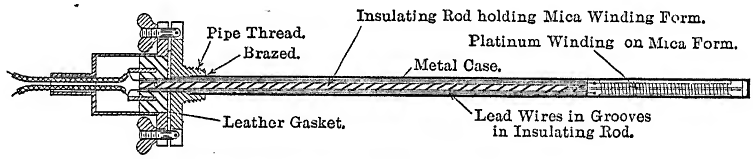

FIG. 1302a.

It is needful to finish the terminals of the wire, especially if short, in such a manner that no local variations in resistance can occur at the joints. As a rule the terminals should be hard silver soldered for low-temperature thermometers, and for high-temperature thermometers all joints exposed to the high temperature must be welded joints.

Generally, the resistance wire should be protected by a casing. When, however, as in the measurement of moderate temperatures of gases or insulating fluids, the wire can be directly in contact with whatever is to have its temperature determined, the resistance thermometer assumes the surrounding temperature very quickly, far surpassing the mercury thermometer in this respect. If a casing must be used, it should be so shaped that the ratio of its surface to its volume is large, and the construction should aim to reduce to a minimum the heat which is conducted along the case or which is distributed by air convection within it. Reproductions are here given of two types of resistance thermometers designed for the measurement of low or moderate temperatures.

The thermometer shown in Fig. 1302a was constructed for use

in measuring and recording with great precision the temperature differences between two brine mains. The average temperature of the brine was about $-37°$ C. and the average difference of temperature between the two mains was about 1.5° C. The allowable error was 0.01° C., and hence great care in the construction of the thermometers, as well as in the rest of the apparatus, was required. This thermometer was wound with No. 35 platinum wire, of great purity. Its resistance at room temperature was about 80 ohms. It is probable that nickel wire would have served as well, but because of the better known properties of platinum and the importance of the experiment platinum was selected.

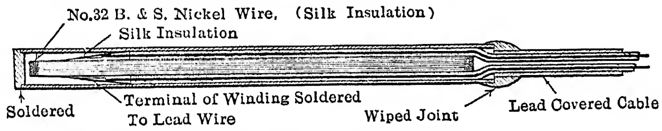

FIG. 1302b.

It should be noted that the steel case is long and small in diameter, that the winding ends well below the nut which screws into the brine main, and that the wire is wound on a light frame of mica, having a minimum of mass. A small sudden change in the temperature of the brine was followed by the thermometer to within about 0.005° C. within two minutes.

Fig. 1302b is a sectional view of a form of resistance thermometer made for the purpose of measuring the temperature of the soil at different depths where the thermometers are permanently buried. The winding is in the form of a skein, and No. 32 silk-insulated nickel wire is used. To insure permanency the wire should be kept immersed, after winding, in hot paraffin for three or four days. The changes in the temperature of the soil are very slow, and hence there is no need to provide against a temperature lag of the thermometer winding.

The resistance is made large, about 100 ohms at 20° C., and the winding is encased in a brass tube filled with paraffin. The lead-covered leads are soldered with a wiped joint to the brass tube, thus preventing the entrance of moisture, which has to be carefully avoided. The resistance of the thermometer being high, the change in the resistance of the leads is entirely negligible.

A somewhat similar construction would be suitable for measur-

ing the temperature of the interior of stored material, such as grain, tobacco, hay, wheat, etc., also for measuring the temperature of cold-storage rooms. Any number of such thermometers can be located at different places and be connected by a switch, one at a time, to a single reading device which reads directly in degrees Fahrenheit or Centigrade. The methods of reading these and other resistance thermometers will be presently described.

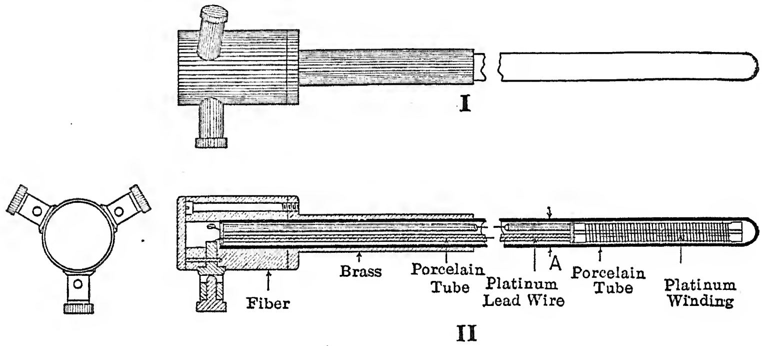

FIG. 1302c.

Resistance thermometers give often the most accurate and convenient means of measuring high temperatures up to 1000° C. or possibly more. It is stated by Le Chatelier* that experiments carried out at the National Physical Laboratory, England, showed that throughout the temperature range of 1000° C. the agreement between the scales of the platinum-resistance and the thermoelectric pyrometers tested was within 0.5° C.

Such statements as the above, however, are true only when the resistance thermometers have been constructed in a particular manner to avoid alterations and deteriorations in the wire that are sure to result at high temperatures with improper construction. Platinum heated red hot and exposed to certain gases, as hydrogen or metallic fumes, absorbs impurities which permanently alter its resistance and often render it extremely brittle.

Accumulated experience has shown that for temperatures above a red heat (525° C. to 600° C. for all materials) the design of the thermometer should embody the general features shown in the illustration, Fig. 1302c, I and II.

In the thermometer here illustrated, the winding is a pure

* "High Temperature Measurements," page 105, 1904 edition.

Haræus wire, its purity being shown by its temperature coefficient, which is about 0.0039° at 100° C. This wire, No. 35 B. & S., is wound bare, on a frame of thin mica, in such a manner as to touch only the edges of the mica. The winding is 36 turns to the inch. The mica frame is made by matching together at right angles two pieces of mica sheet, of the shape shown in Fig. 1302d.

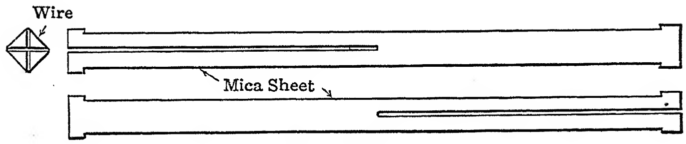

Fig. 1302d.

As the winding touches only at the edges of the mica, only a small percentage of its length can become contaminated by any possible action of a solid material. The lead wires, by a method of compensation to be later described, do not enter into the resistance which is measured, and may be of a less pure platinum than the resistance winding. These lead wires are either three or four in number, according to the method of compensation adopted. They are insulated from each other by being passed through tubes of porcelain.

For temperatures above the fusion point of hard glass, porcelain tubes especially constructed for this work by the Royal Berlin Porcelain Works are the most satisfactory material for a casing.

The interior parts of the thermometer shown in Fig. 1302c, II, are designed to be easily withdrawn from the tube for examination, and again replaced. In the particular case shown, the winding was made 13 cms long, to give the thermometer a high resistance. This is generally an advantage, where the conditions permit, as the contact resistances in the measuring device are then small in comparison, and greater sensibility is more easily obtained.

It was legitimate to make the winding long for the case shown, as this thermometer was designed to measure the temperature of hot gases which would surround the porcelain tube more than half way to its head. If, however, the temperature of the place to be measured is uniform over a small space only, then the winding should be as short and as much concentrated at the end of the tube as possible, and so permit of placing the entire winding in the hot place, the temperature of which is to be measured.

Thermocouples have in this respect an advantage over a resistance thermometer as above designed, for the end of the thermocouple is a very small body, that may be closely located at the place where the temperature is to be observed. This consideration led the author to design another form of resistance thermometer which will be shown to combine the advantages of both. A description of this is best given, however, under methods of reading resistance thermometers, which we shall now consider.

1303. **Methods of Reading Resistance Thermometers.** — As previously stated, the National Bureau of Standards at Washington will furnish the instrument maker with a certificate giving the connection between the electrical resistance and the temperature of a selected standard resistance thermometer, and the calibration of other thermometers is reduced to comparing their resistances with that of the standard when all are brought to equal temperatures. In the case of high temperatures, a specially constructed electric furnace is used for the purpose. The problem, then, of reading temperatures with thermometers thus calibrated resolves itself into measuring their resistance in a simple manner when subjected to different temperatures.

The resistance being known, the temperature may be taken from a previously plotted curve, or the resistance-measuring device may be constructed to read directly in degrees Centigrade or Fahrenheit. The convenience, simplicity, precision, and reliability with which these measurements can be made largely determine the practical and commercial usefulness of resistance thermometers. The continuous recording of temperatures given by resistance thermometers is another, but closely related, problem, but one which cannot here receive our attention.

The available and useful methods of determining resistances for measuring temperatures may be classified as follows:

Slide-wire bridge method.

Differential galvanometer method.

By resistance-thermometer bridge with two traveling contacts.

By use of dial bridges.

Kelvin-double-bridge method of reading temperatures.

Direct-deflection method of reading temperatures.

1304. **Slide-wire Bridge Method.** — This is a very convenient zero method to employ, especially when the reading instrument has a scale calibrated to read directly in degrees. The slide-wire

bridge may have its connections arranged in either of two useful ways. The first is less precise, but more convenient. The connections are given diagrammatically in Fig. 1304a.

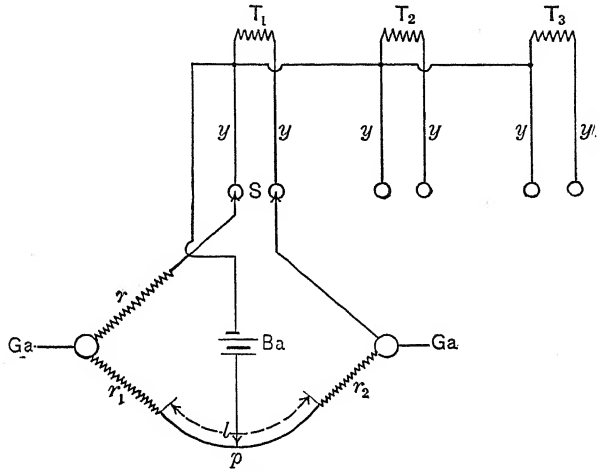

Fig. 1304a.

T_1, T_2, T_3, etc., represent any number of resistance thermometers; y, y are the thermometer leads, which should be alike but may be of any length. Contact can be made with any thermometer by means of a simple sliding switch S. The resistances r, r_1, r_2, should be about equal to each other and to the resistance of the thermometer when at a mean temperature. The resistance of the slide wire l should be such as will take care of only the *variation* in resistance of the thermometers.

In an actual construction, the contact p would move over a circularly disposed wire and scale. This scale may be divided into arbitrary divisions, and reference be made to a curve, to obtain the temperature of any thermometer corresponding to a given setting for a balance. In this case, the different thermometers need to be made of only approximately the same resistance. The scale may, however, without great difficulty, be graduated to read directly in degrees when used with a thermometer of a particular resistance and temperature coefficient.

If, however, many thermometers are to be read on the same scale, they must be adjusted to exact equality both in respect to resistance and temperature coefficient. This last adjustment can

be made by using a certain resistance of manganin in series with those thermometers which have too high a coefficient.

The arrangement of connections shown does not entirely compensate for changes in the resistance of the leads. The error, however, would not exceed from this cause 0.1° C. in an ordinary case. The obvious advantage of making the connections in this way is that while nearly complete compensation is obtained, each thermometer has only two lead wires and one common terminal connecting all the thermometers to the galvanometer. The manner of making the bridge connections according to the second arrangement is shown in Fig. 1304b.

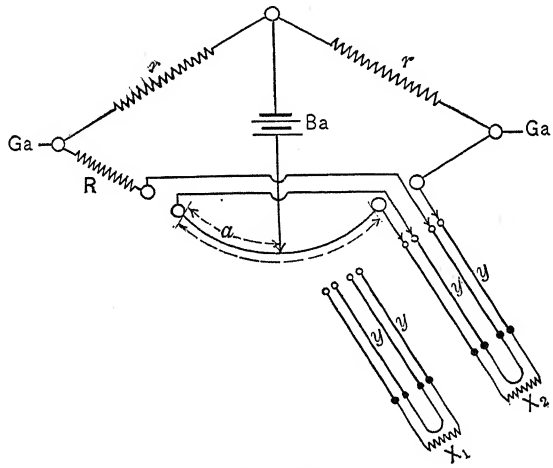

FIG. 1304b.

By connecting the bridge in this manner and choosing the ratio arms equal, the resistance of the leads y, y, entirely eliminate. Thus, the value of any resistance X_1, X_2, etc., is

$$X = R - (l - 2a) = \text{a constant} + 2a.$$

This method, while perfectly compensating, requires that two pairs of leads shall be carried to each thermometer. This is a decided disadvantage where many thermometers are to be read at a distance on one bridge. The method recommends itself when the highest possible precision is required. In this method also the scale may be calibrated in degrees, if desired.

The balance point on the wire in either of the above methods may be found with a telephone, but preferably with a galvanometer.

A pointer galvanometer of portable type, such as that described in par. 1501, is amply sensitive for the purpose.

The illustration, Fig. 1304c, shows a completed instrument, designed for portability.

Fig. 1304c.

A temperature measurement is made by slightly depressing the button, which closes the battery circuit, and then rotating the pointer until the galvanometer shows a balance. The position of the pointer then gives on the scale beneath it the temperature in degrees Fahrenheit or Centigrade. The same kind of instrument is equally well adapted to reading low or high temperatures.

One mechanical feature of this instrument deserves mention as being of extreme usefulness. This is the construction of the slide wire for use with high-resistance thermometers. If made in the customary way, which is to use a single fine wire, it would have to be very small in diameter and delicate in order to secure the necessary resistance in the length that can be used. Thus, a 100-ohm nickel thermometer would change about 40 ohms with

100° C. This objection is entirely overcome by winding in a spiral an insulated wire for the slide wire, as described in par. 401. By the use of such a spiral, some 30 times the resistance of a single wire may be obtained. It gives extremely fine variations in resistance, as the slider moves over it, and makes a contrivance that is mechanically substantial and not liable to wear or become loose.

1305. **Differential-galvanometer Method.** — Another arrangement for reading the resistance of resistance thermometers is that in which a differential galvanometer is used in the manner already very fully described in pars. 301 and 302. The author is inclined to believe that, all things considered, this differential galvanometer arrangement offers more advantages than any other. The scale and mechanical parts would not differ essentially from those shown in Fig. 1304c.

1306. **Resistance Thermometer Bridge with Two Traveling Contacts.** — It is possible, however, to gain the advantage of

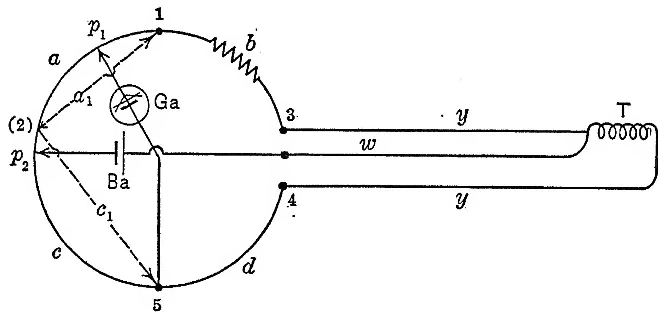

Fig. 1306.

using only three wires leading to the thermometer and have the lead wires entirely compensate and yet use a Wheatstone-bridge arrangement with a galvanometer of ordinary type which is not differential. The way in which this can be done without introducing contact resistances in the measuring circuit was first proposed by Mr. Morris E. Leeds. It is as follows:

Represent a Wheatstone-bridge network as in Fig. 1306. Here T represents the resistance of a resistance thermometer and y, y represent the equal resistances of two lead wires to the thermometer. *Ga* and *Ba* are respectively a galvanometer and a battery. b, a, c, and d are resistances. Let p_1 and p_2 represent two contact

points which can be moved to different points upon the resistances included between points 1 and 2 and between points 2 and 5. Let b always be the resistance between point 3 and p_1, a the resistance between p_1 and p_2, and c the resistance between p_2 and point 5.

Then, for any positions of p_1 and p_2, we have, by the ordinary law of the Wheatstone bridge,

$$\frac{a}{b+y} = \frac{c}{d+y+T}, \tag{1}$$

from which we derive

$$T = \frac{cb - ad + y\,(c-a)}{a}. \tag{2}$$

From Eq. (2) it appears that, if matters are so arranged that the resistance a is always maintained equal to the resistance c, the term $y\,(c - a)$ will disappear, and we shall have

$$T = b - d, \tag{3}$$

where d is constant and may be made zero.

This result may be accomplished by a mechanical arrangement which will cause the contact points p_1 and p_2 to always move together. Assume that the bridge is balanced when p_1 is at point 1 and p_2 is at point 2 and that in this case the resistance between p_1 and p_2 is a_1 and equal to the resistance c_1 between p_2 and point 5. Now let T increase so that in order to maintain a balance of the bridge b must be increased or p_1 must be moved towards the point 2 by a distance δa_1 and at the same time let the contact p_2 move toward the point 5 by a distance δc_1. Then the resistance included between p_1 and p_2 is now $a_1 - \delta a_1 + \delta c_1$ and the resistance included between p_2 and point 5 is $c_1 - \delta c_1$. But to make the term $y\,(c - a)$ disappear we must so move p_2 that

$$c_1 - \delta c_1 = a_1 - \delta a_1 + \delta c_1. \tag{4}$$

If, originally, we have made $c_1 = a_1$, we obtain

$$\delta a_1 = 2\,\delta c_1. \tag{5}$$

Eq. (5) shows that if the resistance of the slide wire 1, 2, 5 is uniform thruout then p_1 must be moved twice as far as p_2. In practice the resistance of the slide wire from point 1 to point 2

would be given twice the resistance per unit length of the slide wire from point 2 to point 5, and thus the actual distances thru which p_1 and p_2 would have to move would be the same. With this arranged the arms which carry the contacts p_1 and p_2 may be rigidly fastened together, and the value of T will always be given by the difference of the resistances b and d regardless of the value of the lead resistances y, y, provided these are alike.

1307. Use of Dial Bridges for Temperature Measurements. — The slide-wire bridge directly calibrated in degrees is a very useful and rapid reading device, but for precision work, combined with robustness of construction, some easily read form of dial or plug Wheatstone bridge may be more useful.

When the connections are made as shown in Fig. 1307a, and the resistances r_1 and r_2 are equal, the resistances of the leads y, y eliminate. This requires that the total resistance in the rheostat shall equal the resistance of the thermometer, which for this reason, as well as for sensibility, etc., should be high, and that the brush or plug contacts used should be well made and of negligible resistance. Since no resistance varies with temperature in a strictly linear way, a dial or plug bridge cannot be calibrated to read directly in degrees and also accurately. The studs or plug holes should be numbered decimally, and, from the setting obtained for a balance, the temperature is gotten by referring to a curve. Thus, each thermometer in an installation has its own curve, and new thermometers may be added without reference to the old. This method is very convenient for an installation of a large number of thermometers, because of the small number of wires that have to be installed.

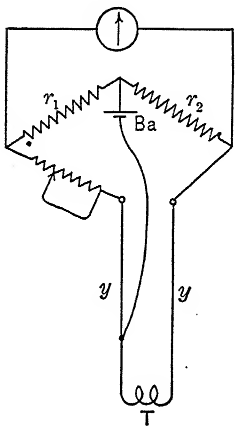

FIG. 1307a.

Fig. 1307b shows four thermometers of a large installation used for measuring the temperature of gases up to 650° C. in the plant of a large chemical company manufacturing sulphuric acid. Here it was desired that ignorant workmen should take the temperatures without gaining information as to what they were. The dial settings for obtaining a balance were reported to the office, where a clerk looked up the corresponding temperatures on the curves.

The thermometers used in these installations were of the form illustrated in Fig. 1302c.

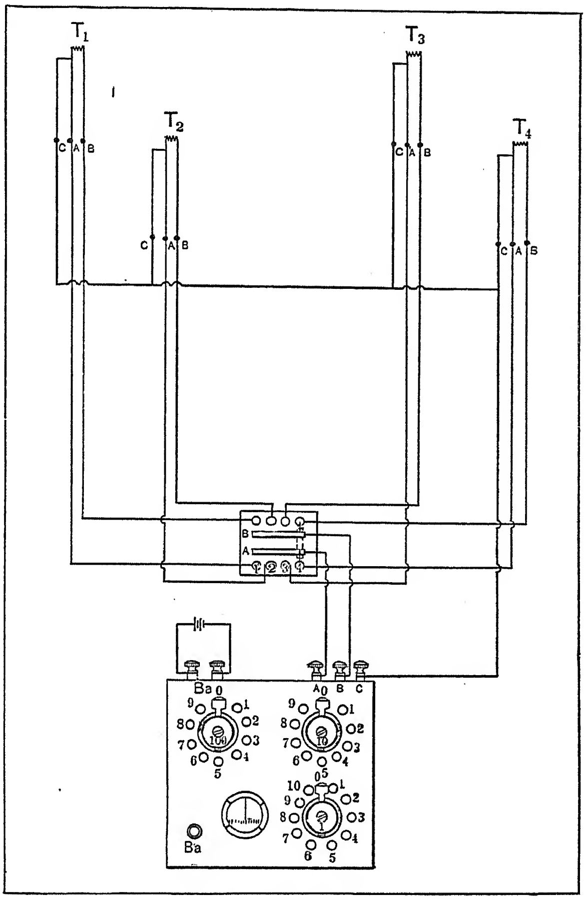

FIG. 1307b.

1308. Kelvin Double-bridge Method of Reading Temperature. — The resistance thermometer as designed for high-temperature work, if wound to a suitable resistance, is necessarily of considerable size. This unfits it, as compared with thermocouples, for taking the temperatures of small places or at points. Moreover, the thermometers, besides requiring considerable skill to construct, are costly and more or less fragile. These disad-

vantages are largely overcome by a method developed by the author which will now be described.

The principle made use of is to measure the changing resistance of low-resistance platinum thermometers by means of the Kelvin double bridge already fully discussed in par. 609.

With these bridge connections a resistance thermometer of 0.01 ohm can be measured with precision. By taking advantage of this bridge as a reading device, a high-temperature thermometer can be constructed as shown in Fig. 1308a.

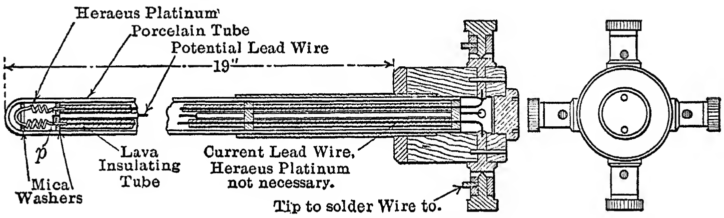

Fig. 1308a.

The small spiral of resistance wire to be measured shown at the end of the porcelain tube is of No. 20 Heræus platinum. The current and potential leads are of a cheaper grade of platinum. In fact, it is a positive advantage to have the potential leads of an impure platinum, because of its low temperature coefficient, which may be about 0.6 that of pure platinum. The connections as arranged for measuring a number of thermometers would be as shown in Fig. 1308b.

To measure a temperature with this arrangement, the terminals p, p' are moved by a switch to the potential terminals of the thermometers to be measured, while the thermometers to the right of the one being measured are cut out of circuit by y, which keeps the resistance of the "yoke" low as required by theory. A balance on the galvanometer is obtained by moving the plug N and the slider S. The slide wire on which S moves would consist of a substantial manganin wire lying over a scale, marked off in degrees Centigrade, if it is desired to make the bridge direct-reading. The only uncertain element in the method is the possibility of the ratio $\frac{a}{b}$ and $\frac{a'}{b'}$ becoming variable in an unknown way through a change in the resistance of that portion of the potential

leads which lie in the thermometer tube. This uncertainty, however, is practically avoided if the resistance a is made sufficiently high. Calculation shows that if a is chosen as high as 250 ohms, the maximum error from this cause, with a thermometer constructed like that shown in Fig. 1308a, will not exceed 0.1° C. The resistance a may, however, be as high as 1000 or even 5000 ohms, thus practically reducing the error to zero.

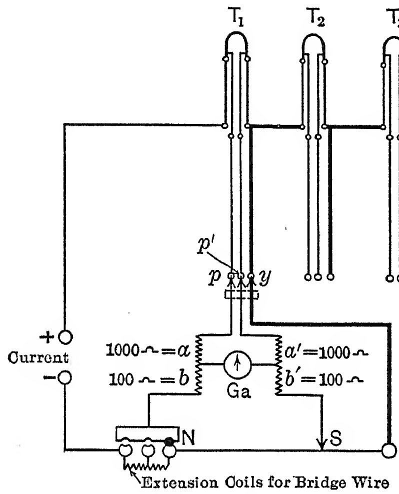

FIG. 1308b.

The necessity of having high resistance in the ratio coils requires that the galvanometer used shall have a greater sensibility than can easily be gotten in a portable pointer instrument. There are, however, several very convenient forms of semi-portable suspended-coil types of galvanometers, having an attached telescope and scale which are amply sensitive for the purpose.

1309. **Direct-deflection Method of Reading Temperatures.**—Direct-deflection methods of measuring physical quantities, as well as temperatures, depending as they do upon the calibration of a scale, seldom have the precision of zero methods. They possess, however, the advantage of showing more clearly the variations as they occur in changing quantities, while no manipulative action is required on the part of the observer. For these reasons, largely, all commercial electrical-measuring instruments

are of the deflection type, though inferior in precision to the null methods used for calibration and other purposes in the laboratory.

A principle may be employed, however, by which the convenience and rapidity of the deflection method is in part combined with the precision of the null method. This principle consists in setting with dials or plugs so a balance with the quantity being measured is nearly obtained. The deflection of a calibrated deflection instrument then gives the additional small quantity which must be subtracted or added, to obtain the exact value of the quantity measured, in this case a temperature.

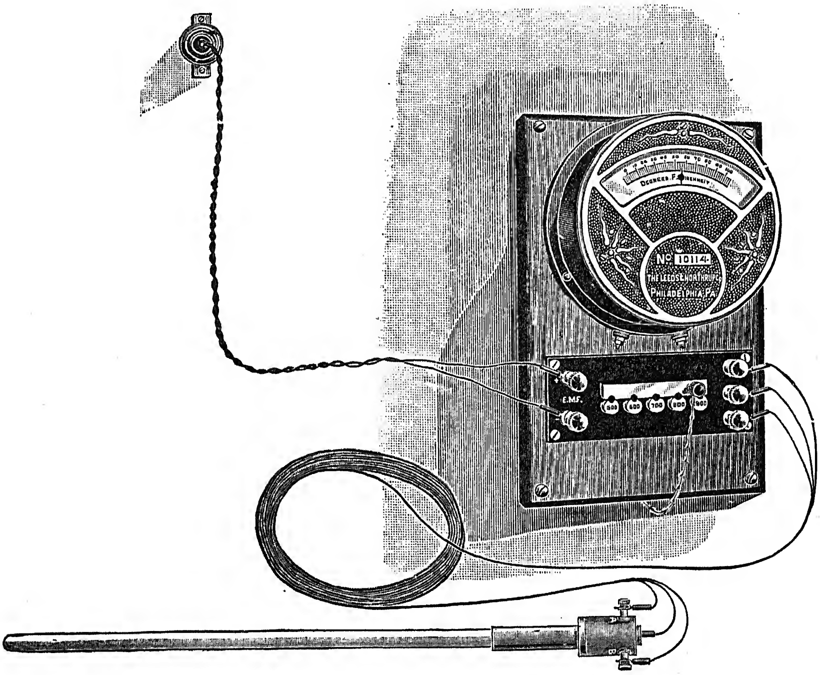

Fig. 1309.

The realization of this principle is embodied in various commercial instruments, one of which is illustrated in Fig. 1309.

The instrument operates upon the principle shown by the connections given in Fig. 1307a. The deflection indicator is of the same construction as a switchboard voltmeter. When the plug is in any one block of the row of blocks the temperature indicated is that marked upon the block plus the reading of the deflection

instrument. The range of the scale is 100° C., so if the plug were in the block stamped 500 and the pointer stood at 57 the temperature would be 557° C., or, if the plug were in the next block to the right, the reading would be 600° plus the scale reading, and so on. Instruments of this kind can be given various ranges up to 1000° C. and are very convenient for the use of workmen in manufacturing establishments. They should be operated upon a fairly constant voltage.

1310. **Deflection Methods; Using Constant Currents.** — It is often required to read and to record temperature differences, possibly very small differences, which must be determined with high accuracy. A thermo-couple is customarily employed for this purpose, placing one junction in the place of higher and one in the place of lower temperature. Resistance thermometers may, however, be employed to advantage, especially if the temperature difference is small and great precision is required.

The author has been connected with the design, calibration and use of a temperature-difference recording apparatus which embodied the highest refinements in this kind of measurement, commercially applied. A brief description of this apparatus will best explain the methods to employ, the precautions that should be used to obtain precision, and the results which may be obtained in this class of work.

The requirement was to obtain a continuous photographic record of the temperature difference between two brine mains carrying brine for refrigeration purposes. The temperature of the brine in one main was about −38° C. and in the other about −36.5° C. It was sought to have the error at all times not greater than 0.01° C. By taking a photographic trace of the temperature difference at each instant, and by obtaining with a planimeter the average height of the ordinates expressing temperature differences, the average temperature difference for any period of time could be found. This result was secured.

An ordinate 500 mm. high corresponded to 5° C. The deflection instrument used was a D'Arsonval galvanometer of the mirror type, of special construction. The record was traced on photographic paper, known to the trade as "rotograph" paper. This paper was wound on a brass cylinder about 55 cms. long and 12.5 cms. in diameter. By means of a clock movement the cylinder made one revolution in 12 hours. Two cylinders were

provided, so that an exposed one could be immediately replaced by an unexposed cylinder.

The galvanometer was placed in one end of a box about 1.2 m long. By suitable optical devices, the spot of light, about 1 mm in diameter, was reflected from the galvanometer mirror upon the slowly rotating cylinder covered with the sensitive paper. The movements of the spot of light were parallel to the axis of the cylinder and proportional to the temperature difference. The source of light was an incandescent lamp. Another optical device cast another spot of light upon a translucent scale, where the deflections could at any time be observed.

Two platinum-resistance thermometers, exactly alike, were used, one being placed in each brine main at a distance of several hundred feet from the recorder, being connected with it by lead-covered compensated leads. One of these thermometers is described in connection with Fig. 1302a. Each thermometer with its leads formed an arm of a Wheatstone bridge. The two other arms were made of equal manganin resistances, each 200 ohms. When the thermometers were at the same temperature, the bridge was balanced, and the galvanometer deflection read zero; that is, the spot of light fell on the scale at the same point as it would with the circuit open. A fixed mirror reflected a spot of light which made a trace near the center of the paper, which served as a reference line from which the extent of the deflections could be measured. When the two thermometers were at different temperatures, the deflections were very nearly proportional to the difference in temperature between them, whatever might be the mean value of the two temperatures, but depended upon the current entering the bridge. Hence, to maintain the empirical calibration of the scale, it was necessary to provide that the current thru the bridge should remain constant to within the percentage precision at which the apparatus was designed to operate. The manner of doing this, as well as the general plan of the method, is best explained by referring to Fig. 1310a.

The source of current was a battery of eight storage cells. This current could be held constant within a fraction of one per cent by occasionally varying the rheostat resistance in the battery circuit. The current was known to have the standard value when the galvanometer G in the standard-cell circuit gave no deflection. It was found in practice that the rheostat resistance had to be

changed only a few times in a day, and then only by small amounts. Since the scale was calibrated so that 1 mm. was equal to 0.01° C., the galvanometer had to be fairly sensitive. Only 90 ohms, R in the diagram, could be used in its circuit. The galvanometer coil had 333 ohms of copper-wire winding, and since copper changes about 0.4 of 1 per cent in resistance per degree C., it was neces-

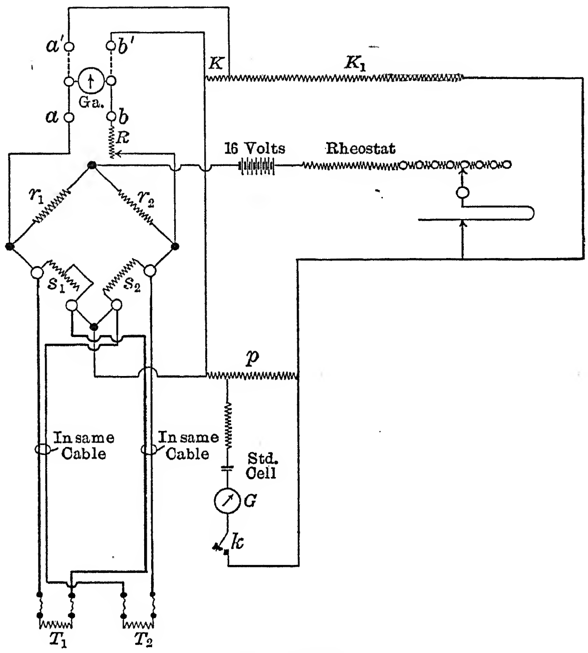

FIG. 1310a.

sary to avoid changes in the galvanometer sensibility due to room temperature changes. The temperature of the box enclosing the galvanometer was, therefore, held constant within about 0.5 of 1 degree by means of an electric thermostat.

Since it is impossible to adjust two resistance thermometers to exact equality, when at low temperatures, the difficulty was avoided by shunting each thermometer, one with 10,000 ohms, and the other with a resistance near 10,000 ohms, which thus made both thermometers act in balancing the bridge as if they were exactly equal when at the same temperature.

In calibrating the apparatus, a necessary adjustment was made

by placing both thermometers in a tank containing well-stirred brine at about $-35°$ C., and then varying one of the shunts S_1 and S_2, until the galvanometer showed no deflection. Another adjustment was made by placing one thermometer in one brine tank and the other in another brine tank. The temperature difference between these brine tanks could be controlled, and this difference was accurately measured by taking a great many readings with specially constructed mercury thermometers with Reichenstalt certificates. The corresponding galvanometer deflections being noted, the scale became calibrated. By adjusting the resistance R in the galvanometer circuit, the value of the scale could be varied as desired.

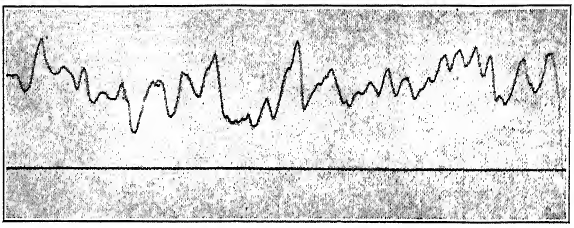

FIG. 1310b.

The calibration thus briefly outlined was worked on for about three weeks, many refinements not mentioned here were used, hundreds of readings were recorded, and many checks made upon the observations taken. It was demonstrated, as a net result, that this apparatus and method gave continuous temperature difference records that were not in error over 0.01° C., the mean temperature measured being about $-37°$ C., and the average difference about 1.5° C. In Fig. 1310b is given on a much reduced scale one of the record curves obtained in a run of 12 hours.

1311. The Measurement of Extremely High Temperatures. — Many scientific investigations and industrial operations now require that temperatures shall be measured at which all materials deteriorate or become fused or altered. The electrical methods directly applied must fail to be of service here, and one must resort to radiation pyrometry. The various methods proposed for measuring high temperatures by means of the radiation given off from a hot body have recently received much study, and very

successful developments have followed along this line. The subject, however, as well as that of thermoelectric pyrometry, does not fall within the scope of this work, and reference must be made to the above-mentioned treatise of Burgess and Le Chatelier, and to an excellent summary of this and other subjects relating to high-temperature measurements by Dr. C. W. Waidner.*

* "Methods of Pyrometry." Printed in the *Proceedings of the Engineers' Society of Western Pennsylvania*, September, 1904.

CHAPTER XIV.

INSTRUMENTS USED FOR MEASURING RESISTANCE. SOME GENERAL PRINCIPLES CONSIDERED.

1400. Proposed Treatment of Subject. — It would be beyond the scope of this work to give a detailed consideration of the requirements, the types, the design and the construction of the instruments used in the measurement of resistance. It will be profitable, however, to consider broadly some general principles which pertain to this class of apparatus.

The apparatus which is involved chiefly in the measurement of resistance consists of:

Resistance standards (medium, low, and high), resistance boxes and Wheatstone bridges, and deflection instruments of the mirror and pointer types, used both as null and deflection indicators. In addition to the more essential instruments there are generally required rheostats, switches, keys, batteries, etc., but these need no further mention here.

1401. Conformity in the Parts of an Outfit. — In selecting a resistance-measuring outfit, when there is latitude of choice, the apparatus should be chosen so its various parts are in conformity. Thus, if resistance boxes and the samples to be measured have a large watt-dissipating capacity, then a galvanometer of moderate sensibility, quick in action and robust in construction, will conform better to the rest of the equipment than a delicate, highly sensitive instrument which deflects slowly and requires, perhaps, repeated adjusting. In the above case, if the magnitude of the measuring current is properly chosen, a relatively insensitive galvanometer of the pointer type will enable the samples to be measured with all the precision which may be obtained from the rest of the apparatus, while the speed and convenience will be greater than if a highly sensitive moving magnet galvanometer with lamp and scale is selected. On the other hand, if the samples to be measured, as, for example, tungsten lamp-filaments, are small, have a high temperature coefficient, and are incapable of dissipating

much energy without heating, then the galvanometer must be highly sensitive. But if the galvanometer is sensitive and the samples will carry little current, it is a waste of room and expense to use a Wheatstone bridge of large size with massive brass blocks and with resistance coils of large heat-dissipating capacity.

An exercise of judgment is very desirable, in regard to this matter of securing conformity in the various parts of an outfit.

1402. **Sensibility and Accuracy.** — The sensibility of a resistance-measuring outfit is determined by two factors, the constant of the detector or deflection instrument and the magnitude of the measuring current. Both of these are limited, but the limitation of the first, when there is latitude of choice, is rarely reached. The limitation put upon the second may result from the limitations in the watt-dissipating capacity of the resistance spools in the bridge (generally to be taken as 0.5 watt per spool), or from the watt-dissipating capacity, without appreciable heating, of the samples which are measured. No approximation to a rule can be given for this last. If the samples have a negligible temperature coefficient, they can dissipate much more heat without changing in resistance than they can, other things equal, if they have a high temperature coefficient. For this reason much greater sensibility can generally be obtained in measuring the resistance of the alloys than in measuring the resistance of the pure metals. It is a mistake, however, to suppose that the possible sensibility is much less with a low resistance of a given length of sample than with a high resistance of the same length. It is easily seen that, if the sample is in the form of a ribbon of a given length, doubling the width halves the resistance, but the heat-dissipating capacity is also doubled and hence the current may be doubled. The result is that the fall of potential between two points on the ribbon can be kept constant whatever its width. Hence, the sensibility with which the narrow, high resistance, and the wide, low resistance, ribbon may be measured remains the same. In the case of round wire, as used in a platinum-resistance thermometer for example, the advantage is slightly in favor of the fine wire, as the cross-section increases with increase in diameter more rapidly than the surface from which the heat must be dissipated.

In general, bridges which are to be used with samples, such as resistance thermometers and lamp filaments, may have very small resistance spools, while bridges for general work and where alloys

are to be measured should have larger spools. In this latter case a pointer galvanometer of 0.5 megohm sensibility will serve for most requirements.

The absolute precision of a resistance measurement must primarily depend upon the accuracy of the standards. The spools in a Wheatstone bridge become for the time being the standards employed. It is always possible, if one has one standard resistance, the precision of which is known, to check up or calibrate a Wheatstone bridge so that the true resistance of all its coils becomes known. The systematic procedure for doing this will generally be given to customers by the makers of the bridges — or for a relatively small fee a bridge will be tested and certified to by the National Bureau of Standards.

No resistance measurement, however, can be considered as very precise unless careful attention is paid to the magnitude of the measuring current and the temperature of the surroundings in which the sample is placed at the time it is measured. For much work the watt capacity of the bridge spools will exceed that of the samples and the magnitude of the current must be adjusted to the latter.

In general, it is considered very good work if a Wheatstone bridge will measure resistances from 10 to 10,000 ohms with a precision of 0.02 of 1 per cent. Most Wheatstone-bridge work will range about 0.05 of 1 per cent.

1403. **Resistance Standards.** — In considering resistance units attention should be drawn to the distinction between resistance units used for standards of resistance and resistance units which are intended to carry considerable current when used with potentiometers and like instruments. These latter should be spoken of as current-resistance standards. Resistance standards proper do not need to have much watt-dissipating capacity. Their requirements are unchangeableness with time, low temperature coefficient, and small thermo-electromotive force against copper. They should be susceptible also of immersion in kerosene oil, so their temperature may be accurately ascertained. The development of resistance standards has passed thru quite an evolution, and the latest type as designed by Dr. E. B. Rosa and endorsed by the National Bureau of Standards, will now be briefly described.

A paper entitled "The Variation of Resistance with Atmospheric

Humidity" was read before the American Physical Society at Washington, April 21, 1907, and later published in the Bulletin of the Bureau of Standards, Vol. 4, 1907–8, page 121. In this paper by E. B. Rosa and H. D. Babcock, it was demonstrated "that the shellac" (covering the wire of wire-wound resistance spools) "absorbs moisture from the surrounding atmosphere and expands, stretching the manganin wire and thereby increasing the resistance." This variation in resistance under circumstances exceptionally favorable for stability may amount, in different seasons of the year, to as much as 2 parts in 10,000, and under less favorable circumstances to 7 or more parts in 10,000.

The discovery of this property of shellac of swelling with moisture and straining the wire has led to the present type of construction of resistance standards adopted by the Bureau and executed by prominent instrument makers. The construction referred to is fully described by Dr. E. B. Rosa in a paper entitled "A New Form of Standard Resistance," and published in the Bureau of Standards' Bulletin, Vol. 5, 1908–9, page 413. The essential feature of the construction consists in hermetically sealing the brass cylinder, upon which the shellacked resistance wire is wound, in a metal cylinder filled with kerosene oil. In this manner all access of moisture to the resistance winding is effectually prevented. This construction so improves the constancy of the resistance that of 28 coils kept under close observation there was "*only one coil of the above twenty-eight that has changed as much as two parts in 100,000 during the past twelve months.*" *

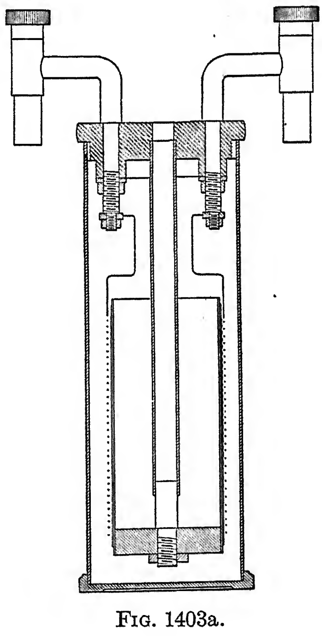

Fig. 1403a.

This type of standard, for resistance units of 10 ohms or more without potential terminals, is shown in section in Fig. 1403a.

"The new form of resistance standard is much smaller than the

* *Bulletin*, Vol. 5, page 427.

Reichsanstalt type, so long and so favorably known throughout the world as a standard of resistance. It weighs only about 400 g filled and measures only 7.5 cm across the terminals instead of 16 cm. For measurements up to an accuracy of .001 per cent it is measured as it stands, its current capacity being ample when using reasonably sensitive galvanometers, and the small thermometer in the central tube giving its temperature with all needed accuracy. The temperature coefficient is generally not greater than .002 per cent per degree, so that a quarter of 1 degree uncertainty in the temperature would cause an error less than that allowed. * * * * * *"

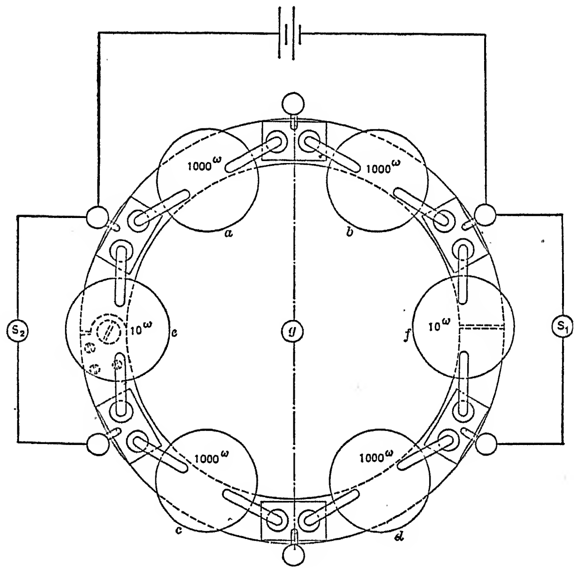

FIG. 1403b.

" The apparatus employed in comparing the coils of any denomination with one another is shown in Figs." 1403b and 1403c. "It is a Wheatstone bridge with the coils arranged in the circle, having two extra coils inserted, on which shunts may be applied for balancing the bridge instead of shunting directly the ratio coils. Or these two openings may be closed by heavy links and the shunts applied directly to the ratio coils. The circular frame is so hinged that it may be opened far enough to admit a larger coil, as, for example, one of the Reichsanstalt form, which may thus

be directly and conveniently compared with one of the new Bureau of Standards form, Fig." 1403c. "The apparatus is very convenient, is compact, requires a relatively small oil bath, or may be used without an oil bath except in comparisons of extreme precision, and will accommodate any kind of a resistance standard that is provided with terminals for dipping into mercury cups."

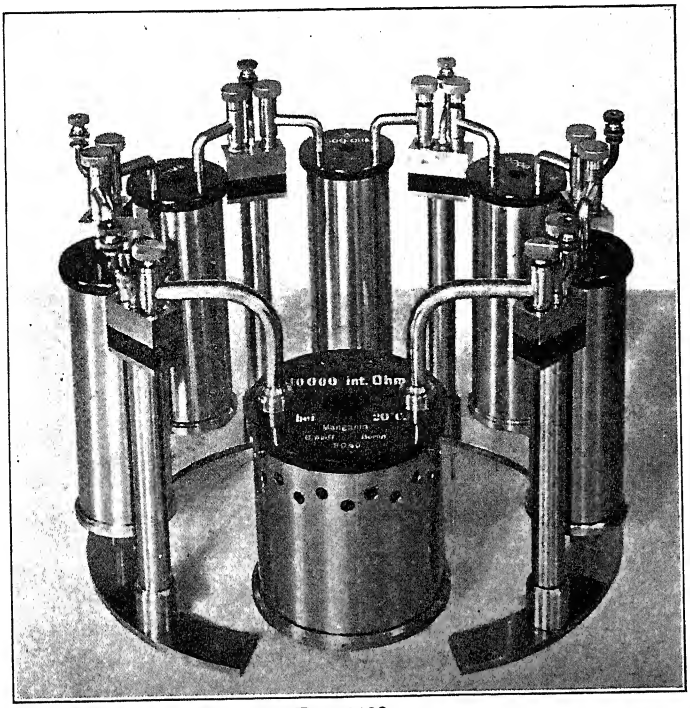

FIG. 1403c.

When a resistance standard is 1 ohm or less it is necessary to provide it with potential terminals. This leads to a different construction than that adopted for the higher denominations. The resistance material for these low-resistance standards is in the form of heavy manganin wire or sheet which is too massive to be strained by the swelling of shellac. In Fig. 1403d is shown a 0.001-ohm standard resistance of the Reichsanstalt form.

This form of standard is guaranteed by its makers to an accuracy of 0.02 of 1 per cent but may be made more accurate. "When immersed in oil and used as a precision standard of resist-

ance it has a current-carrying capacity of 32 amperes. Used for measuring current to a lesser degree of accuracy it will carry 100 amperes or more."

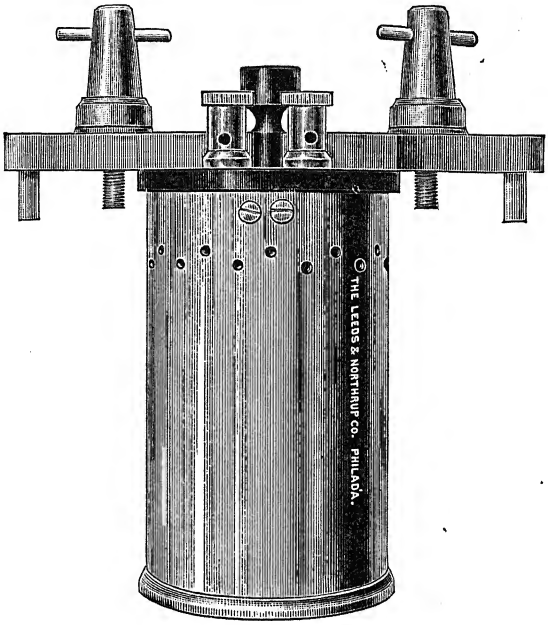

FIG. 1403d.

Otto Wolff of Berlin supplies a line of standard resistances which have the following denominations and current-carrying capacities when used for precision measurements and as current resistance-standards.

Resistance, ohms	Current capacity for precision measurements, amperes	Current capacity as current-resistance standards, amperes
100,000	0.003	0.01
10,000	0.01	0.03
1,000	0.03	0.1
100	0.1	0.3
10	0.3	1
1	1	3
0.1	3	10
0.01	10	100
0.001	30	300
0.0001	100	1000

These standards are called "Small Pattern" standards. They are all arranged to be dipped into mercury cups by means of copper terminals which in all types are separated the standard distance of 16 cms from center to center. The resistance proper is mounted inside a brass case with a hard rubber top which carries the terminals. The case is perforated to permit free circulation of oil when the standard is immersed in a bath of petroleum. The general rule may be applied that, when used in oil for high precision work, a load of 1 watt is allowable, while for less accurate work a load of 10 watts may be applied.

1404. Resistance Boxes and Wheatstone Bridges; General Remarks. — In determining the value of one resistance in terms of another it is very desirable to have a series of known values of resistances which can be varied thru a wide range by known amounts. The fundamental purpose of all resistance boxes, whether employing plug contacts or dials, is to provide means for obtaining the largest possible number of values from the fewest possible number of known resistance units. The very varied forms of construction that instrument makers have given to resistance boxes, have had, more or less, the above object in view. Where the number of coils or units is made greater than the least number required for varying resistance thru a given range, it is done to serve some purpose of convenience of working, or to increase the facility with which resistance values may be changed and mentally added up.

The advantages of being able to obtain a specified number of resistance values with a minimum number of resistance units pertain both to the user and to the maker of the set. The advantages to the user are reduced cost, a smaller number of coils to get out of adjustment and to measure when checking up a set, economy of space, a smaller number of contacts and, often, increased simplicity in forming combinations of resistance values.

The advantages to the manufacturer are in the nature of reduced cost of construction which must, of course, be reflected to the purchaser.

In short it is because of the gain in the above particulars that it is desirable to combine a small number of units of resistance to obtain a great many values. If it were otherwise there would be no occasion for the construction of resistance boxes and one would employ a separate coil for every value which he might wish to use.

The various methods that have been devised for combining resistance units have been discussed already and it may only be remarked here that the so-called decade plan is considered so very superior to all others that its further adoption is urged.

1405. **Watt Capacity of Resistance Units.** — The current-carrying capacity of a resistance unit in a resistance set will depend not only upon the ability of the unit to dissipate heat but also upon the ohmic resistance of the unit. It is therefore meaningless to speak, as is often done carelessly, of the current capacity of coils. The watt-dissipating capacity of a coil or unit is, on the other hand, a definite matter, and in a well-constructed resistance set will be the same for each coil in the set. The watts which a spool will dissipate will be approximately proportional to the square of the voltage at the terminals of the coil divided by the resistance of the coil; or it will be approximately proportional to the square of the current thru the coil multiplied by the resistance of the coil. No definite statements can be made of the number of watts which each coil in a set may safely carry. Often a temperature which will not injure the insulation will permanently impair the precision of the coil. The manganin wire of which resistance coils are mostly wound is generally permanently diminished in resistance — probably by the release of the molecular strains in the wire — when the temperature is considerably elevated. To make this effect as small as possible resistance coils should be artificially "aged" before their final adjustment by being given a prolonged baking (from 24 to 48 hours) at a temperature of from 130° to 140° C. It is, therefore, unsafe to load a precision resistance box so that any of its spools attain a temperature above that at which they were aged. A general rough rule for resistance boxes, the spool windings of which are on brass, is to limit the watts per spool to 3 watts. Thus one should limit the current thru the spools so that $\frac{v^2}{r}$ is not greater than 3, v being the volts at the spool terminals and r the resistance in ohms of its winding. In using a Wheatstone bridge, in which the resistance of the ratio arms at any moment may be one time 1 ohm and 1000 ohms, and then again 1000 ohms and 1000 ohms, it is well to keep from 10 to 100 ohms in the battery circuit all the time. An E.M.F. may then be employed which, acting thru this resistance, will not overheat the low-resistance coils.

In the best boxes the windings are not only upon brass spools but these are in direct metallic connection with the brass blocks upon the top of the box. By this arrangement much of the heat developed is conducted to the brass blocks and there dissipated. Some boxes are provided with outside cases of perforated metal so the entire set may be immersed in kerosene oil. A set used in this way will have the watt-dissipating capacity of its coils increased several times.

It is rare, however, in using a resistance set for the measurement of direct-current resistance that one has any occasion to reach, even approximately, the watt capacity of the set. Plug and dial rheostat boxes and sometimes the rheostats of Wheatstone-bridge sets are employed as auxiliary apparatus and are required to carry as great a load as is allowable. For this reason chiefly it is desirable, in selecting resistance sets, to require that their heat-dissipating capacity shall be as large as practicable without undue increase in the size of the sets.

1406. Construction of Resistance Spools. — The type of resistance spool shown in the illustration, Fig. 1406, combines those features which experience has shown to be desirable. The winding is upon a brass tube. The wire is chosen of such a size (whenever this is possible) that the required resistance is obtained by winding it in a single layer which extends the entire length of the spool. The brass spool is mounted, with good metallic connections, upon the brass shaft, the upper end of which serves as a stud for the brushes of a dial switch to rest upon. The winding itself is of manganin wire and is wound bifilar. This wire is chosen because of its high specific resistance (about 26 times that of copper), small temperature coefficient (about 0.00002 per degree C.) and small thermal E.M.F. against brass or copper. There are other materials which would probably meet the above conditions quite as well but they are of recent development and have not as yet had the thoro trial and endorsement which has been given to manganin.

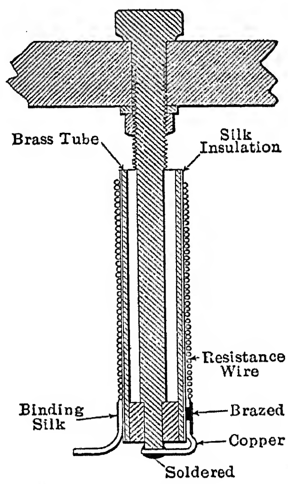

FIG. 1406.

If the spools are 100 ohms or less in resistance the terminals of the resistance-wire proper are not soft soldered directly to the brass. A short length of copper wire is silver soldered to the manganin and this terminal is in turn soft soldered to the brass. In making the soft-soldered connection of the copper terminals, a small variation in the length of the copper terminal affects very little the total resistance of the spool. Thus, adjusted spools may be soldered in place in resistance sets without the necessity of making a further resistance adjustment after the spool is in place.

The prevailing practice is to cover the outside of the spools with a thick covering of shellac which is baked hard and is glossy. As shellac has been shown to absorb moisture from the air and to strain the wire by swelling, a non-hydroscopic material should be sought. If the proper material were found the permanent precision of resistance sets would be considerably improved.

Spools constructed and mounted as above may be made to dissipate safely between three and four watts each. In boxes of cheaper grade the manganin is wound upon wooden spools. These are inferior in heat-dissipating capacity and other respects. When of the same size as the brass spools their watt-dissipating capacity is between 0.25 and 0.5 watt or about 0.1 that of a brass spool mounted as above.

1407. **The Precision of Coils in Resistance Sets.** — The precision of coils in resistance sets is not usually required to be so high as that of individual resistance standards. Even if the adjustments were made originally as high as that of the best individual standards they would only hold for a particular temperature. Furthermore, as different sizes of wire must be used in the same resistance-set, it is impracticable to select all the sizes of exactly the same temperature coefficient; and thus exact temperature corrections for all the coils in a set, for other temperatures than that at which they were adjusted, are hardly possible. The coils, moreover, not being hermetically sealed in oil will not maintain the high precision which might be given to them and which pertains to resistance standards. The usually unknown resistance of leads, the contact resistances under binding posts, and the various plug or brush contact resistances which enter in a resistance set is another reason why the highest possible precision is not demanded. The best makers guarantee conservatively the precision which pertains to the resistance sets which

they list. A Wheatstone bridge of the highest grade, intended for precision work, will have the following guarantee: The rheostat coils will be guaranteed to an accuracy of 0.02 of 1 per cent, with the exception of the 0.1-ohm coils which will be guaranteed to 0.1 of 1 per cent and the 1-ohm coils to 0.04 of 1 per cent. The coils in the ratio arms of such a set will be guaranteed to be adjusted to an accuracy of 0.02 of 1 per cent and to be like each other to 0.01 of 1 per cent. A Wheatstone bridge of more moderate precision and of about one-third the cost would be given an accuracy of adjustment of the coils in the rheostat of 0.05 of 1 per cent and in the ratio arms of 0.02 of 1 per cent. The very cheapest boxes and Wheatstone bridges should not be less accurate than 0.2 of 1 per cent in their rheostat and 0.1 of 1 per cent in their ratio arms.

1408. Some Features of Outside Construction. — The classical method, for many years the only one employed, of making the

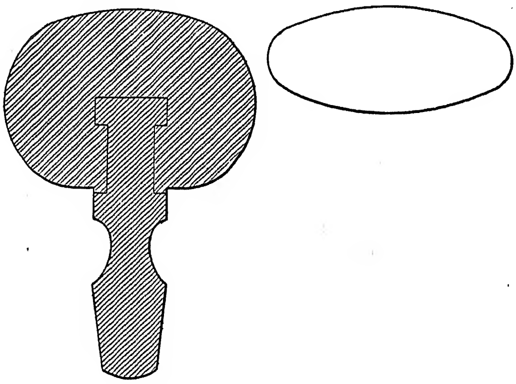

FIG. 1408a.

desired combinations of resistance coils, is to use brass blocks mounted on hard rubber plates, which are connected by plugs inserted between them. In recent years other methods have come more and more into favor, especially where it is required to obtain the desired values rapidly. Among such of special value is the method of dial contacts.

The chief advantage in the use of plug contacts consists in the great number of combinations which can be effected with comparatively few blocks and plugs. The advantage has long been claimed for plug contacts that this form of contact has the least resistance. This advantage, however, cannot be allowed for plug

contacts if certain other forms of contacts are correctly designed and constructed.

In resistance sets of the plug-type it is desirable to use a correct type of plug. This should have a proper taper to fit the holes between the blocks. The hard rubber head should be easy to grasp without hurting the fingers and should not work loose. The type of plug designed by the author, and shown in Fig. 1408a meets these conditions and has been much used on high-grade sets.

Brass blocks when used should be of sufficiently heavy construction to permit undercutting so the base of the blocks will be separated to give sufficient surface insulation over the hard rubber top of the resistance set. This is important because hard rubber, which is exposed to light for a long time, deteriorates in its surface insulation.

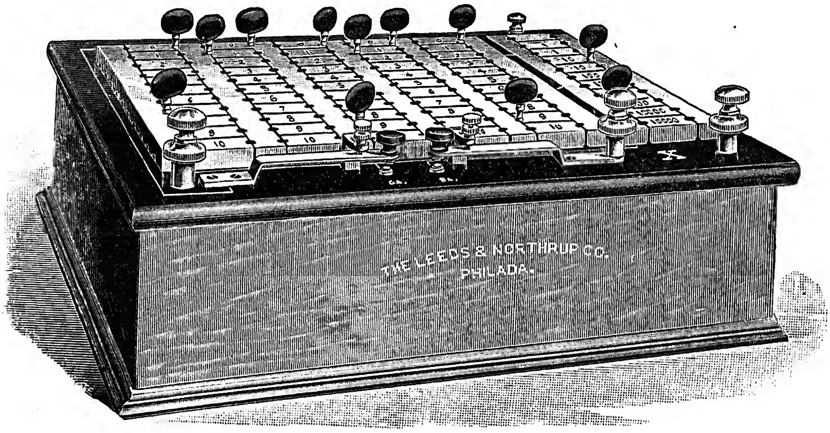

FIG. 1408b.

In Fig. 1408b is shown a high-grade Wheatstone-resistance set of the plug decade type which is intended to embody all of the best and latest features of Wheatstone-bridge construction. This set is of the four-coils-to-the-decade type as described in par. 503. An extra coil is added to the decade of lowest denomination. This extra coil is added to enable the set to be checked up. It is possible to compare every coil in a set with every other coil in the set, and to completely intercheck the different decades.

A particularly fine type of a five-dial Wheatstone bridge, constructed for the author, embodies the following special features·

of construction: The case of the bridge is made of glass which gives a full view of the coils and inside wiring. The rheostat consists of five dials, the one of lowest denomination reading in 0.1 of an ohm. The ratio arms can be plugged to give ratios 1 to 10,000 or 10,000 to 1 with many other ratios lying between these extremes. The four-coils-to-the-decade construction is used in the rheostat (see § 503) and Schöne's arrangement is used in the ratio arms (see § 509). The accuracy of adjustment of the coils in this bridge is better than 0.02 of 1 per cent in the rheostat and still better in the ratio arms.

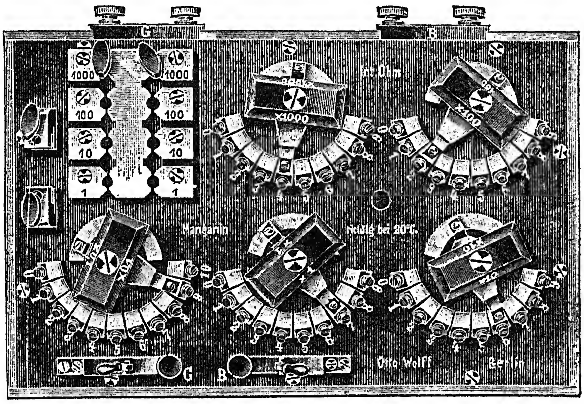

FIG. 1408c.

The form of dial construction which perhaps has met with most favor is the one used by Otto Wolff of Berlin in all his resistance sets and potentiometers. A top view of an Otto Wolff five-dial Wheatstone bridge is shown in Fig. 1408c.

Dial switches are only employed for the rheostat, the values in the ratio arms being obtained with plugs inserted in blocks.

CHAPTER XV.

DEFLECTION INSTRUMENTS AND GALVANOMETERS.

1500. Distinction Between Indicators and Deflection Instruments. — We shall discuss these chiefly from the standpoint of their employment in resistance measurements.

As resistance measurements may be broadly divided into deflection methods — in which the value sought is determined in terms of the deflections obtained with a deflection instrument — and null methods — in which a current detector is used merely to indicate when no current is flowing in its circuit — the instrumental requirements for the two methods are quite dissimilar.

For deflection methods the instruments used are voltmeters and ammeters, pointer galvanometers, and mirror galvanometers with telescope and scale or lamp and scale.

When deflection instruments, voltmeters and pointer galvanometers are used for measuring resistance it is required that the pointer should accurately return to zero after being deflected and that the scale be as long as practicable. This should be either carefully calibrated for correctly measuring equal increments of current, or if the scale divisions are equally spaced the deflections should increase proportionally with the current. If the above requirements are met, then the possible precision obtainable will increase with the number of smallest divisions marked upon the scale.

The damping of the system of such a deflection instrument should be such that the deflection will reach quickly its maximum value without any oscillation. It is much easier to take close readings — when the measuring current is not very steady — if the instruments are aperiodic or critically damped.

It is not generally required that deflection instruments, when used for measuring resistance, shall have a very high sensibility, except when used for insulation testing by direct deflection methods. A good voltmeter of the Weston type will have about 100 ohms

to the volt and consequently, for full scale deflection, will take about $\frac{150}{15,000} = 0.01$ ampere. A current of this magnitude will cause very little heating in any resistance which is measured — tho even this small current will alter the resistance of a tungsten lamp-filament which, consequently, cannot be measured accurately with a voltmeter. One need for sensibility in such an instrument is to be able to use so small a measuring current that resistances will not be appreciably heated.

Portability is a feature which is generally sought and is, of course, obtained with all instruments of the voltmeter or ammeter type.

For measuring insulation resistance by direct deflection specially designed and constructed galvanometers which have high sensibility and proportional deflections are required. These high sensibility insulation-testing galvanometers were formerly of the moving-magnet type, the typical instrument being the mirror galvanometer of Lord Kelvin. The improvements which have been made recently in moving-coil or D'Arsonval galvanometers have been such that these are now much more generally used. Their sensibility is sufficient for most classes of work and they are much more convenient to use. They have a stable zero, their deflections are nearly proportional over a scale a meter long set at two meters from the mirror, and they may be made portable for outdoor work. The best of these instruments represent the highest art in galvanometer design. We shall discuss later their more prominent characteristics.

The instrument most universally employed in resistance measurements, made by any null method, is a galvanometer. Galvanometers are so varied in form and character that we can only consider a few of the most prominent facts regarding them.

The system of a pointer instrument or mirror instrument, whether of the moving-magnet, or the moving-coil type, is essentially an oscillating system. But in resistance measurements — where the null method is used — the possible amplitude of the oscillation need not be large and proportionality of the deflections is not important. In fact, if a galvanometer is to be used expressly for measuring resistance by a null method, the movement of its system may be limited by stops as there is no necessity that its pointer or mirror should move thru more than a very small angle

to indicate when current is passing. This simplifies the design of zero indicators, for the rather difficult requirement of giving the instrument proportional deflections over a long scale does not need consideration.

Zero-current indicators or galvanometers of a portable type which have a pointer that moves in either direction from the center over a scale of 10 or 20 divisions can be obtained that are portable and sufficiently sensitive for the great majority of measurements of resistance between 1 and 100,000 ohms.

Very many resistance measurements are made with a slide wire or Wheatstone bridge in factories and colleges where a pointer galvanometer would be found amply sensitive and far more convenient to use than the suspended coil, mirror type of galvanometer so generally employed. A good pointer galvanometer of 300 ohms resistance should deflect over its scale 1 mm. with 1 volt and with 0.5 to 1 megohm in series. It should be just short of critically damped, and come to rest, after the key is closed, in from two to three seconds. This type of instrument — which can be obtained at half the cost of the better types of suspended coil instruments — is strongly advocated for the use of elementary laboratory students. It will be found suitable for all general Wheatstone-bridge work not requiring accuracies of over 0.05 to 0.1 of 1 per cent. Such instruments are made by Weston, Paul, of London, and others.

The author also has given much attention to the development of this type of galvanometer and the following is a brief description* of the new type which he designed and which is now marketed by the Leeds and Northrup Co.

1501. Pointer Type, Flat-coil Galvanometers. — The galvanometer is called a flat-coil galvanometer and is a somewhat radical modification of the moving coil instrument. The modification was made by the author to make the moving coil instrument into a sensitive, substantial and portable galvanometer in which suspensions might replace the customary jewels. A suspended system may be arranged to withstand rougher handling than a system mounted in jewels, and there is the further

* For a more detailed description of this and other galvanometers consult a paper by the author in the Journal of the Franklin Inst., October, 1910, entitled, "The Comparison of Galvanometers and a New Type of Flat-coil Galvanometer," from which paper much of what is given here is taken.

marked advantage in a suspended instrument that there is nothing which corresponds to an initial pivot friction. If very minute deflections of a pivot instrument are optically magnified, they will be found to be erratic and in no wise proportional to the forces acting. A suspended instrument, on the other hand, will deflect somewhat with the feeblest force, and if its minute deflections are highly magnified by optical means, they will be found to be proportional to the deflecting force. This circumstance especially adapts a suspended instrument to null measurements. Furthermore, the disk-like shape of the moving system lends itself in a portable instrument, with a suspended system, to a construction which requires a comparatively small height.

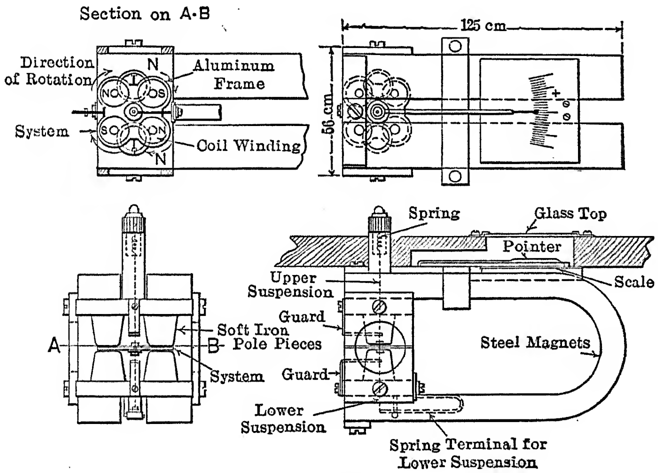

FIG. 1501a.

Fig. 1501a shows the construction of one type of this instrument, used as a pointer instrument which is portable. The cut is made complete and explicit to save lengthy description. The same instrument as it appears when mounted in its case is shown in Fig. 1501b.

It will be observed that when current energizes the four disk-like coils they all contribute to produce rotation in the same direction. A reversal of the direction of the current reverses the direction of rotation, the deflections, with equal current, being the same to either side of the zero.

At first sight this form of system might seem bad for giving a high figure of merit because so large a portion of the moving parts are far removed from the axis. While this is true, there are compensating advantages which practice shows make the galvanometer very creditable in respect to figure of merit, while many features are secured which have practical worth. In the first place, as the galvanometer has no iron core, there are only two air gaps, one above and one below the coil, instead of four, as required by the D'Arsonval galvanometer with an iron core; in the second place *all* of the magnetic field is active at all times in producing a turning moment, regardless of the angular position of the system. In the D'Arsonval type, a magnetic field must be provided for the system to deflect into. This is shown by Fig. 1501c.

FIG. 1501b.

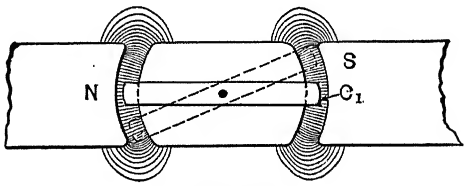

FIG. 1501c.

Thus if the coil section C_1 occupies one fifth of the air gap, or space in which the coil turns, then four fifths of the magnetic lines are at all times idle. Magnets of sufficient strength and capacity must be supplied to furnish five times as many magnetic lines as are at any one moment used. The flat-coil galvanometer has a large advantage in this respect.

Moreover, the aluminum form in which the four coils are held serves admirably as a magnetic damper. Further, if the instrument is to serve as a ballistic galvanometer, its moment of inertia can be made very great out of material which is active in producing a turning moment and for such service the construction is ideal.

The best proportioning of the system and coils to give specific results has received at the hands of the author much thought and calculation.

The accessory features connected with this galvanometer, as magnet, suspensions, etc., deserve a few remarks.

The galvanometer is mounted in a small wooden box and is very portable. For ease in shipping and in manufacture cast-iron magnets are used. These are very suitable, and if properly proportioned they give a field which is as strong as can be used with advantage in a galvanometer in which the sensibility is made large by the use of a weak suspension. There are always magnetic impurities in the coil system. The magnetic field in which the coil swings acts on these impurities, probably, in rough proportion to the square of the field strength. The result is, that the system takes on a polarity and tends, irrespective of the suspension, to set in a particular angular position. To show this strikingly, place the poles of a fairly strong permanent U-magnet near the poles of an ordinary wall form D'Arsonval galvanometer of about 400 megohms sensibility. It will generally show a permanent deflection of from 10 to 20 scale divisions as long as the U-magnet is held on.

The natural zero of the galvanometer is the position taken by the coil, which position is the resultant of the torsional force of the suspension, and the magnetic action of the field upon the magnetic impurities in the coil. When the field is weakened or changed in direction, the resultant controlling force changes, and hence a deflection results. In attempting an accurate measurement, by the Kelvin double bridge, of the resistance of a 3000-ampere standard low resistance, the author recently used a D'Arsonval galvanometer in close proximity to one of the conductors carrying 1000 amperes. Whenever the circuit thru the low resistance was closed the galvanometer deflected one scale division by the influence of the field external to the conductor. This deflection would have produced an error in measuring the resistance of 0.1 of 1 per cent, if it had not been discovered. The galvanometer was a very good one and the circumstance is related to show how precautions against exterior fields must be taken sometimes, even with moving-coil galvanometers. It also shows that little, if any, improvement can be obtained in weakly controlled galvanometers by using magnetic fields greater than

can be obtained with cast-iron magnets. It may be n here, also, that the chief cause of zero shift in sensitive D' galvanometers is due, not to the "set" in the suspensio a "magnetic set" which the coil takes when deflected in field.

If a galvanometer is to have a quick period and a str trol, and be magnetically damped, then it is desirable magnetic field of high intensity, such as can be secured using soft Swedish iron pole pieces and permanent steel of the best quality.

Galvanometers which carry a pointer, or a mirror, intended for use on shipboard or for easy transporta setting down without leveling, must be suspended betwe and lower suspensions which are taut. Guards which movement of the system in all directions, together with spring in both upper and lower suspension, will effectually the suspensions breaking, even when the galvanometer jected to severe jars.

It is desirable, however, to make the tensile strengt suspensions great without undue increase in their torsional The tension on the suspensions may then be made consider it becomes easy to accurately balance the system. This balancing is essential to a steady zero and to make the sys from vibrations. A system, held between two taut sus which is slightly unbalanced about its axis of rotation more influenced by tremors than one which is carefully b

The tensile strength of the suspension can be increased without an increase in its torsional moment by making number of round wires laid together in a cable, instead of wire. Thus:

Let n = the number of wires laid together,
d = the diameter of a single wire.

Then the torsion of one wire is $t \propto d^4 \propto S^2$, where S is th section of one wire. The total torsion of the n wires is

$$T \propto n d^4 \propto nS^2.$$

If these n wires were to be combined into one, the tensile would remain the same, but the torsion would now be

$$T_s \propto (nS)^2.$$

Hence,
$$\frac{T}{T_s} = \frac{nS^2}{n^2S^2} = \frac{1}{n}. \tag{1}$$

This conclusion, that with equal tensile strength the moment of torsion decreases with the number of strands into which a given cross-section of wire is subdivided, has great practical value. It is much employed in galvanometer constructions where the galvanometers are to be made portable. Experience shows that there is no slipping of one strand over the other in a manner to develop friction and uncertain return to zero.

The flat-coil type galvanometer here described is admirably suited to a differential instrument, especially for use in temperature measurements with resistance thermometers. Its applications to this service have been pointed out by the author in a former publication.*

When used as a deflection galvanometer, its deflections are not as proportional to the current flowing as might be desired. Fig. 1501d is a curve which gives the average performance of the instrument in this respect.

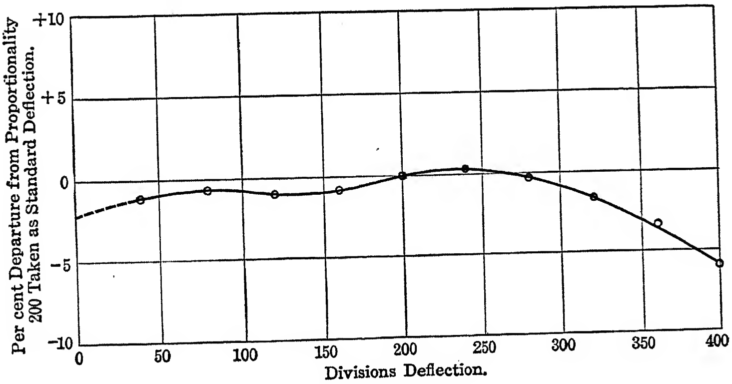

FIG. 1501d.

This type of galvanometer may be given any resistance from 20 ohms to 300 ohms. One made with a steel magnet and of 300 ohms resistance will have a sensibility of about one millimeter deflection upon its own scale with 1 volt and 1 megohm in series, when the period for one complete oscillation of its system is 1.5

* "Cooling Curves and a New Type of Apparatus for their Automatic Registration," *Proc. of the Amer. Electrochem. Soc.*, May 7, 1909.

seconds. The system is magnetically damped by its aluminum coil frame to be slightly less than just aperiodic.

1502. Sensitive Galvanometers for Refined Measurements of Resistance and Insulation Testing. — For precise measurements of low resistance, using a Kelvin double bridge, and for measuring the insulation resistance by direct deflection the pointer type of galvanometer has insufficient sensibility and some type of mirror galvanometer with a suspended system is usually required.

To aid in the selection, understanding and use of galvanometers to meet the above requirements we shall now give a brief outline of the fundamental principles upon which these instruments operate. Some of the principles will also be discussed which will assist in making a rational comparison of the merits and usefulness of the very many different types and forms of mirror galvanometers which the instrument maker supplies.

1503. The Equation of Motion of a Galvanometer System. — The systems of all galvanometers may be considered, and are to be mathematically treated, as oscillating systems. The movements of such systems obey the same laws and are expressed by the same equation of motion as the oscillations of a pendulum bob which is damped in its movements by swinging in a viscous fluid. When, also, the electricity in an electric circuit, that contains a condenser, a self-inductance, and an ohmic resistance, is allowed to oscillate freely its motion can be expressed by an equation of the same form as expresses the oscillations of a voltmeter system. Thus it is that the solutions which are given in many treatises and textbooks for the oscillations of electricity may be applied — by a simple change of constants — to the movements or oscillations of the moving systems of all types of deflection instruments. Those, therefore, who desire to study the laws which govern the motions of deflection instruments will do well to keep the above analogy in mind and study some one of the many treatments which are very fully given for the electrical case.

The equation of motion, which governs all freely oscillating systems, when put in a form that applies to a galvanometer system is expressed as follows:

$$I\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta = 0. \qquad (1)$$

Here

θ = the angle thru which the system has turned,
t = the time,
I = the moment of inertia of the system,
B = a constant called the coefficient of damping, and
K = a constant called the moment of torsion.

The analogous equation for the motion of electricity in a circuit is

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0, \tag{2}$$

where

q = the quantity of stored electricity,
t = the time,
L = the coefficient of self-induction of the circuit,
R = a constant — the ohmic resistance of the circuit — and
$\frac{1}{C}$ = a constant — the reciprocal of the capacity of the circuit.

The forms of Eqs. (1) and (2) being identical their solutions will be the same with the exception of the arbitrary constants involved.

It is not our purpose to give here the solutions of these equations, but merely to point out the useful conclusions to be drawn from them. Readers who are interested in the mathematical side of the subject are referred to the following books and papers:

Lamb's "Infinitesimal Calculus," page 512 and preceding pages. "Instruments et Méthodes de Mesures Électriques Industrielles par H. Armagnat, 1902 edition." *Physical Review*, March, 1903, page 158, where is contained an excellent article by O. N. Stewart on "The Damped Ballistic Galvanometer." Also, "Alternating Currents," by Bedell and Crehore. As shown above, all facts deduced by the two latter authors from the equations of motion concerning the electrical case apply, with modification of the constants only, to the mechanical problem.

Some deductions to be drawn from the differential equations and their solutions may be considered as follows: Suppose we substitute for the scale a photographic film which is drawn along in a vertical direction at a uniform speed while a spot of light is reflected from a mirror on the coil upon this film, it being assumed

that, when the coil deflects, the spot of light moves over the film in a horizontal direction. If the coil has been deflected in any manner thru an angle θ_m and then left to swing freely, the spot of light will trace on the photographic film a certain curve. The form which this curve will have will be entirely dependent upon the values given to the constants I, B, K, in Eq. (1).

In Fig. 1503 are given some characteristic curves which might be obtained in the above way, and which show the effect of varying the magnitude of the constants.

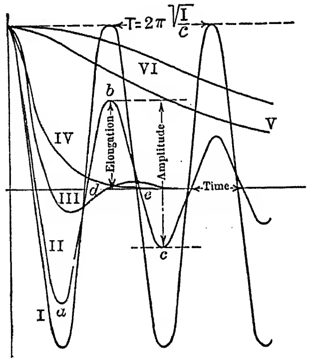

FIG. 1503.

The movement of the system is *periodic* if it passes the zero position, or position of equilibrium. This case is illustrated by curves I, II and III. If there is no damping, that is, if the constant B is zero, the system oscillates indefinitely and equal distances each side of the zero. This is illustrated by curve I. If B is small, but not zero, the damping is only partial, as curve II best illustrates, the magnitude of the oscillations gradually decreasing.

For a certain critical value of B the system returns with the maximum speed possible to zero without passing it. The system is then just *aperiodic*, and is in the most favorable condition for rapid working. Curve IV illustrates this case. We shall designate this as *critical aperiodic* motion. If B is still larger, that is, if the damping is greater than the critical value, the return to zero is slower. As B increases the time of return to zero increases more and more and, when B is very large, the galvanometer is in the most unfavorable condition for rapid working. In this case the zero is not only rendered uncertain but the movement of the system is very slow near the zero position, which makes it uncertain when the zero has actually been reached. The same remarks apply to the outward deflection when current is applied to the galvanometer, the outward and return deflections taking place in the same time and in a similar manner. Curves V and VI illustrate this condition.

The extent to which a galvanometer system is damped is independent, in the case of moving-magnet galvanometers, of the

resistance in the circuit external to the galvanometer, but depends only upon the frictional or air damping of its system. In this respect a moving-magnet galvanometer is more convenient to use than a moving-coil galvanometer, the damping of which varies greatly with the resistance in its external circuit. In any situation or with any type of galvanometer the best conditions of working will be realized when the damping is adjusted so the system returns to zero aperiodically, as exemplified in curve IV. In the use of a moving-magnet galvanometer this adjustment of the damping may be effected by shunting the galvanometer with a certain resistance or by hanging upon its coil a closed metallic circuit, as a rectangle of copper wire, or by both methods combined.

If the galvanometer, of the moving magnet type, is used in low-resistance measurements with a Kelvin double bridge it will always be upon a closed circuit and this closed circuit may have a resistance which is lower than the critical damping resistance. In this case the galvanometer will be overdamped and there is no recourse for reducing the damping other than to increase the torsion of the suspension, or to weaken the strength of the magnetic field; but both of these procedures will diminish the sensibility of the galvanometer. It should be mentioned, however, that sensibility is directly proportional to the field strength while damping is proportional to the square of the field strength, and so a considerable decrease in damping may be effected by reducing the field strength (by magnetically shunting the field magnet) without producing a corresponding decrease in sensibility.

1504. Comparison of Galvanometers. — It should be stated at the outset that "sensibility" is only one of many requisites which a galvanometer, useful for resistance measurements, should possess. Nor will an understanding and solution of the general equation of motion of a galvanometer system, possessed of a moment of inertia, a moment of torsion and a damping factor, be of much aid either in the design or in the selection of a good galvanometer. The merits of any galvanometer should be judged by its adaptability to the work it has to perform. The problem of estimating what is essential to usefulness in any galvanometer must be solved by methods which only experience in use and design can teach.

Every galvanometer design, by necessity, is a series of compromises. No one instrument can possess in full measure all

the qualities desired. Experience and judgment must guide to a proportionate selection of such diverse qualities as the following: Sensibility, short period, accurate return to zero, tensile strength of suspension, freedom from easy disturbance by external influences such as air currents, accurate balancing of the system to lessen the effect of vibrations, proportionality in the deflections, strength and uniformity of the magnetic field, clearness of scale and ease in reading, freedom from magnetic impurities in the system of moving-coil instruments, variable or fixed damping to secure aperiodic return to zero, proper coil clearance for free motion and ease of adjustment, arrangements to facilitate transportation, freedom from parasitic currents resulting from internal thermoelectric forces, proper resistance of coil and suspensions, permanence of the galvanometer constant and magnitude of its temperature coefficient, provisions for easy inspection of system and replacing of suspensions, high insulation where required, facility in mounting, adaptability to general classes of work as well as to a specific service, ease in assembling, grace in appearance, and a construction which is moderate in cost. Besides understanding such features in practical design, one must also understand the fundamental principles which underlie every galvanometer, if the best possibilities of the instrument are to be secured. As a beginning in design or in selecting an instrument for use, particular qualities, required for a particular service, should have special attention. No universally serviceable instrument of this class, equally good for all kinds of work, ever was or ever will be constructed.

These preliminary remarks hint at the scope of the subject of galvanometer construction, and emphasize the necessity one is under, if he would pass judgment upon the merits of a galvanometer, to have first a clear conception of the use for which the instrument is intended. A galvanometer admirably adapted to show the readings of a bolometer giving the energy distribution in the spectrum would make a poor lecture-room instrument for general demonstration purposes, nor would it be suitable to put into students' hands for Wheatstone-bridge work.

A common basis for comparison of the respective worths of galvanometers of diverse pattern, in respect to the single quality which has been termed "the figure of merit," may be reached with a fair degree of satisfaction. A clear physical conception of this feature and what it means will be a useful preliminary

to any estimation of the worth of any current-measuring instrument.

Let (Fig. 1504) G_a and G_b represent, diagramatically, two systems of any type of moving-coil galvanometer. Let these systems be held by suspensions S_a and S_b attached at points p_1, p_2 and p_1' and p_2'.

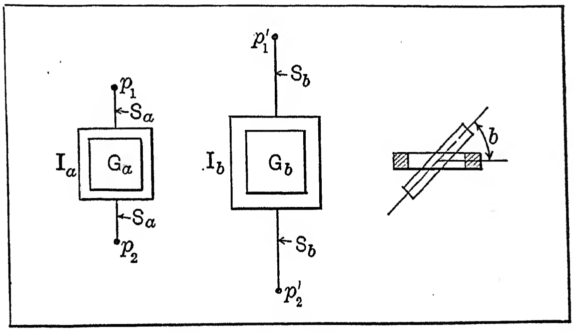

FIG. 1504.

Suppose the coil-winding of each is on a metal frame of such cross-section and conductivity that, when the system rotates in the magnetic field, its return to zero from a deflection is just aperiodic in virtue of the currents induced in the frame. To have this condition always fulfilled, one may conceive the conductivity of the frame to vary whenever the moment of inertia of the system, the torsion of the suspension, and the strength of the magnetic field are varied. This premised, let such a current pass thru each system that it will be rotated thru a standard angle b, which may be made always the same by varying the strength of the current. Evidently the current which will be required to produce this deflection will depend upon many factors, chief among which are the strength and uniformity of the magnetic field, the torsional force of the suspension, the length and number of turns in the coil, and the degree of freedom from magnetic impurities in the system.

The system, starting from rest, will require a certain time T after the current is applied, to complete a certain fractional part of its aperiodic deflection. To be definite, assume that the deflection is practically completed, when it has reached within 0.05 of 1 per cent of its final deflection. This time T will be, for all

purposes of practical computation, the same as the time of a complete oscillation of the same system undamped. If I is the moment of inertia of any galvanometer system, then the square of its time of deflection, as above defined, or of a complete oscillation if undamped, is proportional to this moment of inertia — or

$$T^2 \propto I.$$

Now so arrange matters that the same current thru each of the two systems G_a and G_b will produce in each the standard angular deflection b. Then the *sensibility* S_m of each system is the same and this sensibility will be inversely proportional to the current required to produce the deflection. If, however, the system G_a has a moment of inertia I_a, and the system G_b a greater moment of inertia I_b, then G_a will reach its standard deflection in a shorter time than G_b. The ratio will maintain

$$\frac{T_a^2}{T_b^2} = \frac{I_a}{I_b}.$$

By hypothesis, the sensibilities, that is, the currents required thru each to produce the standard deflection, are equal. Since, however, G_a deflects in a shorter time than G_b its suspension might be weakened until its time of deflection equals that of G_b. But with a weaker suspension it will take less current to produce the standard deflection. Hence, with equal times to make the standard deflection, the system G_a is more sensitive than the system G_b.

Conversely, to make G_b deflect the same amount in the same time as G_a, its suspension must be stiffened, and with a stiffer suspension it will require more current than G_a for the standard deflection. In this respect it is a less sensitive galvanometer than G_a.

As it is always possible to vary within wide limits the torsional force of a galvanometer suspension, a galvanometer which is quick but not sensitive can be made more sensitive at the expense of quickness by changing its suspension, and a galvanometer which is slow but sensitive can be made quicker at the expense of sensibility. If we call S_m the sensibility of a particular galvanometer and T its period, then the product $S_m \frac{1}{T^2}$ cannot be increased by changes of the above character. We shall call this

product proportional to the *useful sensibility* of any particular galvanometer and write

$$U \propto \frac{S_m}{T^2}. \tag{1}$$

By T we must understand the time of a complete oscillation, if the galvanometer is undamped, or the time it takes to reach its final deflection within 0.05 of 1 per cent, if it is magnetically damped to be just aperiodic.

For practical purposes of comparison of galvanometers, the time may be considered the same for the instrument in either of these conditions.

While the sensibility S_m is inversely proportional to the current needed for the standard deflection, this current, if everything else remains the same, will be less as the number of turns in the coil is increased. Of two galvanometers which are to be used on the same *constant-current* circuit, and which are alike in all features except in respect to number of turns, that one which has the more turns will be the more sensitive. We can call, therefore, the sensibility of a galvanometer for use on a constant-current circuit a quantity which is proportional to its number of turns n, and inversely proportional to the current i required to produce a standard deflection, or

$$S_m \propto \frac{n}{i}. \tag{2}$$

Hence, its useful sensibility is

$$U \propto \frac{n}{iT^2} \propto \frac{S_m}{T^2}. \tag{3}$$

To increase the number of turns we may proceed in either or both of two ways; the size of the insulated wire may be diminished and the same winding space be filled, or the wire may be kept the same size and the dimensions of the cross-section of the winding channel may be increased. By the first method the moment of inertia of the system remains nearly the same. It would remain exactly the same, if in altering the size of the wire no alteration were made in the density of the coil by changing the ratio of insulation to wire, thru a change of wire size. By the second method, the moment of inertia, and hence T^2, will be changed unless the length of the turns are also diminished in a proper proportion. We have seen that U, the quantity which we have called the useful sen-

sibility of a galvanometer, cannot be changed by changing the torsional force of its suspension, but it may be changed by changing the coil winding. Thus, in changing n, if the moment of inertia only is changed T^2 will be changed and U will be changed because the period changes, but if n is changed in such a manner as not to change the period then U will change again because S_m, the sensibility, changes. If n is changed in such a way as to vary both S_m and T^2, U will still change unless the exceptional condition is met, that n so changes that $\frac{n}{T^2}$ remains constant. We conclude, by the above line of reasoning, that *useful sensibility* is a constant property of a particular galvanometer with a particular winding, but a quantity which usually varies when the coil windings are changed. But if we divide the useful sensibility by the number of turns in the coil and write

$$F \propto \frac{U}{n} \propto \frac{S_m}{nT^2} \propto \frac{1}{iT^2} \tag{4}$$

we obtain the new quantity F which has been designated the "figure of merit" of a galvanometer.

The figure of merit of a galvanometer is a kind of "specific quantity" which attaches to every galvanometer. As the useful sensibility of a galvanometer cannot be improved by changing the torsional force of its suspension so also we cannot increase the figure of merit of a galvanometer by changing either its suspension or the turns which fill a winding space of fixed volume. A galvanometer with a certain "figure of merit" is potentially, so to speak, capable of having a certain chosen period with a certain accompanying sensibility and number of turns, or a certain chosen sensibility with a certain accompanying period and number of turns, but to increase the figure of merit changes must be made in the field strength or in the proportioning of the galvanometer parts. As "figure of merit" attaches as a specific property to every galvanometer, it serves in a useful way to compare the intrinsic worths of various types of instruments. If, however, we wish to compare the figures of merit of different galvanometers, we cannot do so practically by using the expression above in its present form, because there is no easy way of counting the number of turns in the coils after the galvanometers are built. It is necessary, therefore, to find how the resistance of the coil is related to its

number of turns, for this is a quantity easily measured. To do this write

$$R = \frac{4\,l_1 \rho n}{\pi W^2}, \tag{5}$$

where l_1 = length of mean turn,
W = diameter of wire and
ρ = specific resistance of wire.
If S = cross-section of channel and
d = double thickness of wire insulation, then

$$n = \frac{S}{(W+d)^2} = \frac{\pi S}{\pi W^2 + 2\,\pi W d}, \tag{6}$$

the term $\pi\, d^2$ being neglected, as being very small.

From Eq (6) we derive $\pi W^2 = \dfrac{\pi\,(S - 2\,Wn\,d)}{n}$, and putting this value of πW^2 in Eq. (5) we obtain

$$R \propto \frac{n^2}{S - 2\,Wn\,d}. \tag{7}$$

Solving this quadratic [see appendix II, 7, Eq. (16)] and using the positive sign before the radical, we obtain

$$n \propto \sqrt{RS + R^2W^2\,d^2} - RW\,d. \tag{8}$$

When this value of n replaces n in Eq. (4) we have the resulting expression, not involving n, for the figure of merit of a galvanometer,

$$F \propto \frac{S_m}{T^2[(RS + R^2W^2\,d^2)^{\frac{1}{2}} - RW\,d]}. \tag{9}$$

Except in cases where galvanometers wound with coarse wire are compared with galvanometers wound with very fine wire, the thickness d of the insulation may be neglected. We may consider S constant and we have, when we do this,

$$F \propto \frac{S_m}{T^2\sqrt{R}} \propto \frac{U}{\sqrt{R}} \propto \frac{1}{iT^2}. \tag{10}$$

$\dfrac{S_m}{T^2\sqrt{R}}$ is the usual expression for the figure of merit of any galvanometer. In comparing galvanometers by it, the supposition is made that thickness of insulation is neglected and that the galvanometers compared are wound with wire of the same *specific resistance*. The specification which we shall adopt to define S_m is as follows:

With the scale at 1000 scale divisions from the mirror, of a mirror galvanometer, the sensibility S_m is the number of megohms which must be in the galvanometer circuit so that with an E.M.F. of one volt in the circuit, there will result a deflection of one scale division. With this understood, we can write

$$F = \frac{S_m}{T^2\sqrt{R}}. \qquad (11)$$

A galvanometer would have, then, a unit figure of merit, if its time of a complete oscillation is one second (or which is practically the same thing, if its time of a periodic return to zero within 0.05 of 1 per cent of its previous deflection is one second), and the resistance of its winding is one ohm, and if, with one megohm in series and one volt in circuit, its deflection, on a scale 1000 scale divisions from its mirror, is one division.

As this unit has received no name, we shall call it, for convenience, a *D'Arson.*

We can say, also in accord with the above definition of sensibility, that the sensibility is unity when one microampere produces the standard deflection. Hence, if i_m = the microamperes in the galvanometer circuit,

$$S_m = \frac{1}{i_m}.$$

If E_m microvolts are applied at the terminals of the galvanometer of resistance R, we have

$$i_m = \frac{E_m}{R} \quad \text{or} \quad S_m = \frac{R}{E_m}.$$

Putting this value of S_m in Eq. (11) gives

$$F_v = \frac{\sqrt{R}}{E_m T^2}. \qquad (12)$$

The relation (12) defines the figure of merit of a galvanometer in terms of its resistance, period, and the number of microvolts applied at its terminals to produce the standard deflection.

As an example of the use of relation (11) suppose we have a galvanometer with a complete period of 5 seconds, a coil resistance of 400 ohms, and which deflects one scale division with one volt acting thru 500 megohms, then its figure of merit is

$$F = \frac{500}{5^2\sqrt{400}} = 1 \text{ D'Arson.}$$

If this galvanometer were given a longer period, by using a weaker suspension, its sensibility would be larger, but F would not be altered by this change. It is evident that the coil might be wound, using the same size wire, to a smaller resistance, but if this were done the mass of the coil would be less and hence T^2 would be smaller. Both of these changes would contribute to a greater figure of merit. On the other hand, a smaller resistance would mean a smaller number of turns which would reduce the sensibility and hence diminish the figure of merit. Thus it is always open to the designer to so choose the winding and proportion the coil and to so arrange the strength of the magnetic field and other factors that F shall be large. The success with which he does this determines in considerable measure the perfection of his design.

It must not be forgotten, however, that the number of D'Arsons possessed by a galvanometer is not necessarily a final measure of its fitness for actual service. It may possess faults of many kinds which more than offset a large figure of merit. Chief among such is zero shift and magnetic impurities in the system, which two, in fact, generally go together. There are also other common defects, as small coil clearance, inaccessibility of the parts, a poor optical system, an unproportional scale, a provoking tendency of the system to respond to small tremors, and a host of other minor defects which the user soon observes and condemns.

If it were not necessary to load a galvanometer system with a mirror or a pointer for the purpose of reading the deflections, it would be possible by proper designing and by a great diminution in the size of the moving parts to realize an instrument which would possess an enormous figure of merit as compared with an ordinary moving-coil galvanometer. This has in fact been done in the case of the Einthoven String Galvanometer, which has about 3000 times the number of D'Arsons of a good moving-coil galvanometer using a mirror and scale. We are led thus to the general consideration of what are the possibilities of obtaining a great figure of merit for galvanometers of the deflection type.

In every galvanometer of this type we may consider the moment of inertia of its moving system as made up of two parts: One part is the moment of inertia which is contributed by the mirror, the pointer, or whatever device may be attached to the system which is required for reading the deflections of the instrument. We

may make this reading device small but we cannot dispense with it altogether and preserve the instrument as a galvanometer of a type to which the name is ordinarily applied. Indeed, there are many practical considerations which soon put a limitation upon a continual diminution of mass of these parts. The other part is the moment of inertia which belongs to the moving wire or magnets, which constitute the effective working element of the system. This part of the total moment of inertia can be modified at will by the designer with the object of making the figure of merit of the galvanometer as large as possible. The question then arises, has the figure of merit a maximum value which the most skilful designing cannot exceed? If there were no " dead parts " attached to the system for reading deflections, then, theoretically, a galvanometer could be given, by proper designing, an indefinitely great figure of merit. This realization is obtained practically in the Einthoven String Galvanometer. But as long as galvanometers continue to be instruments the deflections of which are read with mirrors or pointers, there will be a theoretical maximum figure of merit which cannot be exceeded. The author has shown* that when we have chosen the moment of inertia of the winding of a rectangular coil, equal to the moment of inertia of the " dead parts," mirror or pointer, the resistance of the coil remaining always the same, we have designed the proportions of the coil such, that the galvanometer, in this respect, has the greatest figure of merit which it is possible to give it.

Every consideration shows that in starting out to design a galvanometer of any type which is to have a large number of D'Arsons, one should begin by carefully considering the selection of the mirror, pointer, or other contrivance essential to reading the deflections, so that this contrivance may have the least possible moment of inertia; for it is the mass of these " dead parts " which ultimately sets a limit to the maximum figure of merit obtainable. The dispensing of " reading parts " in the Einthoven String Galvanometer is the essential reason for its enormous number of D'Arsons. Similar considerations hold for oscillographs, wattmeters, pointer voltmeters and ammeters and other deflection instruments in which a high figure of merit is desired.

* See pages 255–256 *Jour. of the Franklin Institute*, October, 1910, "The Comparison of Galvanometers and a New Type of Flat-coil Galvanometer."

In the above discussion attention has been confined chiefly to galvanometers intended for use on constant current or nearly constant-current circuits. This is the condition which applies when galvanometers are used for measuring insulation resistance by direct deflection methods. It also applies, tho to a less extent, when galvanometers are used, with considerable resistance external to themselves, in Wheatstone and Kelvin double-bridge measurements of resistance. When, however, galvanometers are designed for use with thermocouples and for reading millivolt or microvolt drops over low resistances, many points of design, as the galvanometer resistance, damping, etc., should receive attention; but our limits will not permit a discussion of this phase of the subject. When dealing with galvanometers for use on constant potential circuits, it is preferable to use, as the expression for the figure of merit, the relation (12) given above. In this expression when $E_m = 1$ microvolt, $T = 1$ second, and $R = 1$ ohm, the figure of merit is unity, and then we may call the unit, a *microvolt D'Arson.*

We have collected in a table the essential characteristics of sixteen different well known types of galvanometers, for which see table at end of paragraph.

Number 16 is a galvanometer of the D'Arsonval type made by The Leeds and Northrup Company, which has the very high figure of merit of 2.80 D'Arsons. Its megohm sensibility, however, is but 121.8. Theoretically, a very fine suspension could be used until its sensibility reaches that of No. 13 which is 1750 megohms. Practically, however, this would not be feasible, because the suspension would be finer than any wire on the market except Wollaston wire. But were such a fine suspension used, the more serious difficulty would arise that the coil would then be influenced to a relatively great degree by traces of magnetic matter in the coil. This would produce a large zero shift on reversed deflections. Thus, for high sensibility work, as in cable testing, where the longer period is not too serious a disadvantage, galvanometer No. 13 would be a much better instrument to use, altho its figure of merit is but 11.4 per cent of that of No. 16.

In respect to Table I the following remarks may be added. For the Einthoven String Galvanometer (No. 1) the resistance of the string was reduced to its copper equivalent, and a magnifica-

tion of 100 was taken as the equivalent of a scale at 1000 scale divisions from a mirror.

The data for No. 2 was taken from Siemens and Halske's reprint No. 30, and the data for No. 4 was taken from an article by Dr. H. Sack in the same reprint. The data for No. 5 is from The Cambridge Scientific Instrument Company's catalogue. The data for No. 16 is based on the average of five instruments designed by the author, and made by The Leeds and Northrup Company in August, 1910.

The Weston Voltmeter (No. 7) shows a figure of merit of 0.115 D'Arson. In considering the meaning of this low figure, it must be remembered that a light system has to carry a pointer which must be heavy enough to be perfectly rigid. The same system, fitted with a mirror, would show a much higher figure of merit. The design of this instrument, as every one knows, is most scientifically worked out, and the fact that its figure of merit is low simply emphasizes the fact that one must exercise great caution against estimating the real worth of an instrument by this feature alone.

No.	Type of instrument	Method of reading	Critical resistance for damping	Instrument resistance R	Megohm sensibility S_m	Complete period T	Figure of merit in D'Arsons $F=\frac{S_m}{T^2\sqrt{R}}$
*1	Einthoven string.....	Microscope, 100 fold magnification	Aperiodic	287	5.2	0.01	3000.00
*2	Dubois Rubens, iron-clad moving magnet...............	Mirror	Aperiodic	290	12200.0	6.0	20.00
3	Siemens & Halske, high sensibility....	Mirror	120	200	2500.0	12.0	1.24
*4	Siemens & Halske, high sensibility....	Mirror	....	290	1220.0	6.0	2.00
*5	Ayrton-Mather......	Mirror	Undamped	20	52.6	3.5	0.96
6	R. W. Paul, single pivot millivoltmeter.............	Pointer	25	50	21.4	3.0	0.33
7	Weston, voltmeter...	Pointer	Aperiodic	75	0.25	0.5	0.115
8	Weston, portable galv	Pointer	3000	280	13.8	1.5	0.37
9	L. & N., type P......	Mirror	140	124	85.0	8.5	0.10
10	L. & N., marine type	Mirror	Aperiodic	1660	21.4	2.0	0.131
11	L. & N., No. 2300 standard four coil..	Pointer	300	300	15.54	2.5	0.14
12	L. & N., type H.....	Mirror	Aperiodic	544	295.0	7.0	0.26
13	L. & N., No. 2280 wide coil..........	Mirror	Aperiodic	1420	1750.0	12.0	0.32
14	L. & N., steel magnet	Pointer	500	230	8.8	1.3	0.344
15	L. & N., special small four coil...........	Mirror	500	85	6.4	1.0	0.695
16	L. & N., No. 2280 narrow coil...........	Mirror	200	180	121.8	1.8	2.80

* Data not obtained by author.

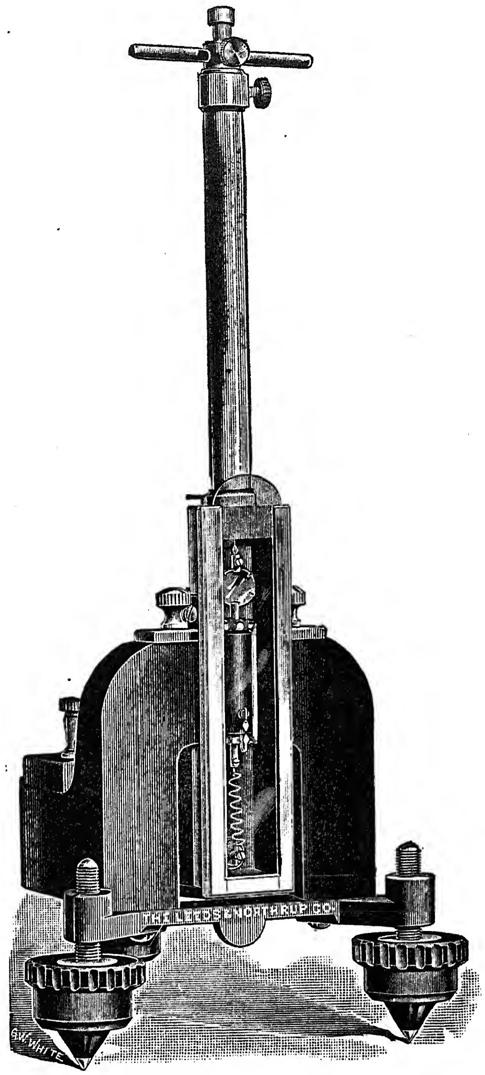

FIG. 1505.

1505. Description of One Type of High-sensibility Galvanometer. — As it is not our purpose to discuss the structural details of galvanometers as made by the various instrument makers we can do no more in this respect than briefly describe, and give an illustration of, a single type of high-sensibility galvanometer

(galvanometer 2280; Nos. 13 and 16 in the above table). The instrument is shown in Fig. 1505. It is furnished with a coil of medium width when intended for use in insulation testing and where high sensibility is more important than a quick working period. It may also be furnished with a very narrow coil, when it becomes best adapted to Wheatstone-bridge work, low resistance measurements by the Kelvin double-bridge principle and general laboratory work in connection with potentiometers, thermocouples, etc.

Its more prominent features may be summarized as follows:

The tube which contains the coil system makes one unit and the magnet another, so different tubes with coils of different characteristics may be fitted to the same magnet.

The coil system is always exposed to full view.

The deflections are very closely proportional to the current passed thru the coil.

The damping of the system may be varied thru wide limits by removable copper rectangles which fit upon the coil.

The suspended system may be locked by a clamping device when the instrument is to be carried about.

The instrument as a whole is very highly insulated from ground by hard rubber, petticoat-insulated leveling screws.

The figure of merit is high, the narrow coil type reaching 2.8 D'Arsons, while the wider coil type of 1500 ohms resistance has a megohm sensibility of 1700 megohms with a period of 12 seconds.

The suspension is easily replaced if broken, an event which is made unlikely by means of a protecting spring at the top of the suspension tube.

APPENDIX

I.—TABLE.

(1) *Values of* $\dfrac{a}{1000 - a}$.

100	10	Units										
			0	1	2	3	4	5	6	7	8	9
	0	.00	0000	1001	2004	3010	4016	5025	6036	7049	8064	9082
	1	.0	1010	1112	1214	1317	1420	1523	1626	1730	1833	1937
	2	.0	2041	2145	2250	2354	2459	2564	2670	2775	2881	2987
	3	.0	3093	3199	3306	3413	3520	3627	3735	3843	3950	4058
	4	.0	4167	4275	4384	4493	4602	4712	4820	4931	5042	5152
	5	.0	5263	5374	5485	5596	5708	5820	5932	6044	6156	6269
	6	.0	6383	6496	6610	6724	6838	6952	7066	7180	7296	7412
	7	.0	7527	7643	7759	7875	7992	8109	8225	8342	8460	8578
	8	.0	8696	8814	8933	9051	9170	9290	9408	.9528	9649	9770
	9	.0	9890	.1001	.1013	.1025	.1037	.1050	.1062	.1074	.1086	.1099
1	0		.1111	.1123	.1136	.1148	.1160	1173	.1186	.1198	.1211	.1223
1	1		.1236	.1248	.1261	.1274	.1287	.1300	.1312	.1325	.1338	.1351
1	2		.1364	.1377	.1390	.1403	.1416	.1429	.1442	.1455	.1468	.1481
1	3		.1494	.1507	.1521	.1534	.1547	.1561	.1574	.1587	.1601	.1615
1	4		.1628	.1641	.1655	.1669	.1682	.1695	.1710	.1723	.1737	.1751
1	5		.1765	.1778	.1792	.1806	.1821	.1834	.1848	.1862	.1876	.1890
1	6		.1905	.1919	.1933	.1947	.1962	.1976	.1990	.2005	.2019	.2034
1	7		.2048	.2063	.2077	.2092	.2106	.2121	.2136	.2151	.2165	.2180
1	8		.2195	.2210	.2225	.2240	.2255	2270	.2285	.2300	.2315	.2331
1	9		.2346	.2361	.2376	.2392	.2407	.2423	.2438	.2454	.2469	.2485
2	0		.2500	.2516	.2532	.2547	.2563	.2579	.2595	.2610	.2625	.2642
2	1		.2658	.2674	.2690	.2706	.2722	.2739	.2755	.2772	.2788	.2804
2	2		.2820	.2837	.2853	.2870	.2887	.2903	.2920	.2937	.2954	.2971
2	3		.2987	.3004	.3020	.3038	.3055	.3072	.3089	.3106	.3123	.3140
2	4		.3157	.3175	.3192	.3210	.3228	.3245	.3262	.3280	.3298	.3316
2	5		.3333	.3351	.3369	.3387	.3405	3423	.3440	.3459	.3477	.3495
2	6		.3513	.3532	.3550	.3568	.3587	.3606	.3624	3643	.3662	.3681
2	7		.3699	.3717	.3736	.3755	.3774	.3793	.3812	.3831	.3850	.3869
2	8		.3889	.3908	.3928	.3947	.3966	.3986	.4005	.4024	.4044	.4064
2	9		.4084	.4104	.4124	.4144	.4164	.4185	.4205	.4225	.4245	.4265
3	0		.4285	.4306	.4326	.4347	.4368	4389	.4409	.4430	.4450	.4471
3	1		.4493	.4514	.4535	.4556	.4577	.4598	.4619	4640	.4661	.4683
3	2		.4705	.4727	.4749	.4771	.4793	.4814	.4836	.4858	.4881	.4903
3	3		.4925	.4947	.4969	.4992	.5015	.5038	.5060	.5083	.5106	.5129
3	4		.5152	.5174	.5197	.5220	.5244	.5267	.5290	.5313	.5336	.5360
3	5		.5384	.5407	.5431	.5455	.5480	.5504	.5528	5553	.5576	.5600
3	6		.5625	.5650	.5674	.5698	.5723	.5748	.5773	5798	.5823	.5848
3	7		.5873	.5899	.5924	.5949	.5974	.6000	.6025	6051	.6077	.6103
3	8		6129	.6155	.6181	.6207	.6233	.6260	.6286	.6313	.6340	.6367
3	9		.6394	.6420	.6447	.6474	.6502	.6529	.6557	.6584	.6611	.6638
4	0		.6666	.6694	.6722	.6750	.6778	.6806	.6834	6862	.6891	.6920
4	1		6949	.6978	.7007	.7036	.7065	.7094	.7123	.7152	.7181	.7211
4	2		.7241	.7271	.7301	.7331	.7361	.7391	.7421	.7451	.7482	.7512
4	3		.7543	.7574	7605	.7636	.7667	.7698	.7729	.7760	.7792	.7824
4	4		7857	.7889	.7921	.7953	.7986	.8018	.8050	.8084	.8117	.8150
4	5		8182	.8215	8248	.8282	.8316	.8349	.8382	.8416	.8450	.8484
4	6		.8518	.8552	8586	.8620	.8655	.8691	.8727	.8762	.8798	.8834
4	7		8868	.8904	.8939	.8975	.9011	.9048	.9084	.9120	.9157	.9194
4	8		.9231	.9267	.9304	.9341	.9379	.9417	.9454	.9493	.9531	.9570
4	9		9609	.9649	9687	.9725	.9764	.9803	.9842	.9881	.9920	.9960

Values of $\dfrac{a}{1000 - a}$.

100	10	Units 0	1	2	3	4	5	6	7	8	9
5	0	1.000	1.004	1.008	1.012	1.016	1.020	1.024	1.028	1.032	1.036
5	1	1.041	1.045	1.049	1.053	1.058	1.062	1.066	1.071	1.075	1.079
5	2	1.083	1.088	1.092	1.097	1.101	1.105	1.110	1.114	1.119	1.123
5	3	1.128	1.132	1.137	1.141	1.146	1.151	1.155	1.160	1.165	1.169
5	4	1.174	1.179	1.183	1.188	1.193	1.198	1.203	1.208	1.212	1.217
5	5	1.222	1.227	1.232	1.237	1.242	1.247	1.252	1.257	1.262	1.267
5	6	1.273	1.278	1.283	1.288	1.294	1.299	1.304	1.309	1.314	1.320
5	7	1.326	1.331	1.336	1.342	1.347	1.353	1.359	1.364	1.370	1.375
5	8	1.381	1.386	1.392	1.398	1.404	1.410	1.415	1.421	1.427	1.433
5	9	1.439	1.445	1.451	1.457	1.463	1.469	1.475	1.481	1.487	1.494
6	0	1.500	1.506	1.513	1.519	1.525	1.531	1.538	1.544	1.551	1.557
6	1	1.564	1.571	1.577	1.584	1.591	1:597	1.604	1.611	1.618	1.625
6	2	1.632	1.639	1.645	1.652	1.659	1.667	1.674	1.681	1.688	1.695
6	3	1.703	1.710	1.717	1.724	1.732	1.740	1.747	1.755	1.763	1.770
6	4	1.778	1.786	1.793	1.801	1.809	1.817	1.825	1.833	1.841	1.849
6	5	1.857	1.865	1.873	1.882	1.890	1.899	1.907	1.916	1.924	1.933
6	6	1.941	1.950	1.958	1.967	1.976	1.985	1.994	2.003	2.012	2.021
6	7	2.030	2.039	2.048	2.058	2.068	2.078	2.087	2.096	2.106	2.115
6	8	2.125	2.135	2.145	2.155	2.165	2.175	2.185	2.195	2.205	2.215
6	9	2.225	2.236	2.247	2.257	2.268	2.278	2.289	2.300	2.311	2.322
7	0	2.333	2.344	2.355	2.367	2.378	2.389	2.401	2.413	2.425	2.436
7	1	2.448	2.460	2.472	2.485	2.497	2.509	2.521	2.534	2.546	2.559
7	2	2.571	2.584	2.597	2.610	2.623	2.636	2.650	2.663	2.676	2.690
7	3	2.703	2.716	2.731	2.745	2.759	2.774	2.788	2.802	2.817	2.831
7	4	2.846	2.861	2.876	2.891	2.907	2.922	2.937	2.953	2.968	2.984
7	5	3.000	3.016	3.032	3.049	3.065	3.081	3.098	3.115	3.132	3.150
7	6	3.168	3.185	3.202	3.220	3.237	3.255	3.273	3.291	3.310	3.329
7	7	3.348	3.367	3.386	3.405	3.425	3.445	3.464	3.484	3.505	3.525
7	8	3.546	3.566	3.587	3.608	3.630	3.652	3.674	3.695	3.717	3.740
7	9	3.762	3.785	3.808	3.831	3.854	3.878	3.902	3.926	3.950	3.975
8	0	4.000	4.025	4.050	4.075	4.102	4.127	4.154	4.181	4.209	4.236
8	1	4.263	4.290	4.319	4.348	4.376	4.405	4.435	4.464	4.494	4.525
8	2	4.556	4.587	4.618	4.650	4.682	4.715	4.748	4.781	4.814	4.848
8	3	4.882	4.917	4.953	4.988	5.025	5.061	5.097	5.135	5.173	5.211
8	4	5.250	5.290	5.330	5.370	5.411	5.451	5.493	5.536	5.580	5.623
8	5	5.666	5.711	5.757	5.803	5.850	5.898	5.945	5.994	6.043	6.093
8	6	6.143	6.194	6.247	6.300	6.353	6.407	6.463	6.519	6.576	6.634
8	7	6.693	6.752	6.812	6.873	6.937	7.000	7.064	7.129	7.196	7.264
8	8	7.334	7.403	7.474	7.546	7.620	7.696	7.772	7.849	7.928	8.009
8	9	8.091	8.175	8.259	8.346	8.434	8.524	8.616	8.709	8.804	8.901
9	0	9.000	9.101	9.204	9.309	9.416	9.526	9.638	9.753	9.870	9.989
9	1	10.11	10.23	10.36	10.49	10.63	10.76	10.90	11.05	11.19	11.34
9	2	11.50	11.66	11.82	11.99	12.16	12.33	12.51	12.70	12.89	13.08
9	3	13.28	13.49	13.71	13.93	14.15	14.38	14.62	14.87	15.13	15.40
9	4	15.66	15.95	16.24	16.54	16.86	17.18	17.52	17.87	18.23	18.61
9	5	19.00	19.41	19.83	20.28	20.75	21.22	21.73	22.26	22.81	23.38
9	6	24.00	24.64	25.32	26.03	26.77	27.57	28.41	29.30	30.25	31.26
9	7	32.33	33.49	34.70	36.04	37.46	39.00	40.67	42.48	44.44	46.62
9	8	49.00	51.63	54.55	57.83	61.50	65.67	70.43	75.93	82.33	89.91
9	9	99.00	110.1	124.0	141.9	165.7	199.0	249.0	332.3	499.0	999.0

II. — MATHEMATICAL QUANTITIES AND RELATIONS.

(1) *Functions of π and e.*

π is the ratio of the circumference to the diameter of a circle.
e is the basis of the natural logarithms.

$$\pi = 3.14159$$
$$\frac{1}{\pi} = 0.31830$$
$$\pi^2 = 9.86960$$
$$\sqrt{\pi} = 1.77245$$
$$\log_{10} \pi = 0.4971498$$
$$\log_e \pi = 1.1447298$$
$$e = 2.71828$$
$$\frac{1}{e} = 0.36788$$
$$\log_{10} e = 0.4342944$$
$$\log_e e = 1$$
$$\log_e 10 = 2.3025850$$
$$\log_{10} x = 0.43429 \log_e x.$$
$$\log_e x = 2.30258 \log_{10} x.$$
$$\text{If } x = e^y, \text{ then } y = \log_e x.$$

(2) *English-Metric and Metric-English Conversions.*

1 inch = 2.54001 centimeters.
1 centimeter = 0.3937 inch.
1 foot = 0.304801 meter.
1 meter = 3.28083 feet.
1 mile = 1.60935 kilometers.
1 kilometer = 0.62137 mile.

1 square inch = 6.452 square centimeters.
1 square centimeter = 0.1550 square inch.

1 cubic inch = 16.3872 cubic centimeters.
1 cubic centimeter = 0.0610 cubic inch.

1 U. S. liquid gallon = 3.78543 liters = 231 cubic inches.
1 liter = 0.26417 U. S. liquid gallon.

1 avoirdupois pound = 0.45359 kilogram.
1 kilogram = 2.20462 avoirdupois pounds.
1 avoirdupois ounce = 28.3495 grams.
1 gram = 0.03527 avoirdupois ounce.

(3) *Formulæ for the Conversion of Temperature Scales.*

If F = Fahrenheit degrees and C = Centigrade degrees, then

$C = \frac{5}{9}(F - 32).$ $0°$ C. $= + 32°$ F.

$F = \frac{9}{5}C + 32.$ $0°$ F. $= - 17.777°$ C.

$20°$ C. $= + 68°$ F.

$- 40°$ C. $= - 40°$ F.

$100°$ C. $= 212°$ F.

(4) *Formulæ for Temperature Coefficients.*

If a linear relation exists between electrical resistance (or resistivity) and temperature (only true, usually, to a first approximation and for short ranges of temperature) then these relations hold,

$$\rho_t = \rho_0 (1 + \alpha t), \tag{1}$$

$$\rho_{t_1} = \rho_0 (1 + \alpha t_1), \tag{2}$$

$$\alpha = \frac{\rho_{t_1} - \rho_t}{t_1 \rho_t - t \rho_{t_1}}, \tag{3}$$

$$\alpha = \frac{\rho_t - \rho_0}{t \rho_0}, \tag{4}$$

where ρ_0 = resistance (or resistivity) at temperature 0 degrees,
ρ_t = resistance (or resistivity) at temperature t degrees,
ρ_{t_1} = resistance (or resistivity) at temperature t_1 degrees,
and α = temperature-resistance coefficient which, by the assumption made, is a constant quantity.

If a linear relation between resistance and temperature does not hold, then the resistance (or resistivity) at temperature t can be expressed, sometimes, in terms of the resistance (or resistivity) at 0 degrees by the relation

$$\rho_t = \rho_0 (1 + at + bt^2), \tag{5}$$

where a and b are constants which can be determined by experiment.

Let
$$\sigma_t = \sigma_0 (1 - \beta t) \tag{6}$$

express the variation of electrical conductivity with temperature, upon the assumption that relation (1) above holds, then

$$\beta = \frac{\alpha}{\alpha t + 1}, \tag{7}$$

and

$$\alpha = \frac{\beta}{1 - \beta t}. \tag{8}$$

(5) *Relations between Resistance and Conductivity.* (Consult Chapter VII.)

$$C_s = \frac{0.0203\, l\theta}{R_t d^2} 100, \tag{1}$$

or

$$C_s = \frac{0.01594\, l\theta}{R_t S} 100. \tag{2}$$

C_s = per cent conductivity by Matthiessen's meter-millimeter standard.

l = length of conductor in meters.

d = diameter in millimeters of conductor of uniform and circular cross-section.

S = cross-section in square millimeters of conductor of uniform cross-section.

R_t = resistance in ohms of a length l meters at temperature t degrees C.

$\theta = \frac{\rho_t}{\rho_0}$ = a temperature coefficient, or the ratio of the resistivity of the material at temperature t degrees C. to the resistivity of the material at 0 degrees C.

When the temperature is 20° C. $\theta = 1.07968$ for copper.

Use for calculation the value $\theta = 1.08$.

It is nearly the same for all pure metals.

If the cross-section is given in circular mils and is called V and we write

$$S = \frac{V}{1973.52}, \tag{3}$$

then

$$C_s = \frac{31.457\, l\theta}{R_t V} 100, \tag{4}$$

and

$$C_w = \frac{0.14173\, l^2\theta}{R_t W} 100. \tag{5}$$

C_w = per cent conductivity by Matthiessen's meter-gram standard.

W = weight of l meters expressed in grams.

$$C_w' = \frac{8.89}{\delta} C_s', \tag{6}$$

where C_w' and C_s' stand for conductivities instead of per cent conductivities and

δ = density of the material of the conductor.

Example (1).

A copper wire is 2 mm in diameter and 4 m long, and has a resistance at 20° C. of 0.0223 ohm. By Eq. (1) its per cent conductivity by Matthiessen's meter-millimeter standard is

$$C_s = \frac{0.0203 \times 4 \times 1.08}{0.0223 \times 4} 100 = 98.76 \text{ per cent conductivity.}$$

Example (2).

An aluminum wire is 2 m long and weighs 50 grams. Its resistance at 0° C. is 0.005585 ohm. Then $\theta = 1$ and by Eq. (5) its per cent conductivity by Matthiessen's meter-gram standard is

$$C_w = \frac{0.14173 \times 4 \times 1}{0.005585 \times 50} 100 = 203 \text{ per cent conductivity.}$$

Example (3).

The density of aluminum is 2.7 and its conductivity is 0.615 by the meter-millimeter standard, then by Eq. (6) its conductivity by the meter-gram standard is

$$C_w' = \frac{8.89}{2.7} \times 0.615 = 2.0249 \text{ conductivity.}$$

For bimetallic conductors, as copper or aluminum conductors, with a steel core we have the following relations:

$$C_s = \frac{SC_s'' + sC_s'}{S + s} 100. \tag{7}$$

S = cross-section of the outside metal and
s = cross-section of core of the bimetallic conductor.

S and s may be expressed in terms of any convenient unit provided the same unit is chosen for each.

C_s'' = conductivity of the outside metal and
C_s' = conductivity of the metal of the core, both by the meter-millimeter standard.

Example (4).

A conductor is made up of a steel core which has a conductivity, by the meter-millimeter standard, 0.16, and of an aluminum covering which has a conductivity 0.61. The cross-section of the core is 2000 circular mils and that of the outside metal is 4000 circular mils. Then by Eq. (7) the per cent

conductivity by the meter-millimeter standard of the bimetallic conductor is

$$C_s = \frac{4000 \times 0.61 + 2000 \times 0.16}{4000 + 2000} 100 = 46 \text{ per cent conductivity.}$$

$$C_w = \frac{W_1 C_w'' + W_2 C_w'}{W_1 + W_2} 100. \tag{8}$$

W_1 = the weight of the outside metal and
W_2 = the weight of the core.

These weights may be expressed in any unit, provided the same unit is used for both.

C_w'' = conductivity of outside metal and
C_w' = conductivity of core; both by the meter-gram standard.

Example (5).

A conductor has a steel core of conductivity 0.20 by the meter-gram standard, and an aluminum covering of conductivity 2 by the same standard.

For a given length of the conductor the steel core weighs 10 lbs., and the aluminum covering 8 lbs. Then by Eq. (8)

$$C_w = \frac{8 \times 2 + 10 \times 0.20}{8 + 10} 100 = 100 \text{ per cent conductivity.}$$

$$C_s = \frac{q\, \delta_2 C_w'' + \delta_1 C_w'}{8.89\,(q + 1)} 100. \tag{9}$$

q = ratio of the cross-section of the outside metal to the cross-section of the core.
δ_2 = density of the outside metal.
δ_1 = density of the metal in the core.
C_w' and C_w'' have the same meaning as in Eq. (8).

Example (6).

In the conductor under example (5) the density δ_1 of the steel core is 7, and the density δ_2 of the aluminum covering is 2.7. The ratio of the cross-section of the outside metal to that of the core is, for the case chosen,

$$q = \frac{S}{s} = \frac{W_1 \delta_1}{W_2 \delta_2} = \frac{8 \times 7}{10 \times 2.7} = 2.074 +.$$

As in example (5) taking $C_w' = 0.2$ and $C_w'' = 2$ then by Eq. (9) the per cent conductivity by the meter-millimeter standard is

$$C_s = \frac{2.074 \times 2.7 \times 2 + 7 \times 0.2}{8.89 \times (2.074 + 1)} = 46.10 \text{ per cent conductivity.}$$

$$R_t = \frac{0.01594\, l\theta''}{SC_s'' + sC_s'}. \tag{10}$$

$\theta'' = \frac{R_t}{R_0}$ = the temperature coefficient of the bimetallic conductor.

R_t = ohmic resistance in ohms at t degrees centigrade of l meters of the bimetallic conductor.

Other quantities have the same meaning as above.

Example (7).

A bimetallic conductor 304.8 m long has a steel core 16.72 sq. mm cross-section and a copper covering 25.59 sq. mm. cross-section. The conductivity by the meter-millimeter standard of the core is 0.16, and of the covering 1.00. If at 20° C. the coefficient θ'' is found to equal 1.09, then by Eq. (10) the resistance of this conductor at 20° C. is

$$R_{20} = \frac{0.01594 \times 304.8 \times 1.09}{25.59 \times 1 + 16.72 \times 0.16} = 0.1873 + \text{ohm.}$$

$$R_t = \frac{0.14173\, l^2\theta''}{C_w''W_1 + C_w'W_2}. \tag{11}$$

Example (8).

A bimetallic conductor is 1000 m long. The weight W_1 of its steel core is 180,000 grams, and the weight of its aluminum covering is 59,100 grams. The conductivity of the core by the meter-gram standard is 0.2, and of the covering 2.03. At 20° C. its temperature coefficient is found to be 1.09. Then by Eq. (11) its resistance is

$$R_{20} = \frac{0.14173 \times 1000^2 \times 1.09}{2.03 \times 59100 + 0.2 \times 180000} = 0.9903 \text{ ohm.}$$

$$\$ = \frac{\$_1}{W_1} WC_w. \tag{12}$$

\$ = value in dollars of a certain length of a certain conductor which weighs W lbs., and has a conductivity C_w by the meter-gram standard, if $\$_1$ = the value in dollars of the same length of 100 per cent conductivity (by the meter-gram standard) conductor which weighs W_1 lbs.

Example (9).

Suppose that 100 lbs. of a certain length of 100 per cent conductivity copper is worth \$20, then by Eq. (12) 250 lbs. of the same length of aluminum wire of 2.03 conductivity or 203 per cent conductivity, by the meter-gram standard, is worth

$$\$ = \frac{20 \times 250 \times 2.03}{100} = \$101.50.$$

This is the value of the aluminum as compared with the copper wire when considered *only* as a conveyer of electric power. Equal weights of the same length of aluminum and copper have values, as conveyers of electric power, in the ratio of about 2 to 1.

$$\$' = \frac{\$_{11}}{S_1} SC_s. \tag{13}$$

$\$'$ = the value in dollars of a certain length of a conductor which has a cross-section S and a conductivity C_s by the meter-millimeter standard, when $\$_{11}$ = the value in dollars of the same length of 100 per cent conductivity conductor (by the meter-millimeter standard) which has a cross-section S_1.

Example (10).

Suppose that 100 feet of copper conductor of 105,625 circular mils is worth \$64, then by Eq. (13) 100 feet of aluminum conductor of conductivity 0.6, by the meter-millimeter standard, and 133,225 circular mils is worth

$$= \frac{64}{105625} \times 133225 \times 0.6 = \$48.44.$$

Equal cross-sections of the same lengths of aluminum and copper have values as conveyers of electric power in the ratio of about 6 to 10.

(6) *Conversions from Practical, to Electrostatic, to Electromagnetic Units.* (C.G.S. System.)

v = velocity of light = 3×10^{10} centimeters per second.

N = any number. Elst = electrostatic. Elmg = electromagnetic.

N coulombs = $N\,3 \times 10^9$ Elst units = $N\,10^{-1}$ Elmg units.

1 Elst unit of electricity = v^{-1} Elmg unit.

N volts $= \frac{N}{3} \times 10^{-2}$ Elst units = $N\,10^8$ Elmg units.

1 Elst unit of potential difference $= v$ Elmg units.
N amperes $= N\, 3 \times 10^9$ Elst units $= N\, 10^{-1}$ Elmg units.
1 Elst unit of electric current $= v^{-1}$ Elmg unit.

N ohms $= \dfrac{N}{9} \times 10^{-11}$ Elst units $= N\, 10^9$ Elmg units.

1 Elst unit of resistance $= v^2$ Elmg units.
N farads $= N\, 9 \times 10^{11}$ Elst units $= N\, 10^{-9}$ Elmg units.
N microfarads $= N\, 9 \times 10^5$ Elst units $= N\, 10^{-15}$ Elmg units.
1 Elst unit of capacity $= v^{-2}$ Elmg unit.

N henrys $= \dfrac{N}{9} \times 10^{-11}$ Elst units $= N\, 10^9$ Elmg units.

1 Elst unit of self-induction $= v^2$ Elmg units.

Examples: 25 ohms $= \dfrac{25}{9} \times 10^{-11}$ Elst units $= 25 \times 10^9$ Elmg units.

5 microfarads $= 5 \times 9 \times 10^5$ Elst units of capacity,

or 5 Elst units of capacity $= \dfrac{5}{9 \times 10^5}$ microfarads.

(7) *Approximation Formulæ.** *Certain other Expressions.*

In a mathematical expression it sometimes occurs that some quantities are very small compared with others. In such cases the expression may often be given a form which is more convenient for calculation if formulæ of approximation are used.

In the expressions considered let a, b, c, d, etc., be magnitudes which are very small as compared with unity. The formulæ of approximation may then be given forms such that the corrections are contained in terms which are added to or subtracted from 1.

The following formulæ can be shown to hold upon the above supposition, and will often be found convenient to use. Where the sign $\pm$ or $\mp$ is used before a quantity, either the upper or lower sign must be taken all thru the formula.

In general (when a is very small compared with 1)

$$(1 + a)^m = 1 + ma; \; (1 - a)^m = 1 - ma. \qquad (1)$$

For $m = 2$

$$(1 + a)^2 = 1 + 2a; \; (1 - a)^2 = 1 - 2a. \qquad (2)$$

For $m = \frac{1}{2}$

$$\sqrt{1 + a} = 1 + \tfrac{1}{2}a; \; \sqrt{1 - a} = 1 - \tfrac{1}{2}a. \qquad (3)$$

* The substance of what is given under this head has been selected from "Physical Measurements," by Dr. F. Kohlrausch.

For $m = -1$

$$\frac{1}{1+a} = 1 - a; \quad \frac{1}{1-a} = 1 + a. \tag{4}$$

For $m = -2$

$$\frac{1}{(1+a)^2} = 1 - 2a; \quad \frac{1}{(1-a)^2} = 1 + 2a. \tag{5}$$

For $m = -\frac{1}{2}$

$$\frac{1}{\sqrt{1+a}} = 1 - \tfrac{1}{2}a; \quad \frac{1}{\sqrt{1-a}} = 1 + \tfrac{1}{2}a. \tag{6}$$

$$(1 \pm a)(1 \pm b)(1 \pm c) \ldots = 1 \pm a \pm b \pm c \ldots \tag{7}$$

$$\frac{(1 \pm a)(1 \pm c) \ldots}{(1 \pm b)(1 \pm d) \ldots} = 1 \pm a \pm c \ldots \mp b \mp d \ldots \tag{8}$$

For the geometrical mean of two quantities, which are very nearly alike, the arithmetical mean may be used. Thus

$$\sqrt{p_1 p_2} = \frac{p_1 + p_2}{2}. \tag{9}$$

If δ signifies a small angle measured in radians (1 radian = 57.2958 degrees) then,

$$\sin(x + \delta) = \sin x + \delta \cos x; \ \sin \delta = \delta, \tag{10}$$

$$\cos(x + \delta) = \cos x - \delta \sin x; \ \cos \delta = 1, \tag{11}$$

$$\tan(x + \delta) = \tan x + \frac{\delta}{\cos^2 x}; \quad \tan \delta = \delta. \tag{12}$$

Also, if a quantity a is very small compared with a quantity $x > 1$, then

$$\log_e(x + a) = \log_e x + \frac{a}{x}; \quad \log_e(1 + a) = a. \tag{13}$$

The true value of $(a + b)^n$ is given by the expansion

$$(a + b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{\underline{|2}}a^{n-2}b^2 + \cdots \tag{14}$$

The exact value of $(1 \pm a)^m$ when $a^2 < 1$ is given by the expansion

$$(1 \pm a)^m = 1 \pm ma + \frac{m(m-1)}{\underline{|2}}a^2 \pm \frac{m(m-1)(m-2)a^3}{\underline{|3}} + \cdots \tag{15}$$

Any quadratic may be put in the form $x^2 + px = q$. Its solution is then

$$x = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} + q}. \tag{16}$$

III. — WIRE DATA AND FORMULÆ.

(1) *Wire Table for Pure Copper; from Standard Underground Cable Co.*

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B. & S. G. No.	Diameter in mils	Area circular mils	Length, feet per ohm	Resistance in International ohms at 68° F.=20° C. Ohms per 1000 feet
0000	460.0	211,600	20,440	0.04893
000	409.6	167,800	16,210	0.06170
00	364.8	133,100	12,850	0.07780
0	324.9	105,500	10,190	0.09811
1	289.3	83,690	8,083	0.1237
2	257.6	66,370	6,410	0.1560
3	229.4	52,630	5,084	0.1967
4	204.3	41,740	4,031	0.2480
5	181.9	33,100	3,197	0.3128
6	162.0	26,250	2,535	0.3944
7	144.3	20,820	2,011	0.4973
8	128.5	16,510	1,595	0.6271
9	114.4	13,090	1,265	0.7908
10	101.9	10,380	1,003	0.9972
11	90.74	8,234	795.3	1.257
12	80.81	6,530	630.7	1.586
13	71.96	5,178	500.1	1.999
14	64.08	4,107	396.6	2.521
15	57.07	3,257	314.5	3.179
16	50.82	2,583	249.4	4.009
17	45.26	2,048	197.8	5.055
18	40.30	1,624	156.9	6.374
19	35.89	1,288	124.4	8.038
20	31.96	1,022	98.66	10.14
21	28.46	810.1	78.24	12.78
22	25.35	642.4	62.05	16.12
23	22.57	509.5	49.21	20.32
24	20.10	404.0	39.02	25.63
25	17.90	320.4	30.95	32.31
26	15.94	254.1	24.54	40.75
27	14.20	201.5	19.46	51.38
28	12.64	159.8	15.43	64.79
29	11.26	126.7	12.24	81.70
30	10.03	100.5	9.707	103.0
31	8.928	79.70	7.698	129.9
32	7.950	63.21	6.105	163.8
33	7.080	50.13	4.841	206.6
34	6.305	39.75	3.839	260.5
35	5.615	31.52	3.045	328.4
36	5.000	25.00	2.414	414.2
37	4.453	19.83	1.915	522.2
38	3.965	15.72	1.519	658.5
39	3.531	12.47	1.204	830.4
40	3.145	9.888	0.9550	1047.0

No. 10 pure copper wire weighs 31.43 pounds per 1000 feet.

(2) *The Ohm.*

1 international ohm = 1.06300 Siemens units.
1 international ohm = 1.01348 British Association (B.A.) units.
1 international ohm = 1.00283 legal ohms.
1 international ohm = resistance at 0° C. of column of pure mercury, 106.3 cms long, weighing 14.4521 grams, and of uniform cross-section.
1 British Association unit (B.A.U.) = 0.986699 international ohm.
1 legal ohm = 0.997178 international ohm.

(3) *Resistance of Wire Wound in a Channel.*

The following formula will enable the resistance to be calculated for any wire with any thickness of insulation when wound to fill a channel of any given volume. The formula assumes the wire is wound in square order and in a regular manner. The formula is approximate to the extent that terms which involve the square of the thickness of the insulation are neglected We have

$$R_t = \frac{1.27\ V\rho_t}{d^4\left(1 + \frac{2\,h}{d}\right)}. \tag{1}$$

In relation (1) R_t = ohms at $t°$ to which wire winds,
V = total volume of channel filled with wire,
ρ_t = resistivity of wire at $t°$,
d = diameter of wire,
h = twice the thickness of the insulation.

Example. — To what resistance will No. 26 B. & S. copper wire wind per cubic centimeter, when its temperature is 20° C., and the double thickness of its insulation is 5 mils?

In this case

$V = 1$ cu. cm,
$\rho_{20} = 1.594 \times 1.08 \times 10^{-6}$,
$d = 0.0405$ cm,
$h = 0.0127$ cm (5 mils).

These values placed in relation (1) give

$R_t = 0.499$ ohm.

(4) *Certain Formulæ for Wire.**

The wire is pure copper and at 20° C.; bare.

(Ohms per 1000 feet) = 10354 ÷ (area in cir. mils).

(Pounds per 1000 feet) = 0.0030269 × (area in cir. mils).

(Feet per pound) = 330,360 ÷ (area in cir. mils).

(Ohms per pound) = 3,420,400 ÷ (area in cir. mils)^2.

(Feet per ohm) = 0.096585 × (area in cir. mils).

Area in circular mils = $\overline{\text{diameter in mils}}^2$.

For approximate calculations it may be easily remembered that

1000 feet No. 10 wire = 1 ohm,
1000 feet No. 13 wire = 2 ohms,
1000 feet No. 16 wire = 4 ohms, etc.,
1000 feet No. 7 wire = $\frac{1}{2}$ ohm,
1000 feet No. 4 wire = $\frac{1}{4}$ ohm,
1000 feet No. 1 wire = $\frac{1}{8}$ ohm, etc.

To change microhms per centimeter-cube to ohms per mil-foot multiply by 6.014. To change ohms per mil-foot to microhms per centimeter cube multiply by 0.166. (Statement taken from Catalogue J, Driver-Harris Wire Company.) †

IV.—PHYSICAL DATA.

(1) *Resistivity of Mercury.*

The following data was furnished the author by the Bureau of Standards. (In his own work the author has used the value, volume resistivity of mercury at 20° C. = 95.782.)

The values for the resistivity of mercury are those computed from the temperature formulæ of F. E. Smith of the National Physical Laboratory of England (*Phil. Trans.* 204, p. 112; 1904), C. E. Guillaume of the International Bureau (*Comptes Rendus*, 115, p. 414; 1892), and Kreichgauer and Jæger of the Reichsanstalt (*Wied. Ann.*, 47, 527; 1892). In the following table volume resistivity is given in microhms per centimeter cube and mass resistivity is given in ohms per meter-gram.

* The constants under this heading are reproduced with the permission of the Standard Underground Cable Company, of Pittsburgh, Pa., from their "XVII Hand-Book, Standard Underground Cable Company, Copyright, 1906."

† Since the material given in Appendix III was prepared, the Bureau of Standards has issued circular No. 31, entitled "Copper Wire Tables." This circular should be consulted for the most authoritative information upon wire sizes, standards, and temperature coefficients.

	Volume resistivity			Mass resistivity		
	0° C.	20° C.	100° C.	0° C.	20° C.	100° C.
Smith............	94.073	95.783	103.410	12.7898	12.9751	13.8076
Guillaume........	94.073	95.782	103.379	12.7898	12.9749	13.8034
Kreichgauer and Jæger...........	94.073	95.782	103.56	12.7898	12.9748	13.828

The 0° C. values are computed from the quantities given in the definition of the international ohm, 14.4521 grams mass and 106.300 centimeters length; and, in the case of volume resistivity at 0° C., the density is assumed to be such as to make the cross-section 1 mm.[2] Smith's temperature formula was obtained from observations between 0° C. and 24° C., Guillaume's between 0° C. and 61° C., and Kreichgauer and Jæger's between 14° C. and 28° C. Accordingly the values given for 100° C. are based on extrapolation.

The temperature formula for mass resistivity is of course not the same as for volume resistivity. Thus, letting δ represent mass resistivity, ρ volume resistivity, and d density,

$$\frac{\delta_t}{\delta_0} = \frac{\rho_t}{\rho_0} \cdot \frac{d_t}{d_0}.$$

The temperature formula of resistance, as measured in a glass tube, is different from either of the temperature formulas for resistivity. Thus, letting γ denote the resistance as measured in a glass tube, and γ_g the linear coefficient of expansion of the glass,

$$\frac{\gamma_t}{\gamma_0} = \frac{\rho_t}{\rho_0} \cdot \frac{1}{1 + \gamma_g t}.$$

(2) *Resistivities at* 20° C.; *Densities and Melting Points of the Solid Elements.*

Resistivity or the specific resistance is given in microhms. (To change to ohms multiply by 10^{-6}.) The resistivity is the ohmic resistance between opposite faces of a centimeter cube of the substance. Resistivity is quite dependent upon the purity of the material, which is not generally recorded. It is not useful, therefore, to record values beyond three, or at the most four, figures. Where values given by different observers differ a mean value is given here. The table is arranged in the order of decreasing resistivity.

Element	Resistivity in microhms at 20° C.	Density, grams per cubic centimeter	Melting temperature, degrees C.
Bismuth	119*	9.80	269
Mercury	95.782†	13.5462	−38.80
Silicon	59.5	2.3	1200(?)
Antimony	41.3	6.62	629.2±0.5 F. P.
Thorium	40.9‡	11.3	1690
Arsenic	38	5.73	volatilizes.
Strontium	25	2.54	900
Tellurium	21	6.25	450
Lead	20.96	11.37	327
Steel (1% C.)	20	7.8(?)	
Thallium	19§	11.9	301
Tantalum	14.7	16.6	2910(?)
Rubidium	12.3	1.53	38.5
Iron (0.1% C.)	12.1	7.86	1505 (about)
Tin	11.4	7.29	232
Tungsten (wire)	6–12	18.8	3002
Palladium	10.78	11.4	1549.2±2° F. P.
Calcium	10.5	1.55	780
Platinum	10.2	21.5	1755.0±5° F. P.
Cobalt	9.71	8.6	1489.8±2.0 F. P.
Nickel	9.52	8.9	1452.3±2.0 F. P.
Osmium	9.5	22.5	2200
Lithium	9.07‖	0.534	186
Indium	9.0	7.12	155
Cadmium	7.57	8.64	320.2±0.3 F. P.
Potassium	7.11¶	0.862	62.5
Zinc	6.1	7.1	418.2±0.3F.P.
Rhodium	6.04	12.44	1907
Iridium	5.34	22.41	2290
Sodium	4.87¶	0.971	97
Magnesium	4.57¶	1.723¶	633
Molybdenum	4.0	8.6	very high.
Aluminum	3	2.65	657
Gold	2.44	19.32	1062.4±0.8 F. P.
Copper	1.721	8.89	1082.6±0.8 F. P.
Silver (99.9%)	1.65	10.5	960.0±0.7 F. P.
Barium	.....	3.75	850
Beryllium	.....	1.93	1430
Boron	.....	2.5(?)	above 2000.
Cæsium	.....	1.87	26.4
Graphite, carbon	.....	2.3	vaporizes at about 3720.
Lanthanum	.....	6.12	810
Manganese	.....	7.39	1200 (about)
Neodymium	.....	6.96	840
Sulphur (amorphous)	insulator	1.92	444.6 (boils).
Titanium	.....	3.54	2500
Vanadium	.....	5.5	1620
Zirconium	.....	4.15	1300

* Changes in a magnetic field.
† See Appendix, IV, 1.
‡ Estimated. Value at 15° C. = 40.1.
§ Estimated. Value at 0° C. = 17.6.
‖ Estimated. Value at 0° C. = 8.4.
¶ Author's value.

The *density* of a substance may depend upon its previous treatment and its physical state; hence values are generally not given closer than three figures. The densities given are the grams per cubic centimeter of the substance at room temperature and at atmospheric pressure. When authorities differ a mean value is given.

The *melting point* is given in degrees centigrade at which the substance melts. In the case of several substances the melting points are very sharp and have been determined with great care. They serve as fixed points of temperature which are used for reproducing the temperature scale. In these cases the symbol, F.P., follows the recorded temperature; the numerical figure, as $\pm$ 0.8 states that the recorded temperature of melting is known within that number of degrees centigrade.

(3) *Data on a Few Alloys.*

Alloy	Composition	Maker	Resistivity in microhms at 20° C.	Temperature coefficient over small range	Density
Nichrome II.........	....	Driver-Harris Wire Co.	109.6	0.00016	8.02
Nichrome...........	....	"	96.6	0.00044	8.15
Climax..............	Nickel — steel	"	87.1	low	8.14
Constantan..........	60 Cu+40 Ni	....	49	0.00001	8.88
German silver, 30% ..	30 Ni+Cu+Zn	....	48	0.00023	....
Therlo..............	Cu+Mn+Al	"	46.7	0.0000056	8.15
Manganin............	84 Cu+4 Ni+12 Mn	....	44.5	0.00002	8.5
German silver, 18% ..	18 Ni+Cu+Zn	"	36	0.0003	8.5
Yankee silver........	....	"	33	0.00028	8.6
Platinoid............	62 Cu+15 Ni+22 Zn	....	32.5	0.00025	9
Ferronickel..........	....	"	28.2	0.002	8.2
Brass................	70 Cu+30 Zn	....	6 to 9	0.001	8.4 to 8.7

(4) *Standard Solutions for Calibrating Purposes.* (See § 1121.)

The table gives the resistivity ρ_t at six temperatures t of NaCl and KCl solutions.

The numbers in the columns are resistivities. They express the resistance in ohms of the solution (at the temperature in degrees C. heading the column) between opposite faces of a centimeter-cube of the solution. The reciprocals of the resistivities expressed in ohms are conductivities expressed in mho, cubic centimeter, units. [Note that the resistivity of a saturated solution is roughly one million times the resistivity of a pure metal.]

n = normal solution.

Thus, $1\ n$ = normal KCl = 74.59 gms. of the salt dissolved in 1 liter, at 18° C., of pure water.

NaCl, Sat. = saturated solution of NaCl at temperature heading a column.

Solution	0° C.	8° C.	12° C.	16° C.	20° C.	24° C.
NaCl, Sat.	7.435	5.924	5.342	4.847	4.425	4.062
KCl, 1 *n*	15.288	12.572	11.509	10.592	9.797	9.104
KCl, 0.1 *n*	139.86	112.61	102.14	93.283	85.690	79.113
KCl, 0.02 *n*	657.9	526.3	478.5	436.7	400.0	369.0
KCl, 0.01 *n*	1282.0	1031.0	934.6	852.5	782.47	721.50

(5) *Resistivities of Insulators.*

These vary so widely with the temperature, the purity and character of the material, the state of the surface,* and the conditions of the test, that the values given below are only roughly approximate and merely serve to indicate the order of magnitude of the quantities. Expressed in *megohms* resistance between opposite faces of a cubic centimeter of the material the resistivities of a few materials at ordinary temperatures are:

Paraffin wax $= 3 \times 10^{12}$,
Mica $= 9 \times 10^{9}$,
Ebonite $= 2 \times 10^{9}$,
Porcelain, 50° C., $= 2 \times 10^{9}$,
Rubber (used as insulation on wire) $= 4.5 \times 10^{8}$,
† Fused Silica $= 2 \times 10^{8}$,
† Glass (soda-lime) $= 5 \times 10^{5}$.

* Consult par. 815.
† National Physical Laboratory, England.

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